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Hence:
\[ \lambda = -m \pm (\mu^2 - n^2)^{\dagger} \]
from which it appears that the real part of \( \lambda \) is always negative, namely that condition (A.22) is satisfied. Therefore, the stationary equilibrium defined by the couple of solutions \( x = \bar{x} \) and \( f(N_1) = \bar{x} \) is stable.

A proof of the instability of the trivial solutions, characterized by \( N_1 = 0 \), can be given in an easier way because in this case the products involving \( N_1 \) itself—besides those involving \( (x - \bar{x}) \)—are of second order of smallness and equation (A.18) may be considered in isolation, as appears by rewriting (A.19) as:
\[ N_1 \frac{d(x - \bar{x})}{dt} + (x - \bar{x} + \bar{x}) \frac{dN_1}{dt} = \varphi(0) N_1 [f'(0) - x + \bar{x} - \bar{x}] \]
and then, neglecting the squares of \( N_1 \) and of \( (x - \bar{x}) \) and their products:
\[ \frac{1}{N_1} \frac{dN_1}{dt} = \frac{1}{\bar{x}} \varphi(0) [f'(0) - \bar{x}] \]
This is a similar sort of result from the neglect of higher-order terms, namely:
\[ \frac{dC}{dt} = k C \]
where:
\[ k = \frac{1}{\bar{x}} \varphi(0) [f'(0) - \bar{x}] \]
and \( C = N_2(0) \).
Since \( (1/\bar{x}) \varphi(0) [f'(0) - \bar{x}] > 0 \), then the solution (A.25) is explosive, which means that the stationary equilibrium defined by the solution \( N_1 = 0 \) is unstable.

II

The economics of effective demand

It is not difficult, at a century and a half’s distance, to see the many deficiencies of Ricardo’s theory. Even leaving aside all the problems connected with his theory of value, there are three major shortcomings that cry out for correction: Ricardo’s naive view of population growth, his under-estimation of technical progress, and his inability to grasp the importance of effective demand.

In the present essay attention will be concentrated on effective demand, the role of which has become clear to us only after the publication of Keynes’ General Theory. Keynes himself explicitly linked up his analysis of effective demand with a long discussion that took place between Ricardo and Malthus. A few words on this famous discussion may be helpful.

1. Malthus on effective demand

Malthus’ Principles of Political Economy Considered with a View to their Practical Applications appeared in 1820 as an answer to Ricardo. Among the views that Malthus attacked was the traditional one that ‘every frugal man is a public benefactor’. He retorted that ‘the principle of saving, pushed to excess, would destroy the motive of production’. And he added that ‘if production is in great excess above consumption, the motive to accumulate and produce must cease from the want of will to consume’. On this basis, Malthus defended the unproductive consumption of the landlords as a remedy to ‘market gluts’, and warned

1 These problems have recently attracted new attention after the publication of Piero Sraffa, Production of Commodities by Means of Commodities, Cambridge, 1960.
against the dire consequences of excessive 'parsimony and thrift'. He devoted ample space to the problem, and summarized it as the 'questions, 1st whether the motive to accumulate may be checked from the want of demand, before it is checked by the difficulty of procuring food; and 2ndly, whether such check is probable'. His answers to both questions were positive. And the prescription followed consequentially: 'we may conclude that a body of unproductive labourers is necessary as a stimulus to wealth'.

Ricardo failed to see any force in these arguments. For him, savings were associated with the capitalists and therefore meant the same thing as capital accumulation. It was, moreover, very easy for him to appeal to the authority of the leading French economist of the time, Jean-Baptiste Say, who had stated that any production generates its own demand - what has then become universally known as la loi des débouchés or 'Say's Law'. Unfortunately, Malthus' replies remained ineffective. Malthus lacked the minimum analytical tools necessary to give expression to his contention. He even failed to stress effectively what was absolutely necessary to his arguments, namely the distinction between the savings of the landlords and the capital accumulation of the capitalists.  

4 These are Malthus' own summary words for his pp. 478-9; ibid. p. 587. Most of Malthus' arguments on the importance of effective demand are contained in section ix of chapter vii of his Principles, and restated in a series of letters to Ricardo. (See, as an example, letter 7 July 1821, in vol. ix pp. 9-11 of The Works and Correspondence of David Ricardo, ed. by Piero Sraffa.)

5 'Say's Law' is enunciated in a few short passages in the chapter 'Des débouchés' (book 1) of his Traité d'économie politique (1st ed. 1803). We find there the famous passage: 'It is worth while to remark that a product is no sooner created, than it, from that instant, affords a market for other products to the full extent of its own value. When the producer has put the finishing hand to his product, he is most anxious to sell it immediately; lest its value should vanish in his hands. Nor is he less anxious to dispose of the money he may get for it; for the value of money is also perishable. But the only way of getting rid of money is in the purchase of some product or other. Thus, the mere circumstance of the creation of one product immediately opens a vent (un débouché) for other products.' (From the English translation of the fourth French edition: Jean-Baptiste Say, A Treatise of Political Economy, London, 1821, vol. i, p. 167.)

6 This failure of Malthus to stress that his arguments applied to the thrift and savings of those classes that do not produce has recently induced some critics to deny him the role of precursor of Keynes. (See, for example, B. A. Corry, 'Malthus and Keynes - a Reconsideration', The Economic Journal, 1935, pp. 717-24; P. Garegnani, 'Note su consumi, investimenti e domanda effettiva', parte prima, Economia Internazionale, 1964, pp. 591-631.) But it seems to me that this is going too far. The fact that Malthus was analytically weak and that he was not always logically consistent may explain why he was not successful; but cannot take away from him the merit of having been aware precisely of those problems of lack of effective demand, which Keynes was able to deal with, in a theoretically better and successful way, a century later.

