Queries about Coincidences and Correlations Between Fisher and Keynes, the Gibson Paradox and the Idea of Duration.

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Interrogation #1.
Q. What is the difference between the Keynesian demand for money and the Classical?
A. The speculative demand for money.

Q. What is the speculative demand for money?
A. The impact of changes in the rate of interest on the capital value of long-term bonds held in investment portfolios. It is a question of interest and capital.

Q. Who was the world's best known expert on the relation between capital and interest in the first half of the 20th century?
A. Irving Fisher.

Q. So, Fisher must have adopted a Keynesian theory of the demand for money?
A. No, he was a pure classical quantity theorist.

Interrogation #2.
Q. Was Keynes a good empirical economist?
A. No, every empirical regularity he identified turned out to be a special case.

Q. Such as?
A. The best known is the relation between prices and output, or the impact of diminishing returns leading to an inverse relation between changes in the level of employment and output on one hand and the level of real wages on the other. Dunlop, Tarshis and Kalecki all provided statistical evidence to the contrary. Fortunately, this had no major consequences for his theory.

Q. Is that all?
A. Well, he got the same thing wrong in a number of different ways. He also considered the positive relation between wholesale commodity prices and yields of British consols one of the most completely established facts in all of empirical economics.

Q. And it is not?
A. Not only does it appear to have no foundation, but Keynes fudged the figures in a way that would have made Milton Friedman proud.

Interrogation #3.
Q. If Keynes and Fisher disagreed about monetary theory they must have disagreed about most things in economics?
A. No, they agreed about the "Gibson Paradox", that there was an unequivocal relation between prices and interest rates.
Q. Then they were both wrong?
A. No.

Q. No?
A. They had different explanations to explain the relationship that didn't exist. Fisher's was wrong. Keynes just had the right explanation for the wrong problem. He needed to find the right problem to explain.

Q. And that was?
A. The decision to hold money or hold bonds.

Q. And what was the difference?
A. You should have guessed by now. The impact of interest rates on capital values.

Interrogation #4.

Q. Where did Fisher go wrong in his explanation?
A. As you can read in any economics or finance textbook, Fisher argued that when there was an increase in the inflation rate, lenders would require an increase in return. If the rate of interest was r, they expected $1+r$ at the end of the investment period. If, during the period prices had risen by p, then they would require that their funds be indexed and the return would have to be $(1+r)(1+p)$.

Q. But, what determined the r that they got before calculating the depreciation of money due to inflation?
A. Ah, there's the rub. r was the "real rate" of interest. If lenders always got the inflation adjustment, then the real rate remained constant.

Q. What has all this got to do with the demand for money?
A. Good question. Remember that the speculative motive is based on changes in capital values caused by changes in interest rates. If all interest rate changes due to changes in inflation are offset by nominal rate adjustment such as to leave the "real value" of capital unchanged, then there are no "real" capital value adjustments because "real" interest rates are unchanged. Or at least this seems a plausible explanation of why the world's expert on interest and capital could overlook the impact.. in real terms there was none.

Q. You mean that if nominal interest rates always adjust so as to offset changes in inflation then the real value of capital assets are constant and have no impact on the demand for money?
A. Now you're catching on. Yes, this seems to have been Fisher's explanation. But, remember, there is no theoretical or empirical foundation for this proposition. It all seems to have been based on the statistical anomaly created by Gibson.

Interrogation #5.

Q. But, how can you be so sure that Gibson was wrong?
A. An exhaustive study of the course of interest rates and bonds yields in the United States since 1856 and of the statistical relations of those rates and yields to one another, to stock and commodity prices, to the physical and monetary volume of trade, and to credit and banking conditions published in a 1938 National Bureau of Economic Research Study by Frederick R. Macauley.

Q. And what did Macauley show?
A. First, that the data do not support Gibson -- there is no reason to accept the proposition that there is an empirical relation. Second, that there are severe logical inconsistencies with the explanations for its existence.

Q. But then both Keynes and Fisher were wrong?
A. Yes, Macauley is merciless in showing how both of them continually fudged their positions. He even quotes Keynes' *Treatise on Probability* to show that Keynes once knew better.

Q. What else did Macauley do?
A. Since Keynes and Fisher used slightly different versions of the Gibson idea, Keynes dealing with yields on consols, Fisher with short-term rates, Macauley raised the question of just exactly what was meant by yield on long and short term instruments.

