ISLM: The abc model

Algebra of the model

\[ S = a + bY + ci \]
- \(a (<0)\) is autonomous savings,
- \(b (>0)\) is the marginal propensity to save
- \(c (>0)\) is the interest elasticity of savings

\[ I = d + eY + fi \]
- \(d (>0)\) is autonomous investment
- \(e (>0)\) is the marginal propensity to invest
- \(f (<0)\) is the interest elasticity of investment (mec schedule)

\[ L = A + BY + Ci \]
- \(A (>0)\) is autonomous money demand
- \(B (>0)\) is transactions demand
- \(C (<0)\) is speculative demand (relative to bond price)

\[ M = \bar{M} \]
- Exogenous money supply (Fed)

Equilibrium condition: Goods market

\[ I = S \]

Solve for \(i_{IS}\)

\[ a + bY + ci = d + eY + fi \]
\[ ci-fi = d-a + eY - BY \]
\[ i(c-f) = d-a + (e-b)Y \]
\[ i_{IS} = \frac{d-a}{c-f} + \left(\frac{e-b}{c-f}\right)Y \]

Solve for \(Y_{IS}\)

\[ a + bY + ci = d + eY + fi \]
\[ bY - eY = d - a + fi - ci \]
\[ Y(b-e) = d - a + (f-c)i \]
\[ Y_{IS} = \frac{d-a}{b-e} + \left(\frac{f-c}{b-e}\right)i \]

Equilibrium condition: Money market

\[ L = M \]

Solve for \(i_{LM}\)

\[ A + BY + Ci = M \]
\[ Ci = M - A - BY \]
\[ i_{LM} = \frac{M-A}{C} - \frac{B}{C}Y \]

Solve for \(Y_{LM}\)

\[ A + BY + Ci = M \]
\[ BY = M - A - Ci \]
\[ Y_{LM} = \frac{M-A}{B} - \frac{C}{B}i \]
Equilibrium in both markets

Solve for $i_e$:

$$\frac{d-a}{b-e} + \left(\frac{f-c}{b-e}\right)i = \frac{M-A}{B} - \frac{C}{B}i$$

$$\left(\frac{f-c}{b-e}\right)i + \frac{C}{B}i = \frac{M-A}{B} - \frac{d-a}{b-e}$$

$$\left(\frac{f-c}{b-e} + \frac{C}{B}\right)i = \frac{M-A}{B} - \frac{d-a}{b-e}$$

$$i_e = \frac{\frac{M-A}{B} - \frac{d-a}{b-e}}{\frac{f-c}{b-e} + \frac{C}{B}}$$

$$i_e > i_{lt}$$

Solve for $Y_e$:

$$\frac{d-a}{c-f} + \frac{e-b}{c-f}Y = \frac{M-A}{C} - \frac{B}{C}Y$$

$$\left(\frac{e-b}{c-f}\right)Y + \frac{B}{C}Y = \frac{M-A}{C} - \frac{d-a}{c-f}$$

$$Y\left(\frac{e-b}{c-f} + \frac{B}{C}\right) = \frac{M-A}{C} - \frac{d-a}{c-f}$$

$$Y_e = \frac{\frac{M-A}{C} - \frac{d-a}{c-f}}{\frac{e-b}{c-f} + \frac{B}{C}}$$

$$Y_e < Y_f$$
Adding government

\[ S = a + bY + ci + tY \]  \hspace{1cm} \text{(tax is savings, i.e. a withdrawal)}

\[ I = d + eY + fi + G \]  \hspace{1cm} \text{(G is investment, i.e. an injection)}

\[ a + bY + ci + tY = d + eY + fi + G \]

\[ ci - fi = G + d - a + eY - bY - tY \]

\[ i(c-f) = d+G-a + Y(e-b-t) \]

\[ \frac{i}{JW} = \frac{G+d-a}{c-f} + \frac{e-b-t}{c-f} Y \]