The controversy dragged on, and died out in the end in a sterile dispute on how long in time was the term under discussion? Ricardo and Malthus remained, of course, of their original opinion; but it was the analytically stronger Ricardian theories that carried the professional opinion. And it was of no avail for Malthus to appeal to the factual evidence of common experience and to stress the practical importance of the theory (explicit reference to 'practical applications' is made in the very title of his book4). Stronger theory prevailed.

A century later, at the peak of his enthusiasm for Malthus, Keynes could exclaim: 'If only Malthus, instead of Ricardo, had been the parent stem from which nineteenth-century economies proceeded, what a much wiser and richer place the world would be today!' But Keynes' hypothetical wish was illusory. Malthus' weak arguments on effective demand could never have carried the burden of nineteenth-century economic theory. What would have been necessary was a logically consistent theory that could embody and express Malthus' intuitive ideas. This we could get only from Keynes a century later.

2. The principle of effective demand

The basic principle which has remained hidden for so long and which may be called 'the principle of effective demand' can be stated straightforwardly in a very concise form.

Among the peculiarities which an industrial society has acquired, with respect to more primitive (agricultural) societies, there is one that requires us to make a sharp distinction between productive capacity and actual production. In primitive (agricultural) societies, each farmer tries to produce as much as he can. He will then take whatever amount of his produce is in excess of his needs to the market. And there this produce will fetch the price the market makes. In an industrial society it is not so. At any given point of time, productive capacity is indeed what it is - it cannot be changed. But productive capacity does not mean...

7 'It appears to me that one great cause of our difference in opinion, on the subjects which we have so often discussed, is that you have always in your mind the immediate and temporary effects of particular changes - whereas I put these immediate and temporary effects quite aside, and fix my whole attention on the permanent state of things which will result from them.' (From Ricardo's letter to Malthus, 24 January 1817, in vol. vii, p. 120, of The Works and Correspondence of David Ricardo, ed. by Piero Sraffa.) Here Malthus would have needed Keynes' knack of repartee: 'in the long run we are all dead!'

8 See footnote 3 above.

9 Keynes, Essays in Biography, pp. 120-1.
production – it only means potential production. In order that there may be actual production, there must be effective demand.

The point may be expressed with the help of a well-known diagram introduced by Alvin Hansen, in which total net production (or net national income $Y$) appears on the abscissa and total effective demand ($D$) on the ordinate, and in which a 45° line is drawn. (See fig. 11.1.)

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32 See his A Guide to Keynes, New York, 1953, and all his textbooks on Keynesian economics.

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difference that the additions to income will only be in monetary terms, since in real terms production cannot exceed productive capacity.11

It must be stressed that all this is meant to refer to an industrial society (as against the case of a more primitive agricultural and artisan society, where fluctuations of demand normally determine fluctuations of prices, as at the given amount of production). What this typical behaviour of an industrial economic system depends on is a question which would take too long to go into here, and has not even been completely and successfully investigated so far by the economists themselves. Many factors may be mentioned, all acting in a cumulative way: stickiness of prices due to oligopolistic situations, mark-up pricing rules in manufacturing business, contractually fixed wage-rates, etc., and many other factors which are continually evolving. The basic feature remains, by contrast with more primitive societies, that among the factors concurred to determining prices, fluctuations of demand have become unimportant. Therefore, the traditional response mechanism of price changes having become inoperative, another response mechanism is brought into use. To changes in demand, producers respond by changing production.

This has a very serious consequence. Changes in production entail changes in the utilization of existing productive capacity and in the employment of labour. A fall of total demand generates unemployment and a slump – a bitter reality so often experienced in capitalist economies. There are machines, and there are workers able to man them, but they all remain idle for lack of effective demand.

3. Theories of under-consumption (and over-production)

The principle of effective demand may appear very simple; so simple in fact as to make one wonder why it has taken so long to emerge. But the plain answer to this is that many authors did perceive it in the past, though they never succeeded in making of it an accepted tenet of official economic theory.

11 This way of presenting the problem is, of course, a simplification. In practice full capacity utilization may be reached at different moments in different sectors. There may be sectors working at full capacity even in a prevailing slump situation, and there may be sectors with idle capacity even when total demand as a whole has out-grown total existing productive capacity. This means that full employment production must not be rigidly taken as coming at a singular point. There may be a band around $Y$, in which fluctuations of demand induce changes in production in some sectors and changes in prices in other sectors. But the proportion of the quantity changes, with respect to the price changes, will be the higher the farther the system is on the left of $Y$, and will be the lower the farther the system is on the right of $Y$. 
The unsuccessful attempts have a long history. At the time of the Ricardo-Malthus discussions, the Swiss-Italian economist, Sismondi, who criticized Ricardo rather violently, had among his major contentions precisely the claim that capitalist systems tend to situations of under-consumption.13 Later Karl Marx, the sharpest of all critics of capitalism, indicted capitalist societies, among other things, also for over-production - a situation which in his theory is expressed by crises in the 'realization of surplus value'.13 And at the turn of the century, John Hobson insisted on the existence of under-consumption in capitalist systems in almost all his (numerous) works.14 Moreover, one may say that practically all the economists who have been concerned with 'crises' and business cycles (e.g. Lauderdale, Tugan-Baranowski, Aftalion, Spithoff, etc., and again all the Marxists, such as Hilferding, Rosa Luxembourg, Bukharin, etc.) did come, at one stage or another, to stress the possibility and the disastrous consequences of a gap between potential production and effective demand.