Q. And what did he discover?
A. First, that the concept of yield to maturity commonly used to classify interest returns is virtually meaningless. Since it is based on discounted value it presumes what now would be called a flat yield curve, that all interest payments can be reinvested at the common yield to maturity, no matter how far in the future they may lie. Yield to maturity thus more resembles a "promised" rate of return than a "secure" or certain return.

Q. You mean that when I buy a bond with a 10% yield to maturity I'm not certain of the 10% return?
A. Not at all. You get 10% only if you manage to reinvest all the coupons from now to maturity at 10%.

Q. With Alan Greenspan at the helm that seems hardly likely.
A. You said it.

Q. But, aside from the implications for my retirement, why is this important?
A. Remember that Gibson's relation linked the yield on consols and wholesale prices. What Macaulay pointed out was that these were simply an average of a whole range of interest rates, what the Austrians would have called an "aggregate" rate with no direct market referent. Thus to argue that the market responded to inflation by adjusting yield was like saying that the economy adjusted to inflation by changing its marginal propensity to consume. It might be true in the aggregate, but there is no individual decision rule that makes it so.

Q. But what interest rates should be used?
A. Here Fisher was probably closer to the mark. Short rates are probably the best response to changes in expected prices. What Macaulay then tried was to work out what would happen if individuals perfectly forecast short-term changes in commodity prices and adjusted their short-term borrowing and lending behaviour accordingly. He argued that this should then have an impact on long term rates since investors always had the choice between operating short term or buying long term and selling before maturity (Note how changes in capital values come back in here). What he discovered was that long term rates did not come anywhere near what would be required on the assumption that inflation could be foreseen and that investors would be indifferent between say a six-month Treasury and buying a twenty-year Treasury bond and selling it after six months.

Q. So that got rid of Fisher's explanation. What about Keynes?
A. Well, one of the things Macaulay tried to do was to provide a measure of what was a long and what a short maturity. His idea was that the difference in investor behaviour between short and long term bonds was
precisely in the fact that changes in capital values did not adjust so as to make the six month bill equal to the holding period return on the twenty year bond over six months. He argued that some bonds of similar maturity may be longer term than others, precisely because of the defect noted above in the use of yield to maturity as a measure of return. Bonds with different coupons or different premia and discount will react differently to changes in the rate at which their interest payments can be reinvested. To take an extreme, what is now called a zero coupon bond pays no coupon and thus has no uncertainty over the rate at which they will be invested. At the other, a bond with a very high coupon gets reinvested more quickly and thus has rates that are closer to the hypothesised yield to maturity. On the other hand, low coupon bonds or deep discount bonds are more like zeros in that their payoff comes at the end of their life. Macauley tried to take these things into account. He developed a concept called "duration" to measure the effective maturity of a bond with respect to the risk of a change in reinvestment rates.

Q. That must have been a very important concept?
A. Yes, but it was almost totally ignored. Hicks, working from the hints in Keynes, developed his own version in Part II of Value and Capital, (1939) but then no one ever reads anything but Part I dealing with Statics. It was more or less rediscovered by Paul Samuelson in a 1945 article dealing with the impact of higher interest rates on the stability of the banking system, and implicitly formed the basis for the analysis in Homer and Leibowitz, Inside the Yield Book, the Salomon Brothers bond traders' bible which launched modern bond trading management in the 1970s and is the basis for modern financial innovations in risk management. In particular, stripping Treasury bonds into their individual components.

Interrogation #6

Q. OK, I don't want to appear ignorant, especially where Salomon Brothers is involved. But, what is the big deal with the duration idea and the Gibson paradox?
A. Remember that Gibson had used the yield on British consols to compare with the wholesale price index. Macauley reconstructed the series (this was in the days when the NBER did serious work on cycles). What he discovered was that for the years 1901 to 1903 the prices used were for a new stock issue, announced in 1888, to replace the existing 3% consols. This new stock was to pay 3% until April 5, 1889, 2.75% until April 5, 1903 and 2.5% until 1923. After that date it could be called at par. Yet, Gibson assumed that this stock would continue to pay over its entire life the interest rate payable at any particular date, without taking into account the changes in the rate which would occur over the life of the bond. Thus, the rate he calculated for 1901 was based on 2.75%, and ignored the fact that lower rates would be paid after April 5, 1903.

