**Keynes and Duration**

Keynes argues that an investor will choose to hold money rather than assets when the potential for capital loss per annum on assets exceeds their expected coupon yield per annum. The condition is thus \( dP - C = 0 \), where \( dP \) is the annual cumulative change in price of a perpetual bond, and \( C \) its coupon. The price of a perpetual bond is given by its present value, or \( C/r \), where \( r \) is the current yield to maturity. The change in price may then be given by the result of multiplying price times the change in the rate of interest, \( dr \), times the modified duration of the bond. For a perpetual bond duration is given by \( D = (1+r)/r \) and modified duration by \( D/(1+r) \) which reduces to \( 1/r \). Thus \( dP = P*dr*MD \), which can be rewritten as \( dP = (C/r)*dr*(1/r) = dr * (C/r^2) \). \( dP - C = 0 \) thus can be expressed as \( dr * (C/r^2) - C = 0 \) which resolves to \( dr = r^2 \) as the condition under which \( dP = C \) and is simply Keynes' square rule.

The duration of a perpetual bond is given by the formula \( (1+r)/r \) so that a 4% consol selling at par of 100 and paying a £4 coupon has a duration of 26 years. The modified duration, defined as \( D/(1+r) \), which measures the volatility of bond prices, is then 25 years. The change in the price of the 4% par consol is calculated by multiplying modified duration by the current bond price and the result by the change in the bond's yield to maturity. In the case of a 16 basis point rise in the yield from .04 to .0416, the value of the bond will fall by 25*100*.0016 = £4, which is precisely the value of the bond coupon. For any higher increase in interest rates, the fall in the bond's value will exceed the current coupon yield of the bond, producing net losses for the holder. This is a case of complete liquidity preference and forms the basis of what Keynes defines as the liquidity trap, for the investor should prefer to hold only cash and no bonds if his expectation is for a rise in interest rates by more than the square of the rate.

This point can also be described in terms of the break-even point on the bond. Any change in yield on a bond affects both capital value, and reinvestment income, but in opposite directions. Duration gives the point at which the fall in value is just offset by the increased reinvestment income of the bond's coupons. Thus, the lower the rate of interest, the higher the value of the bond's duration and the longer it takes to recover the fall in capital value with increased reinvestment earnings. At 3% the duration rises to 34.33 years and at 2% to 51 years. At 1% it is 101 years. Thus Keynes' dictum that the lower the rate of interest, the more likely the liquidity trap. However, it should be clear that this does not rule out the existence of a liquidity trap at higher rates. At 8% duration is 13.5 years and modified duration, 12.5 years. A rise in the rate of interest to .0864 would produce a fall in the price of a par 8% consol of 100*12.4*.0064 = £8 which is exactly the coupon value. An expectation of a rise in interest rates of more than 8% then leads to the decision to remain liquid. The expected percent rise in interest (or equivalently, the expected percentage fall in the price of the bond) is however, twice as high as in the case of 4% rates. However, these should be judged relative to recent changes in bond prices. If 8% lies outside of the range given by two standard deviations from the mean of rate changes over the recent past, then it would be just as rational to remain liquid at 8% as it was at 4% in similar conditions of volatility.

However, as rates rise, duration measures become increasingly lower. At 20% interest rates
duration has fallen to six years and modified duration to five years. At 60%, similar to rates currently being paid in Mexico, duration is 2.67 years, so an investor could recover any loss in capital value from a rise in interest rates in two years and eight months. If there is any basis for the interest elasticity of the demand for money, this is it. However, it also suggests that the elasticity will be based on expectations, and that expectations will be based on recent volatility of rates. In a volatile rate environment, it will become more difficult to use changes in interest rates to influence the demand for money.

As rates rise, the square rule produces larger and larger absolute changes in the interest rate, and the use of modified duration to calculate the change in price becomes less and less accurate. The full calculation of the change in the bond price will require the calculation of convexity.