Yet, all these authors have had very little success in their times. The attitude of the established economic profession towards them has always been one of great contempt, in the belief that their theories contained all sorts of analytical faults. Under-consumption theories were simply regarded as bad theories. And in fact it must be said that established economic theory made the task of refuting them rather easy. Any well-trained economist could do so with arguments that appeared logically unexceptionable. In the early nineteenth century these arguments would take the form of a simple enunciation of Say's Law; and in the early twentieth century they would take the more sophisticated form of

13 J. C. L. Simonde de Sismondi, *Nouveaux principes d'économie politique*, Paris, 1819. Unlike Malthus, who criticized Ricardo, so to speak, from the right - in favour of the landlords - Sismondi criticized Ricardo, in a sense, from the left - in favour of the workers. But neither of them was obviously the first to realize the dangers of under-consumption. As Keynes points out in chapter 23 of the *General Theory*, much of the Mercantilists' writings can be seen as self-interpreted, as preoccupation with a lack of effective demand. There is moreover the famous case of the satirical *Fable of the Bees*, by Bernard Mandeville, a book convicted as scandalous by the grand jury of Middlesex in 1733, for the praise it made of extravagance as a public virtue.


16 A general equilibrium scheme, in which the total available resources are considered as given, and competition determines the (equilibrium) prices, which are such as to lead to the clearance of all markets. It simply sounded inconceivable, to a professional economist, that any resource could remain unused, except for temporary frictions, as long as its price is positive. And it simply sounded inconceivable that there could be a situation of equilibrium with 'involuntary' unemployment.15

In Great Britain, the most famous expression of these arguments is contained in a Government official paper, representing the Treasury view of 1929. In that official paper, the economists of the British Treasury - mainly well-trained Oxford and Cambridge economists - opposed the proposal, made by Lloyd George in an election campaign, of public works to alleviate unemployment, with the argument that public works would only cause an *increase* of unemployment. For, the available funds being given, their absorption for public works could only subtract them from other (productive) investments,16

This view seems almost incredible today. And yet it was the prevailing professional view only forty years ago. Only in the 1930s did the theoretical scene change radically. The conditions of the time became, of course, favourable to such a change. In 1929-33 the Western industrialized countries went through the most severe and dramatic of all depressions they ever experienced. Two great economists of that time - Kalecki and Keynes - though with entirely different backgrounds (Kalecki in Poland, proceeding from Marx17, and Keynes in England, rebelling against Marshall), independently arrived at almost the same conclusions on effective demand. It was Keynes however who gained universal success. And there is little doubt - it seems to me - that Keynes' success was due in a decisive manner to the fact that -
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Unlike all his under-consumptionist predecessors - he was able to present not merely an effective criticism of traditional theory, but also a coherent, logically consistent, and complete theory as an alternative.

4. J. M. Keynes' 'general theory' of employment

Keynes' theory of income and employment determination, though surrounded with misunderstandings and obscurities at the time of publication, may be expressed today in a very simple and cogent way.

Once the basic process of income generation by effective demand has been stated (as in section 2 above), it is natural to go on to investigate what determines effective demand. Keynes, in a typically Classical manner, divides people into two broad categories: consumers and producers. Total effective demand is therefore the sum of demand for consumption goods (C) and demand for investment goods (I). And since effective demand generates income, we may simply write:

\[ Y = C + I. \]  
(11.1)

One needs, at this point, a theory of consumption and a theory of investment.

Consumption is simply taken to depend on income. Keynes claims that on the whole and on the average, consumers tend to spend only a fraction - more precisely a decreasing fraction - of any increase in income. Therefore:

\[ C = f(Y), \]  
(11.2)
with the properties:

\[ 0 < f' < 1, \quad f'' < 0, \]

where \( f' \) represents the 'marginal propensity to consume', and its complement to unity, \( (1 - f') \), the 'marginal propensity to save'. By using a simplified linear approximation, (11.2) may also be written as:

\[ C = A + aY, \]  
(11.2a)

where: \( A \) = a positive constant, \( a = \) marginal propensity to consume, (and \( 1 - a = s \); marginal propensity to save).

As regards investment Keynes' theory is rather different.\(^{18}\) Investments do not depend on income at all. In any given short-run situation (with a given technology and a given capital structure), the total amount of investment depends on the expected profitability of all possible investment projects and on the rate of interest. We may imagine entrepreneurs ranking all possible investment projects in order of decreasing profitability and then carrying out investments up to the point at which the expected rate of profit from the last project (called the 'marginal efficiency of capital') is just higher than, or equal to, the rate of interest, as expressing the cost of borrowing. (See fig. 11.2.)

![Fig. 11.2](image)

In synthesis we may write:

\[ I = \varphi (E, i), \]  
(11.3)

where \( E \) expresses the falling expected profitability of investments in the given conditions and \( i \) is the rate of interest.

But at this point a new variable has been introduced - the rate of interest, which had been carefully kept out of the previous relations. Keynes is thereby induced, by the very logic of his system, to look for a new theory of the rate of interest. He claims that, for a series of reasons (transactions, precautionary, and speculative motives), there is a certain quantity of money that people are willing to hold at each level of the rate of interest. This quantity of money - the demand for money - is inversely related to the rate of interest, and tends to infinity before the rate of interest reaches zero (the liquidity preference schedule) Given this relation, the rate of interest will therefore be determined by the quantity of money \( M \) issued by the Central Authority - a purely monetary phenomenon. (See fig. 11.3.)
In synthesis we may write:

$$i = \psi(L, \bar{M}),$$  \hspace{1cm} (11.4)

where $L$ represents the falling liquidity preference schedule and $\bar{M}$ the quantity of money issued by the Central Authority.