Q. You mean he treated the rate then current as if it were the yield to maturity?
A. Now you're catching on real fast. That is precisely it. Indeed, one could go so far as to imagine that Macauley discovered the idea of duration by looking at how Gibson had treated a bond in which the reinvestment rate was set out in advance at a rate which was different from the stated yield to maturity. It was clear that a bond that had a rate schedule attached could not have a yield to maturity equal to any of the preannounced coupon rates, and thus its could not have reinvested its coupons on a fixed rate. Indeed, this was the case with any bond, yet this is what yield to maturity refused to contemplate.

Once he had recognised this, it seemed obvious that there could not be much relevance in a relation between rates of commodity price increase and yields to maturity on consols. At the very least, one would have had to have looked at consols with the same duration. Or simply restricted the analysis to short rates.

Interrogation #7
Q. But didn't Keynes and Fisher use different rates? Keynes consol yields and Fisher short rates? What about the relation between short and long-term rates? Isn't there an explanation of long rates which says that they are determined by expected short rates?
A. Good point. This is the part of Hicks' analysis which has in fact survived in the form of what has come to be called the "expectations" explanation of the slope of the yield curve relating the yield to maturity and the time to maturity of a bond.

Hicks points out that a long-term loan can be viewed as a "spot" loan for one week (you remember that Hicks always uses a "week" as the standard period), and a series of forward one-week loan transactions, each involving relending the principal and interest for a further week. A two-week loan will then be compounded out of the spot rate and the rate expected to prevail one-week forward. The long-term rate of interest can thus always be unbundled into the shortest-term spot rate and a series of expected forward rates for the same term.

From this basis, if short rates expected to rule in future weeks are higher than the rate for the current week, then long-term rates will be higher than the current short rate since an investor can always invest for a single week, and then reinvest the proceeds next week at the expected higher rate, giving a compound weekly return which is higher than this week's short rate. Since long rates have to compete with this rate, they will rise until the return from investing long is equivalent to the compound rate achieved by investing week by week for a similar term. Thus, the "yield curve" slopes upward. If rates for future weeks are expected to be lower, then the curve slopes downward. This is what is meant by the idea that long rates are "explained" by anticipated movements in short rates. But, Hicks also points out that short rates can be explained by expected long rates which refer to a finite holding period. The holding period return for a perpetuity (that is, a consol with no redemption date) will be determined by the interest coupons, adjusted for the rise or fall in the market price of the bond. Thus, the rate of interest for a week is determined by the fixed coupon adjusted for anticipated changes in the future prices of the long bond. With a rise in long rates, holding period returns for short periods may fall to zero, and can easily become negative, i.e. the loss in capital value exceeds the interest coupon. Hicks argues that this will lead lenders to use shorter maturity lending whenever possible, and that they would be willing to pay something for eliminating the potential of a reduced or negative return in the form of lower rates of return. So we are back to the relation between capital and interest, or better, how the change in interest affects capital value relative to yield.

The important point, however, is not so much that expected short rates determine long rates, or vice versa, but the process which determines the formation of the expectations of the rates which will prevail in future. Hicks deals with this in terms of the "elasticity of expectations".

Macauley takes a more direct approach. He notes that the explanation of the Gibson paradox in which short rates rise with inflation because lenders require a higher future income to offset their loss in purchasing power implies that investors are able to form perfect forecasts of inflation over the short-term future. That is, that it is the expectation of future prices which determines the expectation of future interest rates.

Q. Does that make a difference?
A. Not much. Macauley adopts the strong assumption that expectations are correct. Today we would say that expectations are "rational". He then argues that this assumption implies that short-term interest rates will be a function of the rate of change of prices, rather than the level of prices.

Q. Why is that?
A. Say the interest rate is 4% and prices are stable, so the real and the money rate of interest are both 4%. Now, assume that prices start to rise by one percentage point per annum, perfectly foreseen. In the first year the rate of interest will rise to (1.04*1.01 -1 = ) 5.04%. The nominal rate will rise by 1.04 percentage points each year. 6.08% for the second year when inflation is 2%, 7.12% when it is 3% and so forth. Now, assume
that inflation peaks in the 6th year at 6%, with an interest rate of 10.24%. The inflation rate starts to fall back by 1 point per year, as does the interest rate. However, the price level, which had risen by nearly 23% over the first six years, will continue to increase. After 12 years prices will be 42% higher, but interest rates will be back at 4%. Interest rates will have peaked halfway through the inflationary cycle, when the rate of increase of prices, not the price level, reached its maximum value.