To conclude, given the consumption function, the marginal efficiency of capital schedule and the liquidity preference schedule, and given the exogenously determined quantity $\bar{M}$, the four equations (11.1), (11.2), (11.3), (11.4), determine the four unknowns $Y$, $C$, $I$, $i$.\textsuperscript{18}

The important novelty of this scheme, with respect to previous theories, is that it shows that there is no reason why the level of net national income should turn out to be precisely that level which entails the full utilization of productive capacity and the full employment of the labour force. When the system is left to itself, only by a fluke will full employment be achieved. The situation which Keynes regarded as normal is in fact one of less than full employment, as is shown in fig. 11.4.

\textsuperscript{18} Equations (11.5) and (11.4) are an elliptical, though effective, way of representing Keynes' determination of $I$ and $i$. A more detailed formalization would be to write each one of them as a set of two equations, the second of which represents an equilibrium relation, namely:

$$I = E(r),$$  \hspace{1cm} (11.3a);

$$r = i,$$  \hspace{1cm} (11.3b);

$$M = \bar{M},$$  \hspace{1cm} (11.4a);

$$M = \bar{M},$$  \hspace{1cm} (11.4b)

where $r$ = expected rate of profit and $M$ = demand for money. In this case, we would say that the 6 equations (11.1), (11.2), (11.3a), (11.3b), (11.4a), (11.4b) determine the 6 unknowns: $Y$, $C$, $I$, $i$, $r$, $M$.

The important warning must, however, be added that these equations are only meant to represent a first approximation to Keynes' theory. Too much weight should not be laid upon them when considering situations which are far away from the equilibrium points. Moreover one should be careful when carrying out arguments which involve shifts of any curve. For the shifts of any one of these curves are not independent of the shifts of the others.

Demand for consumption is a function of income and demand for investment - determined independently of income - is simply added on to consumption. Equilibrium is attained at that point ($B$ in fig. 11.4) where $C + I$ crosses the $45^\circ$ line. At that point, total demand is equal to total production. Hence there is equilibrium between aggregate demand and aggregate supply. But it is an equilibrium of underemployment. And the difference $(\bar{Y} - Y)$ is there to represent idle capacity and unemployment.\textsuperscript{20} This is the type of unemployment (due to lack of effective demand), which has become known by now as 'Keynesian' unemployment.

But the great practical relevance of this analysis is that not only does it give a neat diagnosis of one of the major weaknesses of the capitalist system. It also immediately suggests a powerful remedy. If (11.2) is substituted into (11.1), and $f(Y)$ is expanded in Taylor series (neglecting higher order terms) we obtain:

\textsuperscript{20} In the whole of Keynesian analysis, idle capacity and unemployment are always used as synonymous, since, in the short run, they may be taken as proportional to each other.
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\[ Y(1 - f') = I, \quad \text{(11.5)} \]

and therefore:

\[ dY = \frac{1}{1 - f'} \, dI, \quad \text{(11.6)} \]

or, in the case of the linear consumption function (11.2a),

\[ dY = \frac{1}{1 - a} \, dI. \quad \text{(11.7)} \]

This is a remarkable relation. It says that any increase in the flow of new investment (\( dI \)) will generate an increase in the flow of net income which is \( \frac{1}{1 - f'} \) times as large as \( \frac{1}{1 - a} \) times as large in the linear case. The expression \( \frac{1}{1 - f'} \) (or \( \frac{1}{1 - a} \)) has therefore been called 'the multiplier'.\(^{21}\)

To give an idea of order of magnitude, if the marginal propensity to consume is 80 per cent, the multiplier is 5; meaning that any increase of investment will generate an increase of income five times as large. In the formulation considered here, where no relation contains any time lag, all effects are shown instantaneously ('instantaneous' multiplier)\(^{22}\) and without any complication. Once this is understood, it becomes immediately evident that there is no necessity for the increase in effective demand to come from investment. Any autonomous increase in effective demand will generate precisely the same multiplicative effects. Therefore, when current investment is too low to bring about full employment, the Government can always step in with public expenditure (through deficit spending) and make a net addition to effective demand. Full employment may thereby be brought about by Government policy. As may be seen from fig. 11.4, given any function \( C + I \), there always exists an amount of additional public expenditure — equal to \( (1 - a)(\bar{Y} - Y') \) — which, if carried out, will take the system to the point of full employment.\(^{23}\)

5. The principle of effective demand in a different context

It is important to distinguish, in Keynes' analysis, the principle of effective demand from the analytical tools he used in order to put it across. The basic enrichment that Keynes brought to economics is the principle of effective demand, but what gave Keynes' theory success were his analytical tools. Had he not developed them he would have failed (like all his predecessors) to convey the principle of effective demand. The distinction is important because all the discussions that centred around the General Theory, both at the time of publication and afterwards, have concentrated on Keynes' analytical tools; not on the principle of effective demand. Yet, by modifying Keynes' analytical tools, the critics have tried to slip behind Keynes' equations a quite different conception of economic behaviour, so as to minimize, or even render entirely irrelevant, the principle of effective demand.