Q. OK, but didn't Fisher eventually adopt the link between interest rates and rates of change of prices in his explanation of the paradox?
A. True, but it doesn't help. Macaulay generalises the argument, assuming the inflation rate takes on a periodic motion in the form of a sine curve on a log scale, such that 12 years of rising prices is followed by 12 years of deflation. He first points out that when deflation hits 6%, interest rates should be about -2.24%, although no supporter of the theory had ever suggested that symmetry should prevail and borrowers would force lenders to pay them negative rates to reimburse them for the increase in the real value of their debt! More seriously, he notes that the peak of interest rates occurs after the first quarter (after 6 years) of the full motion when the slope of the price curve is at a maximum, while the price peak occurs after the first half (12 years), so that under these assumptions interest rates would lead prices by a quarter of the cycle period.

Q. And the link between short and long rates?
A. That's easy. Macauley compares them when inflation is stable, i.e. at the inflection points of the periodic motion. A stable six percent inflation rate corresponds to a stable 10.24% short-term interest rate. But, the same inflation coverage could be obtained by an alternative route of buying a long-term bond and selling it at the end of the year. Remember Hicks' representation of the relation between short and long-term yields. In the absence of price changes, the return on bonds over the year would be given by the coupon yield or the current yield. If the current yield on bonds were less than 10.24% then bond prices would be rising throughout the year (causing the current yield to fall) so that the holding period return for the year would be just 10.24%.

On the other hand, if the bond's yield to maturity were less than then short-term rate, the yield to maturity would also have to decline, but its price need not rise, and indeed might fall.

Since short-term rates are likely to be higher than current bond yields when prices have been rising, current yields will be falling and bond prices rising after short-term rates have peaked, but commodity prices are still rising. The same will be true of yield to maturity on bonds (but not necessarily of prices) which will continue to fall as long as short-term yields are higher. Bond yields will only stop falling when prices have reached the top of the sine curve.

Thus, in conditions in which price changes are perfectly foreseen, short rates should lead prices by a quarter of a cycle and the cyclical movement of long rates should show a perfect negative correlation with the price cycle (and thus lag the peaks of price rates of change by a quarter cycle). Thus even if Fisher's hypothesis concerning the real rate of interest is fulfilled, it does not provide an explanation of positive correlation with either short or long rates.

Q. I noticed something interesting here. When short rates are rising to offset inflation, long yields may have to be falling (bond prices are rising) to keep long bonds competitive, so the difference between the two rates or the spread should be falling or negative. Isn't that an inversion of the slope of the yield curve? Or a negative slope? But doesn't Hicks argue that this is based on expectations of falling forward short rates? Isn't that contradictory? And isn't change in the slope supposed to mean something about the future of prices?
A. Good. Lots of questions. A change in the spread is certainly supposed to mean something about the future. But, we are not sure of what or why. Now that Mishkin has become head of the New York Fed research function, everyone is aware that the yield spread between short and long rates is a good predictor of turning points in economic activity. Harvey is also a backer of this hypothesis. It is based on a different version of
a stable real rate of return. Rather, it assumes that agents maximise intertemporal utility when incomes are stable over time. They thus engage in investment activities which produce income smoothing. If I expect my income to fall because of an anticipated fall in future economic activity I will seek investments which produce income to fill the expected future shortfall in income. (Note that the same argument can be made for business firms, who will reduce their longer term investments, and thus longer term borrowing to finance them, if they expect future conditions to be depressed. The supply of longer term securities will then fall as demand rises.) Thus, the demand for financial instruments that cover the period of expected decline in activity will rise, increasing their prices and reducing their yields; if these increased longer holdings are financed by the sale of shorter maturity instruments, short rates will rise. Thus, longer rates fall relative to current yields, causing a decline in the spread; if this continues the yield curve eventually inverts on the expectation of a downturn in activity. Note that we could substitute an anticipated fall in real income caused by expected inflation for a decline in income due to anticipated fall in the level of activity, and find that expectations of rising prices produce inverted yield curves, and vice versa. And that this would produce expectations of lower future short rates relative to current short rates, so Hicks' conjecture is also preserved. In fact, this is just what Macauley deduced.

However, we have to be careful here to distinguish between a change in the slope and a change in the level of rates. The slope of the yield curve may change while both short and long rates are rising, or vice versa or when they are moving in opposite directions. This should also make it clear that the result we get depends on the hypothesis we make about expectations. Hicks was very clear on this point. The inversion of the yield curve in the face of currently rising prices is the result of the assumption that future inflation (or economic conditions) can be perfectly predicted. (Or in the case of the change in the spread on the idea that the anticipations of private agents are more quickly expressed in interest rates than in any other economic variables) Macauley went on to test that too, and discovered that it perfectly cannot be predicted! Thus, there may not be any stable relation between short and long rates.

**Interrogation #8.**

Q. OK, I see how Fisher came to grief if there is no relation between short and long rates, and long rates calculated be means of yield to maturity are not the relevant expression of the interest rate. But, what about Keynes. Isn't he just as badly off as a result of Macauley's duration?

A. Let's look at Keynes's criticism of Fisher's explanation (*Treatise on Money*, II, pp. 202-3). Fisher argues that a rise in the rate of inflation by 2% will cause bond yields to rise by approximately 2%. (If inflation is initially zero and bonds pay 10% the bond holder now needs \((1.10)*(1.02)= 1.122\) or a rate of 12.2% to retain his 10% yield in real terms) But Keynes argued that Fisher overlooked the negative impact on existing holders of rising interest rates on bond prices (a consol with a £100 par pays £10 per annum to yield 10%, at a 12.2% rate of interest its market value falls to £81.96 or a decline of 1.8% and a capital loss of over £18) so that an investor will be worse off "whilst the variations in the rate of interest earned during the year in question are too small to make much difference". We can presume that the last phrase refers to the higher interest rates that can now be earned on reinvesting the coupon interest on the bonds. Instead of reinvesting each £10 interest payment received is now invested at 12.2%, rather than 10%; this covers the decline in the value of money, but not the change in the capital value of the bond. So it makes no difference at all.

His argument is that Fisher's relation goes in the wrong direction. To the extent that a rising inflation rate increases bond yield it causes absolute losses in capital values which more than offset the rise in the rate of interest which is to compensate for the fall in the value of money. Full inflation coverage should provide for an adjustment in the capital value as well as in the income from the bond.

Q. I know inflation is a bad thing, but what does all this have to do with duration?

A. An alternative method of understanding duration is as the time that it takes for the increased interest
earned on the reinvestment of interest due to a rise in the rate of interest to offset the fall in capital value. What Keynes is arguing here is that in the short term (where Fisher set his argument) this cannot in general take place.

Q. So Keynes got it right?
A. Not quite, and not yet. What he forgot to figure in was the coupon itself. At the end of the period the investor has not only his capital position, he also has the year's interest payments to offset against the rise in the price level. The calculation still holds, but it is not quite as extreme as it looks at first sight.

Interrogation #9

Q. And why was Keynes so interested in all of this?
A. Remember that all of this discussion of the Gibson paradox appears in the Treatise on Money. Keynes provides an explanation by means of his "fundamental equations". Also recall that he is trying to explain a slump, and the behaviour of falling prices on interest rates, not an inflation. He is also trying to justify an expansionary monetary policy, even though it seems not to be working. He thus argues that if the money rate of interest is too high, and the monetary authority is too slow in adjusting to the lowered level of the natural rate, then saving will be continually running ahead of investment and there will be what he called a "profit deflation", which eventually leads into an "income deflation". This implies that falling interest rates will accompany a falling price level.

Q. But how does that support the role of expansionary monetary policy?
A. It allows him to argue that the low interest rate policy has not gone far enough, that a policy "à outrance" would be necessary to get the money rate down to the natural rate and bring about a return to stability.

Q. And where does the business of capital values of bonds come into all this?
A: "It happens that on this occasion when, if I am right, one of the conditions limiting the open-market operations à outrance does not exist; for it is not an occasion -- at least not yet-- when bonds are standing at a price above reasonable expectations as to their long-term normal, so that they can still be purchased without the prospect of a loss. Not until deliberate and vigorous action has been taken along such lines as these and has failed, need we, in the light of the argument of this Treatise, admit that the Banking System can not, on this occasion, control the rate of investment and, therefore, the level of prices" (Treatise on Money, Vol. II, p. 387). To get prices to stabilise and then rise, interest rates have to go down. But, Gibson says it can't be done. Keynes thus has to show why Fisher's explanation of inflation adjusted rates will not be valid. This he does by pointing to the beneficial effect of falling interest rates on capital values.