In order to show that this principle is something quite distinct from Keynes' particular tools of analysis, it may be useful to re-state it in an entirely different context and independently of Keynes' theory. Consider an industrial economy as represented by what has become known as the Leontief closed system:\(^{24}\)

\[
\begin{bmatrix}
I & 0 \\
0' & 1
\end{bmatrix} - \begin{bmatrix}
A & y \\
a_0 & 0
\end{bmatrix} 
\begin{bmatrix}
X \\
N
\end{bmatrix} = \begin{bmatrix}
0' \\
0
\end{bmatrix},
\tag{11.8}
\]

where: \( I \) is a unit matrix of the same order as \( A \), the inter-industry commodity coefficient matrix; \( a_0 \) is the (row) vector of labour-input coefficients; \( y \) is the (column) vector of per-capita final demand coefficients; \( X \) is the (column) vector of total production; \( N \) is the available labour force; and \( 0 \) is the (column) zero vector. Expression (11.8) represents a linear and homogeneous system of equations. Therefore a necessary condition for non-trivial (i.e. non-zero) solutions is:

\[
\det \begin{bmatrix}
I & 0 \\
0' & 1
\end{bmatrix} - \begin{bmatrix}
A & y \\
a_0 & 0
\end{bmatrix} = 0.
\tag{11.9}
\]

\(^{21}\) It was discovered by Richard Kahn, in 'The Relation of Home Investment to Unemployment', The Economic Journal, 1931, pp. 173–98. It is, therefore, also referred to as the Kahn multiplier.

\(^{22}\) The whole of Keynes' analysis is always carried out in these terms, i.e. as if the effects of the Kahn multiplier worked out instantaneously. It may, however, be useful, for certain purposes, to introduce a time lag between the reception of an income and the consumption of it. This gives rise to the 'lagged' multiplier, which is dealt with in an appendix to this essay.

\(^{23}\) The 'Keynesian' management of total effective demand has by now become such a common Government policy as to be used sometimes not only for achieving full employment, but also for deliberately causing 'Keynesian' unemployment.

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The determinant of the coefficient matrix must be equal to zero. Since $A$ and $a_i$ contain technical coefficients, which are given, the only coefficients which can adapt, if condition (11.9) is to be fulfilled, are the per-capita demand coefficients $y$. This makes economic sense. Total demand for consumption and investment cannot be higher than, i.e. is limited by, the technological possibilities of the economic system. But (11.9) also means something more. Not only cannot the determinant of the coefficient matrix be higher than zero, it cannot be lower either, if the system of equations is to be satisfied. The economic meaning of this is straightforward. Total demand must be high enough as to require the full utilization of existing commodity stocks and the full employment of the existing labour force. If demand were too low, with respect to requirement (11.9), there would be idle productive capacity and unemployment of the labour force; if it were too high, there would be an inflationary situation. This is nothing but the principle of effective demand.

There is no reason, a priori, why condition (11.9) should always and automatically be satisfied. The trouble with pre-Keynesian economics was that it took for granted that (11.9) is always satisfied (Say’s Law), and left the proof to the contrary to its opponents; who got muddled up. (Here Keynes had a quick answer: by breaking up total effective demand into two parts — consumption, as a fraction of income, and investment, independent of income — he could show that that equality can only be attained by a fluke.)

What misled pre-Keynesian economists is that, if (11.9) happens to be satisfied to begin with, then the total sum of personal incomes (wages and profits in Leontief’s simple scheme) is exactly equal to full-employment net national income. They took this also to mean effective demand. But there is no reason to expect that the total sum of wages and profit should necessarily all be converted into effective demand (should all be spent) in the following period, which is the necessary condition for (11.9) to continue to be satisfied. There is no guarantee that full employment will be maintained, even if it happened to be realized to begin with.

6. Ricardian features of Keynes’ analysis

We may now briefly consider Keynes’ analytical tools. The most striking feature that emerges immediately is Keynes’ clear break with the sixty-year-old tradition of marginal economic theory and his return to the methods of analysis of the earlier Classical economists of the beginning of the nineteenth century.

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The whole conception of an economic system behind equations (11.1)–(11.4) is typically Classical. The use of macro-economic variables, the division of all economic agents into broad categories (consumers and entrepreneurs, in Keynes’ case), the search for the determination of the rate of interest — and by implication of the distribution of income — in a sphere outside that of output — all these are features inherited from Classical economic analysis. Even the marginal-efficiency-of-capital schedule, which might, at a first superficial look, appear as belonging to marginal economic analysis, when examined more deeply turns out to have a rather different origin. Keynes’ ranking of all investment projects in a decreasing order of profitability is more akin to Ricardo’s ranking of all lands in a decreasing order of fertility than to any marginal economic elaboration. And in any case, there is absolutely no need to consider Keynes’ marginal-efficiency-of-capital schedule as an expression of the marginal productivity theory of capital. This theory necessarily entails an inverse monotonic relation between capital intensity and the rate of interest. But that is not the case with Keynes’ ranking of investment projects (fig. 11.3 above). In a slump situation the last project to be implemented might well be the least capital intensive of all, and therefore entail a decrease (not an increase) of the average amount of capital per employed labour.

Coming down to a more specific comparison, the analytical similarities that are most evident are with the Ricardian scheme. In spite of Keynes’ own understandable enthusiasm for Malthus (in view of the latter’s treatment of effective demand), and in spite of Keynes’ frequent harsh remarks on Ricardo, it is basically the Ricardian method of analysis that Keynes has revived.26 The most typical indication of this is to be found in the directness with which Keynes proceeds to state his assumptions. Like Ricardo, he is always looking for fundamentals. He singles out for consideration the variables he believes to be the most important. All the others, giving rise to unimportant complications — though, as he says, are always ‘kept at the back of his head’ for the

26 Schumpeter perceived this very clearly when he wrote: ‘the similarity between the aims and methods of those two eminent men, Keynes and Ricardo, is indeed striking, though it will not impress those who look primarily for the advice a writer tendsers. Of course, there is a world between Keynes and Ricardo in this respect, and Keynes’ views on economic policy bear much more resemblance to Malthus’. But I am speaking of Ricardo’s and Keynes’ methods of securing the clear-cut result. On this point they were brothers in the spirit.’ (Joseph A. Schumpeter, History of Economic Analysis, New York, 1954, p. 473n.)
necessary qualifications\footnote{See Keynes' own description of this process, as contrasted with the 'symbolic pseudo-mathematical methods of formalising a system of economic analysis', on pp. 297–8 of the General Theory.} – are, for immediate purposes, frozen out by simple assumptions.