But this creates an unexpected counterargument to his own theory. What happens if bond yields cannot be further reduced, such that bond values cannot be further increased? The passage just quoted is from the conclusion to the Treatise, and it looks very much like saying, given the level of bond prices relative to what they are expected to be in normal conditions this is not likely to occur, or that 'there is currently no liquidity trap'.

Interrogation #10

Q. So, that's the end of the story?
A. Not quite. You will have recognised that the Gibson paradox has a life of its own and in some form or other is still alive and well in most of the world's central banks and in most bond houses. They all believe that higher inflation produces higher long-term interest rates -- this is what motivates the so-called "bond
market vigilantes" who push up long-term bond interest rates whenever they think inflation is about to rise, and the Fed, which puts up short rates whenever it thinks that inflationary pressure is about to appear, in the hopes of forestalling the rise in long rates, or even causing them to come down.

Q. That's not what I meant. What about the Keynes and Fisher story?
A. Well, in the General Theory Keynes makes the same criticism of Fisher. But, there is also a strange passage in which he actually makes the calculation of the break-even point between the impact of higher interest rates on the capital value of a bond and the increase in interest earned on the reinvested coupon payments. This is the "square rule" which says that if the rate of interest is expected to rise by more than the square of itself, then the loss in capital value on a bond will just offset the coupon interest received from owning the bond, leaving the investor with a zero annual yield. Clearly, if the rise in the rate of interest is imposed in order to compensate for an equal rise in the inflation rate, then the investor clearly is not being very efficient in protecting against the loss in real purchasing power due to the higher rate of inflation.

Q. You mean that the interest rate would have to rise by more than the inflation rate?
A. Yes, but every increase in the interest rate produces an increasing fall in capital value, and since the coupon rate is fixed, you can never get there from here -- it can't work. Thus, in such conditions you cannot insure an investor that his real return will be protected -- Fisher offered a contract which could not be honoured. This is the chain of ideas which eventually produced the liquidity trap, and the motivation for the speculative demand for money.

Q. And duration?
A. Keynes never mentions it. And Macauley never mentions the General Theory. Hicks does mention Keynes, but he never mentions duration, or Macauley.

Interrogation #11

Q. But, Hicks published his book after Macauley. How did Hicks get into all this?
A. Remember that Hicks' contribution appears in his presentation of "dynamic" theory, or better his prelude to dynamic theory in the form of temporary equilibrium in which there are transactions which occur across "weeks". He had already been working on this theory when Keynes's book came out, and he tells us that he quit when he read Keynes, as it had all been done. Well, he eventually changed his mind.

One of the points he makes is that income is not an appropriate concept for dynamic theory. This is because it is extremely difficult to give a precise definition of income when some of income is composed of fixed income securities resulting from long-term borrowing and lending. If income is defined as what an individual can consume without impoverishing himself, there are a number of ways this idea can be expressed. It could be the expenditure that leaves the present value of his future net receipts constant, or it could be the amount that can be spent in perpetuity (recall the assumption of constant income through time in the spread theory), this either in nominal or real terms. Now, what happens to income when there is a change in the rate of interest? Or the rate of inflation? Take the former. A rise in the rate of interest increases the income from investment, so that expenditure may rise if capital is being reinvestment each week. But, if it is invested for a number of weeks, then possible expenditure can rise by less. Thus two individuals with the same earned income and capital position will be affected differently by a change in the rate of interest depending on whether they have invested long or short-term.

Q. How will they be affected?
A. This is the point, and it will depend on the impact of higher interest rates on capital values relative to the
higher incomes from reinvesting at a higher interest rate. To solve this problem, amongst others, Hicks defines what he calls the "average period" to measure a standard stream of income. He takes the actual expected income stream, and calculates the net present value of that stream at the current rate of interest (on the standard assumption that the yield curve is flat). He then asks what constant income stream would produce the same present value, given the same current rate of interest. This is the "standard stream". Changes in the rate of interest will change the present value of both the actual and the standard stream. Income will increase if the change in the rate of interest changes the present value of the actual stream by more for an increase, or less for a decrease, than it changes the standard stream.