The characteristic consequence of this methodological procedure is the emergence in Keynes, as in Ricardo, of a system of equations of the 'causal type', or, as we may also say, of the 'decomposable type', as opposed to a completely interdependent system of simultaneous equations. If we go back to equations (11.1)–(11.4), we notice in them a very definite logical succession according to which the variables are determined (even if some of them may form among themselves smaller interdependent sub-systems). If we indicate the causal ordering\footnote{Since the terms 'cause' and 'effect' have given rise to so many and so heated epistemological discussions, it may be useful to point out explicitly that there is no need to step onto controversial ground here. The term 'causal ordering' is here used simply in the sense of an asymmetrical relation among certain variables, namely as indicating a one-way direction in which, in a formal sense, the variables of the system are determined. This formal and non-controversial meaning of the term 'causal ordering' has been stressed and illustrated at length by Herbert Simon: 'Causal Ordering and Identifiability', in Wm. C. Hood and T. C. Koopmans (eds.) Studies in Econometric Method, New York, 1953, pp. 49–74.} by an arrow, we may actually write:

$$\psi(\overrightarrow{L}, \overrightarrow{M}) \rightarrow i \rightarrow \phi(\overrightarrow{E}, i) \rightarrow I \rightarrow \begin{cases} Y = C + S \\ C = f(Y) \end{cases} \rightarrow Y, \quad C.$$

In other words, function $\psi$ determines $i$ independently of anything else. Then, given $i$, function $\phi$ determines $I$ independently of anything else; and finally, given $I$, equations (11.1) and (11.2) form among themselves a smaller interdependent sub-system, which simultaneously determines $Y$ and $C$.

As against the attitude – so common to marginal economists theorists – that 'everything depends on everything else', Keynes (as Ricardo) takes the opposite attitude that it is one of the tasks of the economic theorist himself to specify which variables are sufficiently interdependent as to be best represented by simultaneous relations, and which variables exhibit such an overwhelming dependence in one direction (and such a small dependence in the opposite direction) as to be best represented by one-way-direction relations.\footnote{Schumpeter, whose ideal hero was neither Ricardo nor Keynes (but Walras) has found this method of analysis so contrary to his aesthetic sense of symmetry as to call it 'the Ricardian Vice'. (J. A. Schumpeter, op. cit. p. 473.) But there is no justification for such an abuse. Aesthetics is not necessarily the best criterion to use for economic analysis, and even less for economic policy.}

\footnote{See the appendix to this essay.}

$\psi(\overrightarrow{L}, \overrightarrow{M}) \rightarrow i \rightarrow \phi(\overrightarrow{E}, i) \rightarrow I \rightarrow \begin{cases} Y = C + S \\ C = f(Y) \end{cases} \rightarrow Y, \quad C.$

Total savings are, so to speak, an entirely passive variable, which always turns out to be equal to total investments, whatever the decisions to save may be. From Keynes' analysis, which is in terms of the instantaneous multiplier, this result follows immediately. But the same result is confirmed, and shown even better, by the use of the 'lagged' multiplier, which entails a long series of successive steps, during which decisions to save are seen to adapt themselves to investment, through changes in income. In the whole of this process – out of equilibrium as well as in equilibrium – actual savings are always, and at each step, equal to the predetermined amount of investments.\footnote{Thus, if we take conclusiveness as an alternative criterion to aesthetics, what Schumpeter calls 'the Ricardian Vice' might well be called a 'Ricardian Virtue'.}

7. Anti-Keynesian features of some 'Keynesian' literature

The economic literature that has followed the publication of the General Theory is by now immense. Some of it has no doubt contributed
to clarifying obscurities and filling in gaps, before going over to dynamic analysis. But a great deal of it (and it has not been the least successful) has also been aimed at watering down Keynes' innovations – at attenuating the break with tradition and re-casting Keynes' analysis so as to 'digest' it into pre-Keynesian modes of thinking.\textsuperscript{31}

It is not, of course, always easy in considering this 'Keynesian' literature, to pin-point exactly where Keynes' message gets distorted. But if the interpretation put forward in the previous pages is correct, then surely a most reliable sign of distortion shows up any time the original Keynes' 'clear-cut results' are obfuscated by the imposition of interdependences which transform Keynes' causally ordered relations into a system of simultaneous equations.

We may consider, as an illustrative example, the most popular of all textbook presentations of 'Keynesian' theory – the one given by Sir John Hicks.\textsuperscript{32} Hicks is not, after all, an extreme anti-Keynesian. He immediately repudiates Pigou's theory of employment and accepts Keynes' method of stating major relations in aggregate terms. Yet his procedure is, I should say; typically un-Keynesian. After accepting equation (11.1) above as an identity, he proceeds to give equation (11.3) a marginal-productivity-of-capital interpretation. He then turns equation (11.2) upside down by considering savings, instead of consumption, and – very significantly – he modifies it by introducing the rate of interest. Finally, he modifies equation (11.4) by introducing income. In this way, consumption (but he says savings) is made to depend not only on income but also on the rate of interest, and demand for money is made to depend not only on the rate of interest but also on income. At the end of this, apparently innocuous, manipulation, Hicks has in fact broken up Keynes' basic chain of arguments. The relations have been turned into a system of simultaneous equations, i.e. precisely into what Keynes did not want them to be. Hicks exploits his procedure twice over by scolding Keynes for considering only what appears as a

\textsuperscript{31}A critical attitude towards the various interpretations of Keynes has recently been stimulated by the works of the clowers and Leijonhufvud (R. F. Clower, 'The Keynesian Counter-Revolution: A Theoretical Appraisal', in F. H. Hahn and F. Brechling (eds.), The Theory of Interest Rates, London, 1965, pp. 103–25; Axel Leijonhufvud, On Keynesian Economics and the Economics of Keynes, New York, 1968). Yet, while Clower's and Leijonhufvud's critical remarks have been salutary, their more positive suggestion, which amounts to a re-interpretation of Keynes' analysis within a Walrasian scheme, remains as questionable as all the others. For, we know that Keynes' immediate source in traditional economics was not Walras, whose works he knew very little, but Marshall.

\textsuperscript{32}John R. Hicks, 'Mr Keynes and the "Classics": a Suggested Interpretation', Econometrica, 1937, pp. 147–59.

\textsuperscript{33}From a purely formal point of view, any relation can be said to be a 'particular case' of a 'more general' model in this way. It is enough to insert an extra variable, which 'generalizes' it, and then say that the previous relation is a 'particular case' of the new one.

\textsuperscript{34}Hicks, Econometrica, 1937, p. 152n.

\textsuperscript{35}The same conclusion is reiterated by Franco Modigliani, in 'Liquidity Preference and the Theory of Interest and Money', Econometrica, 1944, pp. 45–88.
system, a substitution of interpretative models takes place. The typical features of an industrial society are made to recede and the characteristic features of a rather imaginary 'exchange' economy are imperceptibly slipped in instead. Within such a context, it is not surprising that what is said by Keynes becomes more or less irrelevant, or can only appear as referring to 'imperfections' of the market, 'rigidities' of prices and wages, or 'liquidity traps'.

This is in fact the very basic distortion in so much of the 'Keynesian' literature: that what Keynes says with reference to an industrial society is reinterpreted as referring to something else – to a more primitive society, or sometimes even to a mythical society that never existed – where it can have either no meaning or no relevance. Unfortunately, the consequences are rather serious. The misleading impression is given that all problems of our time would disappear if only the 'rigidities' were to be eliminated. As if the 'rigidities' were the cause and not themselves one of the many inherent consequences of the industrial society in which we live.

8. The principle of acceleration

If a proof were needed at all of the independence of Keynes' theory of effective demand from the liquidity preference schedule, this proof is given by the development – independently of Keynes – of a theory of investment known as the principle of acceleration. This theory is entirely based on effective demand and makes no reference whatever to the rate of interest.

The term 'principle of acceleration' (or simply 'accelerator') comes from the circumstance that an expected change in total demand tends to induce a larger change (an 'acceleration') of investments. If, for example, the capital stock were 400 and annual production were 100 (a capital-output ratio of 4), a change of 10 in annual production would tend to require an addition to the capital stock 4 times as large – i.e. an investment of 40. In general we may therefore write:

\[ I_t = \alpha (Y_{t-1} - K_{t-1}), \quad \beta \leq 1; \]

which reduces to (11.14) and (11.13) in the particular case in which \( \beta = 1 \). The idea is that, when there is a discrepancy between desired and actual capital-output ratio, entrepreneurs may not carry out investments to cover the full difference, but only a fraction of it. More neatly:

\[ I_t = \alpha Y_{t-1} - \beta K_{t-1}, \quad \alpha = r\beta, \]

an expression also known as the 'capital stock adjustment principle'.

From a purely formal point of view, the remarkable property of the principle of acceleration is that it provides a theory of investment

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38 By subtracting equation (11.14), from itself (but shifted by one period) we obtain:

\[ I_t = \beta Y_{t-1} - \beta K_{t-1} \]

\[ I_{t-1} = \beta Y_{t-2} - \beta K_{t-2} \]

\[ I_t - I_{t-1} = \beta (Y_{t-1} - Y_{t-2}) - (K_{t-1} - K_{t-2}) \]

This is nothing but equation (11.13) since \( K_{t-1} - K_{t-2} = I_{t-1} \).

Growth and income distribution

Symmetrical to the Keynesian theory of consumption (i.e. to the multiplier relation). Taken together, the multiplier and the accelerator give an explanation of both parts of total effective demand (consumption and investment) in terms of the same macroeconomic variables. Keynes had explained why an economic system may fall into a depression. The principle of acceleration came also to explain how depressions can recur regularly. The introduction of changes in income immediately carried the investigation over to dynamic analysis. For, a change in income induces a larger change in investment (accelerator), and this change in investment generates an even larger change in income (multiplier). The analysis of this accelerator-multiplier interaction has given rise to practically all modern theories of the trade cycle. These theories represent perhaps the culmination of the theory of effective demand.

9. Final remarks

The warning must be added that, in spite of the formal symmetry of the multiplier and the accelerator functions, there are substantial differences between the two. When income is inserted as the argument of the consumption function, that is precisely what is meant. But when a change of income is inserted as the argument of the investment function, that is not really what is meant. Ideally what should be inserted is the expected change in income. But since expectations cannot be quantified, we must be content with using the past change in income as a proxy variable for the expected change in income. This places the two equations at two quite different levels of abstraction, the one concerning the accelerator relation being the more remote of the two. The difficulty here is that one cannot hope to take this asymmetry into account by any mathematical formulation. The theorist must therefore be keenly aware, in evaluating the conclusions, that at the point of departure there is an investment function that has a weaker theoretical foundation than the consumption function.