Q. And how can you decide this?
A. Hicks's entire theory is built on substitution and complementarity. He thus looks at the interest elasticity of the two definitions of income. The answer thus depends on the relative elasticities. If the actual income has an interest elasticity which is greater, then a fall in the rate of interest (Hicks in fact works with the discount factor, one over one plus the rate of interest) will increase actual income more than standard income and the individual will be better off as a result of the change in the interest rate. This elasticity is defined as the "average period" of the income stream and represents "the average length of time for which the various payments are deferred from the present, when the times of deferment are weighted by the discounted values of the payments" (p. 186). A comparison of the average periods of the standard and actual income will determine the impact of changes in the rate of interest on income. If the average period of actual income is greater, then a fall in interest will increase income, and a rise decrease it.

Q. That's very interesting -- but very complicated, and seems to have nothing to do with investment or the demand for money.
A. What Hicks does is to use the concept of average period to analyse flows of net income from production plans and investment projects. He notes that the Austrian theory is based on a theory of the average period of these income flows which is unweighted. His average period is weighted by its "distance" in time. He thus proceeds to measure the "capital intensity" of production plans in terms of their "average periods" as follows. Calculate the average period of the flows of periodic net surplus at the ruling rate of interest. Allow the rate of interest to change, and allow entrepreneurs to rearrange the production plan. Recalculate the average period of these flows using the old rate of interest. If the average period is longer, then a lower rate of interest leads to a longer "average period" of production. The Austrian theory turns out to be right.

Q. Wait a minute. Aren't we going way off track here? This is Sraffa and Cambridge capital theory controversies stuff.
A. Exactly. Isn't it interesting how one thing leads to another? Hicks sets up his analysis precisely in order to show that a rise in the rate of interest brings about a decrease in the average period (e.g. all periodic surpluses are positive). But, he also notes that there may be exceptions to this result. If surpluses are concentrated at the beginning and the end of the production plan, or are complementary, or if there is a large risk factor attached to the far-dated surpluses such that increasing risk overwhelms the influence of interest rates then the average period may not increase. He considered these as improbable in practice. On analogy with the analysis of the period of individual income, one would have thought that movement in one way was as likely as the other. But, Hicks is so intent on validating the Austrian theory that he discards them. What is even more interesting is that there is no theory that says that duration will rise when interest rates rise. Indeed, if Hicks had put the question that way, we probably all would have been saved the capital theory controversy!

Interrogation #12
Q. But, wasn't Samuelson in on this capital theory controversy? And on the same side as Hicks? Didn't you say he also discovered duration? And he also got the capital theory part wrong?
A. Samuelson looking at the difference between anticipated and unanticipated changes in interest rates. He made the very simple point that the negative impact of rises in rates can be avoided if they are indeed foreseen, but if they are not they can but increase your lifetime wealth. Higher rates must thus be a good thing, even when the future is not foreseen. Of course, Samuelson was arguing in the absence of perpetuities, so that all long-term bonds would eventually pay off at par, and any interest receipts would now be invested at a higher rate, so that lifetime accumulation would be higher.

Q. So where did duration come in?
A. When applied to a financial institution the problem is one of the impact of interest on its receipts and payments. He thus generalised the proposition by defining the average time period of the sum of the discounted receipts and payments weighted by the time period in which they take place. He could then argue that a rise in the rate of interest would benefit any institution which had a weighted average of disbursements greater than its receipts.

Q. And the capital theory debate?
A. Well, if disbursements are given a minus sign and receipts a plus sign, the lower the net value of the average time period, the greater the benefit of a higher rate of interest. This gives an inverse relation between time period and impact of raising the rate of interest, which is similar to Hicks.

Q. Yes, Hicks. What does Samuelson say about his use of duration?
A. Nothing. Despite using the exact same average period terminology. Nor does he say anything about Keynes. Or Macauley, or anyone else! Honour among thieves.

**Interrogation #13**

Q. And bond traders still believe in the Gibson thing AND use duration? And economists use duration AND still believe that capital intensity is inversely related to the rate of interest?
A. Of course. Don't you know the aphorism about consistency and small minds? Besides, they have encouragement. For example, Shiller and Siegel have confirmed that the Gibson relation does hold using spectral analysis, but they do not mention any of the problems raised by Macauley concerning the relation between bond maturity and duration and the relevance of the long rates used to test the relation.