Economics of effective demand

It may, moreover, be remarked that the multiplier relation always remains formally the same, whether the system is in a situation of idle productive capacity or in a situation of full capacity utilization (although in real terms the effects will of course be different). The principle of acceleration, on the other hand, works at its best in a situation of full capacity utilization. When there is idle capacity, the principle may still be working but in an attenuated form, or in any case in a more complicated way.

All that this amounts to is that the results must be interpreted with care and judgment. If the analysis of the interaction of the multiplier and the accelerator has taken the economic theory of effective demand to its peak, it has also clearly strained it to the limits of its possibilities. For, as soon as economic investigation is carried over from the Keynesian short run to movements through time, and the capital stock can no longer be taken as given but is itself changing, then another side of the picture is bound to become relevant – the evolution through time of the physical possibilities of production.

APPENDIX

The ‘lagged’ multiplier

For completeness' sake, it may be useful to devote a few lines to the 'lagged' version, as opposed to the 'instantaneous' version, of the multiplier relation. The 'lagged' multiplier emerged mainly from the discussions of J. M. Keynes with Dennis Robertson and has the interesting property of showing in detail the process through which actual savings adapt to predetermined investments, through changes in income.

Let us consider what has been called a 'Robertsonian' consumption function, in which there is a time-lag between income and consumption:

\[ C_t = A + aY_{t-1}, \]  
\[ (11.2b) \]

and let us take, in conjunction with it, relation:

\[ Y_t = C_t + I_t, \]  
\[ (11.1b) \]

to form a self-contained system in which \( I_t \) is determined exogenously.

44 One way of partially remedying this deficiency has been tried with the introduction of non-linearity. See especially: R. M. Goodwin, 'The Non-Linear Accelerator and the Persistence of Business Cycles', *Econometrica*, 1951, pp. 1–17.
Growth and income distribution

Suppose now that investment, after being zero up to time \( t = 0 \), is then raised to a positive amount \( I \) from \( t = 1 \) onwards. This means \( I_t = 0 \) and \( Y_t = C_t \), for all \( t = 0, -1, -2, \ldots \); but \( I_t = I \) for all \( t = 1, 2, 3, \ldots \). In successive periods, we have:

\[
\begin{align*}
Y_0 &= C_0 + I_0 = A + aY_{-1} = A + aY_0 = C_0, \\
S_0 &= Y_0 - C_0 = 0, \\
Y_1 &= C_1 + I = A + aY_0 + I = Y_0 + I, \\
S_1 &= Y_1 - C_1 = I, \\
Y_2 &= C_2 + I = A + aY_1 + I = A + a(Y_0 + I) + I = Y_0 + aI + I, \\
S_2 &= Y_2 - C_2 = I, \\
Y_3 &= C_3 + I = A + aY_2 + I = A + a(Y_0 + aI + I) + I = Y_0 + a^2I + aI + I, \\
S_3 &= Y_3 - C_3 = I, \\
&\vdots \\
Y_n &= C_n + I = A + aY_{n-1} + I = Y_0 + I(1 + a + a^2 + \ldots + a^{n-1}), \\
S_n &= Y_n - C_n = I.
\end{align*}
\]

As may be seen, savings \( S_t \) are always equal to the predetermined amount of investment \( I \), at each single step, although income is increasing all the time (but at a decreasing rate, since \( a < 1 \)).

If we call \( \Delta Y = Y_n - Y_0 \), the total increase of income from \( t = 0 \) to \( t = n \), we obtain:

\[
\Delta Y = Y_n - Y_0 = I(1 + a + a^2 + \ldots + a^{n-1}),
\]

or, using the formula for the sum of a geometric progression:

\[
\Delta Y = I \frac{1 - a^n}{1 - a},
\]

Since \( a < 1 \), \( \Delta Y \) clearly tends to a finite limit as \( n \to \infty \), i.e.

\[
\lim_{n \to \infty} \Delta Y = \frac{1}{1 - a} I,
\]

which precisely coincides with expression (ii.7) obtained for the instantaneous multiplier. Thus when consumption decisions lag behind income, the total increase in income, given by the multiplier formula \( 1/(1 - a) \), will be obtained not immediately, but asymptotically, as time goes on. The process is illustrated in fig. ii.5. At the beginning of the process towards the new equilibrium situation, there are big leaps—after only four or five steps, the system is already rather near the new

equilibrium level of income \( Y^* \). But since each step is smaller than the previous one, the process slows down as it goes on. The final position, though nearly approached after only the first few steps, is never quite reached exactly.

\[\text{Fig. ii.5}\]

The interesting phenomenon to notice is that, during the whole process, the total savings that people intend to do, or \textit{ex-ante} savings, \( (Y_{t-1} - C_t) \), are different from the total savings they will in fact end up with doing, or \textit{actual} savings, \( (Y_t - C_t) \). \textit{Ex-ante} savings, in fig. ii.5, are the difference between the 45° line and function \( C \), while \textit{actual} savings are the difference between \( (C + I) \) and \( C \), and therefore always coincide with investment \( I \).

To conclude, savings decisions are simply frustrated, as long as they differ from the predetermined amount of investment. Only when the system has actually reached the new equilibrium position, do \textit{ex-ante} savings become equal to actual savings. In other words, savings decisions become effective only when the changes in income have made them yield an amount of savings equal to the predetermined amount of investment.