Q. But, what is the point of all this correlation and coincidence?
A. You should be able to guess by now. First, Keynes appears to be the first to discover duration, and it plays an important role in his theory and his investment activity. Second, that it is much more important than most economists and financial analysts have suspected. And third, that we might have avoided a great deal of debate and misunderstanding if this had been recognised earlier.

**Appendix:** Duration and the Square Rule

Keynes argues that an investor will choose to hold money rather than assets when the potential for capital loss per annum on assets exceeds their expected coupon yield per annum. The condition is thus \( dP-C=0 \), where \( dP \) is the annual cumulative change in price of a perpetual bond, and \( C \) its coupon. The price of a perpetual bond is given by its present value, or \( C/r \), where \( r \) is the current yield to maturity. The change in price may then be given by the result of multiplying price times the change in the rate of interest, \( dr \), times the modified duration of the bond. For a perpetual bond duration is given by \( D = (1+r)/r \) and modified duration by \( D/(1+r) \) which reduces to \( 1/r \). Thus \( dP=P*dr*MD \), which can be rewritten as \( dP=(C/r)*dr*(1/r) \).
= dr * (C/r^2). dP-C=0 thus can be expressed as dr * (C/r^2) - C = 0 which resolves to dr = r-squared as the condition under which dP=C and is simply Keynes' square rule.

The duration of a perpetual bond paying an annual coupon is given by the formula (1+r)/r so that a 4% consol selling at par of 100 and paying a £4 coupon has a duration of 26 years. The modified duration, defined as D/(1+r), which measures the volatility of bond prices, is then 25 years. The change in the price of the 4% par consol is calculated by multiplying modified duration by the current bond price and the result by the change in the bond's yield to maturity. In the case of a 16 basis point rise in the yield from .04 to .0416, the value of the bond will fall by 25*100*.0016 = £4, which is precisely the value of the bond coupon. For any higher increase in interest rates, the fall in the bond's value will exceed the current coupon yield of the bond, producing net losses for the holder. It is this case that Keynes defines as the liquidity trap, for the investor should prefer to hold cash rather than bonds if his expectation is for a rise in interest rates by more than the square of the rate.

This point can also be described in terms of the break even point on the bond. Any change in yield on a bond affects both capital value, and reinvestment income, but in opposite directions. Duration gives the point at which the fall in value is just offset by the increased reinvestment income of the bond's coupons. Thus, the lower the rate of interest, the higher the value of the bond's duration and the longer it takes to recover the fall in capital values with increased reinvestment earnings. At 3% the duration rises to 34.33 years and at 2% to 51 years. At 1% it is 101 years. Thus Keynes' dictum that the lower the rate of interest, the more likely the liquidity trap. However, it should be clear that this does not rule out the existence of a liquidity trap at higher rates. At 8% duration is 13.5 years and modified duration, 12.5 years. A rise in the rate of interest to .0864 would produce a fall in the price of a par 8% consol of 100*12.4*.0064 = £8 which is exactly the coupon value. An expectation of a rise in interest rates of more than 8% then leads to the decision to remain liquid. The expected percent rise in interest (or equivalently, the expected percentage fall in the price of the bond) is however, twice as high as in the case of 4% rates. However, these should be judged relative to recent changes in bond prices. If 8% lies outside of the range given by two standard deviations from the mean of rate changes over the recent past, then it would be just as rational to remain liquid at 8% as it was at 4% in similar conditions of volatility.

However, as rates rise, duration measures become increasingly lower. At a 20% yield to maturity duration falls to six years and modified duration to five years. At 60%, similar to rates paid in Mexico in crisis periods, duration is 2.67 years, so an investor can recover his lost capital value from the reinvested earning on higher interest rates in less than three years. If there is any basis for the interest elasticity of the demand for money, this is it. However, it also suggests that the elasticity will be based on expectations, and that expectations will be based on recent volatility of rates. In a volatile rate environment, it will become more difficult to use changes in interest rates to influence the demand for money.

As rates rise, the square rule produces larger and larger absolute changes in the interest rate, and the use of modified duration to calculate the change in price becomes less and less accurate. The full calculation of the change in the bond price will require the calculation of convexity. It is interesting to note that Hicks, 1939, p. 261, note 2, reproduces the square rule, and notes that it is very likely that returns will be negative, since the fall in capital values will be even greater if changes in risk are taken into account.
References:


