Demand for Money and Expectations
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This article uses the Keynes-Kahn approach to asset-price arbitrage in order to analyze the slope of the liquidity preference function. The approach has also been developed by Robinson (1953) and, more recently, by Kregel (1998). The Keynes-Kahn approach is not mainly an explanation of the shape of the liquidity-preference curve; it is a theory of the yield curve that gives importance to expected long-term rates. However, as Kahn (1954) shows, this analysis can be used to draw conclusions about the shape of the liquidity function.

Traditionally, the demand for money is assumed to have a negative functional relationship with the rate of interest ($dL/di < 0$). It is true that, for the sake of simplification, Keynes argued that one can assume a “smooth curve” that is downward sloped. However, he was also aware that this is a simplification, and that, in fact, portfolio arbitrages imply a complex relationship between the rates of interest, their expected variations, and the demand for money. A rigorous analysis of this relationship implies taking into account at least two important elements: the duration and the breakeven point, and the type of financial-market participants. If these are taken seriously, then there is no reason to think that there is a straightforward relation between the demand for money and the current interest rate given expectations. An upward sloping curve is as conceivable as a downward or a flat curve even if expectations are given. This conclusion has already been reached by Kahn (1954) and more recently by Kregel (1998).

The first part of this article introduces the notions of duration and of breakeven point. The second part looks at the implication of the previous part for the slope of the demand for money regarding interest rates.

1. The Importance of the Notion of Breakeven Point: A Post Keynesian View

Modern portfolio management includes many different tools to try to deal with different risks, among those the notions of duration and breakeven point try to deal with liquidity risk. As shown below, these notions were already present in Keynes’s *General Theory*. In order to understand the importance of the notion of breakeven point for economic theory, it is first necessary to present rapidly some basic concepts of financial theory (Fabozzi 1993).

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1 This led Robertson and Hicks (1946, 164) to argue that Keynes’s theory of the rate of interest hangs on its own bootstraps and led Hicks (1946), Kaldor (1939) and Kalecki to propose an alternative explanation of the yield curve based on expectations of monetary policy. However, Kahn (1954), Robinson (1953) and Kregel (1998) have criticized this approach, and showed the importance of the expectations of the long-term rate for the determination of the current level of long-term rate. Moreover, Keynes already took into account the expectations of the short-term rate for the determination of the long-term rate.
1.1. Basis Financial Management Concepts

1.1.1. Yield to Maturity

A bond provides a yield rate $\bar{i}$ that is a function of the coupons, the interest income on the coupons (or “reinvestment income”), and the resale value of the bond relative to the price paid for it (capital gain or loss). The fair price $V$ of a fix-rate bond $T$ periods of time before maturity is defined by:

$$V \equiv \sum_{t=1}^{T} \frac{C}{(1+i)^t} + \frac{M}{(1+i)^T}$$

with $C$ the periodic coupon payment, $i$ a given periodic market rate, $M$ the par value (or value at maturity), and $T$ the number of coupon-payment periods before maturity. In the following, there is one coupon payment per year. $\bar{i}$ is called the “yield to maturity,” which, because of arbitrage, is assumed to be equal to the market yield at the time of purchase and represents the yield rate that can be obtained at maturity if all coupons are placed (i.e. “reinvested”) at this rate.

The actual reinvestment rate will not necessarily be $\bar{i}$ during the holding period if the required market rate for the same class of risk changes in time. In addition, if one sells before maturity, the resale price is not known and it will depend on the market yield prevailing in the future. The actual yield $\bar{i}$, then, may be either higher or lower than $\bar{i}$.

Therefore, individuals may want to protect themselves against the possibility that $\bar{i} < \bar{i}$. In order to do so, they need to know how the actual yield rate will vary for a given change in $\bar{i}$, and so the sensitivity of the bond price and of the reinvestment income to changes in $\bar{i}$. Indeed, when the market yield rate varies, the income earned on coupons and the fair price change in an opposite direction: a decrease in the rate of interest diminishes the interest income earned on coupons but raises the fair price. An individual, then, may be interested in two kinds of questions that are related. A first set of questions is related to a short-term strategy: what is the sensitivity of the fair price to a change in the rate of interest? Below which volatility of the interest rate is it profitable to buy a bond in the short term? A second type of question is more related to a long-term strategy: How long will it take to be sure to get the yield to maturity or any other targeted yield, whatever the changes in the market yield? In order to answer these questions the notions of breakeven point and duration are very important.

1.1.2. Duration and Breakeven Point

The breakeven point reflects the absolute or relative variation in interest rate for which the capital loss (gain) is exactly compensated by the total gain (loss) from the reinvestment income. This breakeven point can be calculated for different periods of time. The duration term is the time necessary for a capital gain (loss) to be exactly compensated by a reinvestment income loss (gain) so that the actual rate of return is at least equal to a targeted rate: it is the time necessary to reach the breakeven point.
The calculation of the sensitivity of the fair price to the rate of interest requires to
determine the duration of the bond, and to deduce from it the modified duration that
measures the volatility of a bond price. The duration of a bond is equal to (see appendix A):

\[
D = \frac{C}{Vi} \left( \frac{1-(1+i)^{-T}}{1-(1+i)^{-1}} - \frac{T}{(1+i)^T} \right) + \frac{M}{V} \frac{T}{(1+i)^T}
\]

The coupon being paid yearly, this gives a measure in terms of years, and, for all bonds
except zero-coupon bonds, the duration term is always inferior to the time to maturity.
For perpetual bonds \((T \rightarrow \infty)\), the duration is given by

\[
D_P = \frac{1 + i}{i}
\]

and the modified duration is

\[
MD = \frac{D_P}{1 + i} = \frac{1}{i}
\]

Knowing that the fair price of a consol is given

\[
C/i
\]

and knowing that \(dV/V = -MD \cdot di\), then, for a given variation in the market yield, one has:

\[
\Delta V \approx -(C/i^2) \cdot \Delta i.
\]

Of course, for consols, the first derivative of the price gives the same result and no
duration calculation is necessary. But, for more complex bonds, the result is not
straightforward and the formula of duration provides an easy way to approximate the
sensitivity of a bond price.

In addition, the calculation of the duration allows implementing what is called in
portfolio management an “immunization strategy.” Indeed, one portfolio strategy consists
in targeting a yield rate \(\bar{i}\) (and so a certain some of money) for a given holding period, and
to buy bonds that have a duration equal to the holding period for the targeted interest rate.
This will guarantee that reinvestment income and capital gain will compensate at least for
each other so that the actual yield obtained (and so the actual amount of money obtained)
will be equal or higher to the targeted rate (amount of money). We know that the
reinvestment income obtained at the time a bond is sold is:

\[
RI = C \cdot \left[ \frac{(1+i)^h - 1}{i} \right] - hC
\]

with \(h\) the holding period. Therefore:

\[
\Delta RI \approx (C/i^2) \cdot [1 + hi(1 + i)^{h-1} - (1 + i)^h] \cdot \Delta i
\]

And so for \(h = D_P\), \(\Delta RI \approx -\Delta P \approx (C/i^2) \cdot \Delta i\). Thus, one can conclude that, if the rate of
interest goes below (above) a targeted rate, capital gains (losses) will be realized and more (less)
than offset the reinvestment income losses (gains) so that the actual yield \(\bar{i}\) will be
superior to \(\bar{i}\). If \(i\) stays the same then the yield obtained will be \(i = \bar{i}\) (see Appendix B)

The notions of breakeven point and duration are, thus, important to try to cope with
liquidity risk induced by an unforeseen capital losses or an unforeseen decrease in interest
rates. This can also be applied at the balance-sheet level in order to match the cash flows
from the asset and liability sides. Keynes was the first to show the importance of these
notions for economic theory.

\[2\] \text{For large variations in } i, \text{ a better approximation is obtained by adding the convexity (see appendix A).}
1.2. The Breakeven Point in the *General Theory*

The notion of breakeven point is essential to the understanding of the *General Theory*, and was presented in the following way by Keynes:

> Every fall in \( i \) reduces the current earnings from illiquidity, which are available as a sort of insurance premium to offset the risk of loss on capital account, by an amount equal to the difference between the squares of the old rate of interest and the new. For example, if the rate of interest on a long-term debt is 4 per cent., it is preferable to sacrifice liquidity unless on balance of probabilities it is feared that the long-term rate of interest may rise faster than by 4 per cent. of itself per annum, *i.e.* by an amount greater than 0.16 per cent. per annum. (Keynes 1936, 202).

The “square rule” (Kregel 1998) implies that a person who expects that the level of the rate of interest will increase by more than its square in absolute terms should increase his/her preference for money. Indeed, say that an individual bought a perpetual bond and decides to sell it after one coupon period. The nominal return obtained is:

\[
R = C + \Delta V
\]

Which is approximately equal to:

\[
R \approx C - (Ci^2)\Delta i
\]

Consistent with Keynes’s liquidity preference theory in which money rules the roost, one placement strategy consists, then, in determining what change in the nominal market rate is expected to lead to a null nominal return \( (E(R) = 0) \):

\[
C - (Ci^2)E(\Delta i) = 0 \Rightarrow E(\Delta i) = i^2
\]

Thus, for a consol, the short-term breakeven point (corresponding to one coupon period) is reached when the level of the rate of interest varies by its square (*i.e.* when it grows at the level of itself). Therefore, if \( i \) is expected to increase and \( E(\Delta i) < i^2 \), then holding a bond will provide a net gain in the short run because the capital loss is expected to be inferior to the income gain.

As shown above, a longer-term strategy consists in targeting a yield rate for a given holding period, and to buy bonds that have a duration equal to the holding period for a targeted interest rate. This latter strategy is, however, not what Keynes is concerned about because he deals with short-term liquidity problems, whereas the targeting strategy does not assure that the absolute amount of money available before the strategy unfold will be enough to face liquidity problems. Stated alternatively, the targeting strategy assumes that an individual does not have any problem of liquidity before the duration term is complete, but, if this not the case, an individual may have to sell his bonds before the duration period comes to an end and, therefore, may face losses. The timing of events is essential and is a major source of uncertainty. Therefore, even a person involved in the second strategy should care about the first strategy. The short-term breakeven point is, thus, very important for the determination of the current demand for money.
2. Breakeven Points and the Shape of the Liquidity Preference Curve

2.1. The Traditional Speculative Demand for Money

The traditional argument that justifies the downward sloping curve goes as follows. Suppose that there is an increase in the money supply, then, given the normal rate of interest ($i_n$), this generates an excess money supply that pushes some individuals to switch to bonds. As the price of bonds goes up, the difference between $i$ and $i_n$ grows so that some bullish or indifferent agents become bearish and increase their desired demand for money ($L$). Progressively, the price of bonds continues going up (and with it $L$) but at a decelerating rate so that “the market price will be fixed at the point at which the sales of the ‘bears’ and the purchases of the ‘bulls’ are balanced” (Keynes 1936, 170). This process of equilibrium is smooth and more or less fast depending on the elasticity of $L$ relative to $i$.

From the previous explanation, Keynes argues that “[a]s a rule, we can suppose that the schedule of liquidity-preference relating the quantity of money to the rate of interest is given by a smooth curve which shows the rate of interest falling as the quantity of money is increased” (Ibid., 171). Further, he qualifies this more clearly: “in any given state of expectation a fall in $i$ will be associated with an increase in $M_2$” (Ibid., 202) and “$M_2$ may tend to increase almost without limit in response to a reduction in $i$ below a certain figure” (Ibid., 203). Then, for Keynes, a downward sloping curve can be justified because, given expectations, as the interest rate falls the risk of illiquidity increases (the bearishness of the financial community increases and so its propensity to hoard) whereas the reward for being illiquid diminishes (which decreases the incentive to become or to stay illiquid) (Keynes 1936, 202). However, this is a very rough simplification of the role of given expectations and Keynes was aware of this and recognized that “a given $M_2$ will not have a definite quantitative relation to a given rate of interest” (Ibid., 201). In order to understand why, it is necessary to look at the way individuals form their expectations; this implies looking first at the difference between the current rate, the expected rate and the normal rate.

2.2. Normal Rate of Interest, Expectations and Current Rate of Interest

Because the economic situation is uncertain, individuals rely on past experiences and consider the current predominant view regarding the economic situation as the relevant one. In the bond market, this leads to the establishment of a convention that determines the “normal” or “safe” rate of interest. This normal rate of interest is a function of the expectations about the future monetary policies, and about the future levels of the long-

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3 If, before any change in interest rates, everybody is already bull or already bear then the interest rate is undetermined and the elasticity of money demand relative to interest rate is zero. Indeed, all the financial wealth is already devoted toward trying to buy bonds (so that $L = 0$ at the aggregate level) or to sell bonds ($L$ is at its maximum at the aggregate level), therefore, additional changes in the rate of interest will not have a large effect on the desired demand for money for speculation. As Kahn stated it, unanimous opinion is only “a sufficient condition […]. The necessary condition is a high concentration, measured in terms of wealth, in the neighborhood of the margin, of persons with identical view: idiosyncratics can be ignored if their idiosyncracies are based on complete conviction.” (Kahn 1954, 87).
term interest rate (Ibid., 202-203). Then, there is a functional relationship between the rate of interest \( i \), the expected rate \( E(i) \), and the normal rate (Chick 1983):

\[
E(i) - i = f(i_n - i) \quad \text{or} \quad E(\Delta i) = f(i_n - i)
\]

The expectation regarding the rate of interest, and more importantly its expected variation, is a function of the spread between the normal rate and the current rate of interest. If the rate of interest is below the safe rate, an individual who agrees with the convention prevailing in the market will anticipate an increase of the rate of interest \( (E(i) > i) \) toward the normal rate.\(^4\) If an individual expects the rate of interest to go up, it seems relevant for her/him to increase his speculative stock of money because of expected capital losses. Thus, there is a positive relationship between the demand for money and the expected rate of interest, whereas the relationship is negative when the current rate of interest is considered. The question is, then, what will be the impact on the demand for money if the current rate of interest goes up but is expected to go down in the short term? Will the speculative demand for money go up or down? In order to respond, Keynes introduces the notion of breakeven point.

2.3. Implications for the Speculative Demand for Money

In the following, it will be assumed that the horizon of expectations is equal to the coupon period so that bond-market participants compare the change in price \( \Delta V \) to the periodic coupon income \( C \) and the total return is \( R = C + \Delta V \). In addition, all the reasoning is done from the point of view of a bond holder (or a potential buyer) who checks the net benefit of keeping (buying) a bond today and gaining some future income, and potential capital gains, but also facing some potential capital losses.\(^5\)

Say that the interest rate changes to a new level. It is important to note that a current rise (decrease) in interest rates does not imply that an individual will buy (sell) bonds. Indeed, if e.g. \( \Delta i > 0 \), the latter may not be permanent: interest rates may go higher or go back to their previous rate or lower. As shown earlier, an individual is indifferent between bonds or money when he expects that \( E(\Delta i) = i^2 \) even though interest rates changed. In any other cases, the person is not indifferent. One thus needs to take into account the expectations for future interest rate before buying a bond to be reassured that the future total return will be positive. From the point of view of a bond holder (or

\(^4\) “[T]he individual, who believes that future rates of interest will be above the rates assumed by the market, has a reason for keeping actual liquid cash, whilst the individual who differs from the market in the other direction will have a motive for borrowing money for short periods in order to purchase debts of longer term.” (Keynes 1936, 170). If everybody agrees with the convention in the market then \( E(\Delta i) = i_n - i \), but otherwise the expected change will depend only on the difference. This is so because \( i_n \) is an anchor for individuals’ expectations but it does not imply that everybody agrees with \( i_n \): individuals adjust their expectation relative to the market convention.

\(^5\) The reasoning could also be done from the point of view of a bondholder who checks the benefit of selling today. In this case, he compares the cost of losing some income and missing potential capital gains to the benefit of avoiding capital losses. Thus, if he expects that \( E(\Delta i) > |i^2| \), he should not sell today because the net cost is positive: misses capital gains and losses income.
potential buyer), Figure 1 shows that there are five cases possible for a current increase in the rate of interest (and five others for a decrease):

Figure 1: Possible Financial Positions of an Individual (Excluding Indifference)

- but $E(\Delta i) > \bar{i}^2$: expectation of net loss (expected capital loss > income gain): case 1 ($\Delta L > 0$) (BEAR)
- but $E(\Delta i) < \bar{i}^2$: expectation of net gain (expected capital loss < income gain): case 2 ($\Delta L < 0$)
- but $E(\Delta i) < -\bar{i}^2$: expectation of net gain (expected capital gain < income gain): case 3 ($\Delta L < 0$) (BULL)
- but $E(\Delta i) > -\bar{i}^2$: expectation of net gain (expected capital gain > income gain): case 4 ($\Delta L < 0$)
- and $E(\Delta i) = 0$: expectation of permanent increase in $i$ ($\Delta i_n > 0$ so that $i = i_n$): case 5 ($\Delta L = ?$)

Thus, a bear will hold (buy) some consols today except if s/he expects a net loss in the short term: $\Delta V > C$ (case 1). In cases 2, 3, and 4, holding a bond today gives a net benefit, thus, only case 1 leads to an increase in the demand for money today. As interest rates increase, the bearishness of individuals decreases (as large increase in $i$ become less probable) and their bullishness may actually start to take over: more individuals are in cases 3 or 2. For sufficiently high interest rates, bullishness prevails and is strengthened: more and more people are in case 4. Thus, it is fair to assume, as a first approximation, that following an increase in the rate of interest, $M_2$ will decrease because the net gain from holding a bond is positive.

One limit to this conclusion, however, is that it ignores the case of low interest rates; “low” depends on the type of bond involved and can actually be quite high (Kregel 1998, 130). In this situation, cases 1 and 4 have a higher chance to occur, simply because an absolute variation of the interest rate by more than the square of its level is easier to get for low interest rates. Then, the demand for money (bond) can be quite unstable.

In case 5, for which the normal rate adjusts totally to the change in $i$, the effect on the demand for money will depend on the interest- and capital-risk sensitivity of bond holders as shown below. Gentlemen, will be pleased as long as long as the decrease in their net wealth in not too affected; even in the latter case they will stay in bond because they cannot do anything about the loss of wealth. Fund managers will prefer to switch to money and find more active assets, at least as long as the potential loss resulting from the permanent increase in $i$ is not too high, in which case they are stuck with bonds; in any case their preference for money increases (Chick 1983, 207).

Thus, by taking into account the short-term-strategy breakeven point, one can see how the current rate of interest and the normal rate of interest interact to determine the speculative demand for money. The idea that the speculative demand for money is necessarily, or usually, a decreasing function of money is not true even if expectations are given. This indeterminacy of the curve is even more evident when one includes the role of the precautionary motive in the demand for money as a store of wealth. Indeed, for example, how will an indifferent person allocate his portfolio? When one tries to answer this question, the nice “smooth curve” totally disappears.
2.4. Income Risk, Capital Risk and Demand for Money\(^6\)

Keynes recognizes in the *General Theory* that, for an individual, “the transaction-motive and the precautionary-motive are not entirely independent of what he is holding to satisfy the speculative-motive” (Ibid., 199). Kahn is the first to take this remark seriously. Drawing on Robinson (1953), he redefines the precautionary motive and the speculative motive to provide a better explanation of liquidity preference, and of the way actors allocate their portfolio between liquid and illiquid assets.

2.4.1. Microeconomic Implications

A person holds monetary assets (bonds and/or money) for precautionary motive because he thinks that the interest rate will move but he does not know how, or the degree of conviction in his expectation about \( E(\Delta i) \) is low. This precautionary motive implies making a distinction between income-risk sensitivity and capital-risk sensitivity. Following Robinson (1953) and Kahn (1954), indeed, one can make a difference between two types of economic agents; both are concerned with the liquidity of their positions but in different ways. On one side, there are income-risk sensitive agents who buy assets to earn an income (interest, dividend, rent). These “gentlemen” prefer to hold securities in their portfolios because “the fear of income loss more than offsets a mild change of capital loss” (Ibid., 82). On the other side, there are capital-risk sensitive persons who buy financial assets to trade them and make capital gains; they are afraid of increases in interest rates\(^7\) and of missing an opportunity of capital gains by buying too late. This is the typical behavior of “fund managers.” The precaution of fund managers pushes them to stay liquid because “the fear of capital loss more than offsets a mild change of capital gain” (Ibid., 83).

In total, the precautionary motive works differently for the persons who are concerned with income changes (gentlemen) compared to the persons who are concerned with capital-value changes (fund managers). This has an effect on the speculative motive. For example, a fund manager will not place in bonds unless expectations of capital gains are strong enough (case 4); that is to say, until \( i \) is high and is expected to go down steeply during the coupon period. Inversely, a gentleman will not sell bonds until the income gain is more than compensated by an expected capital loss (case 1). If the expected capital loss is small (case 2), a gentleman will stay in bonds whereas a fund manager will switch to money.\(^8\)

Of course, the intensity with which the speculative and the precautionary motive affect portfolio arbitrages depends on the degree of conviction with which an individual holds his expectation. The higher the weight of argument, the more an individual will have the courage to act upon the argumentation that sustained his expectation of the future.

\(^6\) What follows relies on Kahn (1954).

\(^7\) This is so for two reasons: first, an increase in interest rate raises the cost of the funds they borrow to speculate, and, second, it puts a downward pressure on the price of the assets they hold.

\(^8\) This is all the more the case that the increase in interest rates has not been anticipated.
Thus, for a given degree of conviction, and following an increase in interest rate ($\Delta i > 0$) we have Figure 2.

Figure 2: Microeconomic Impacts an Increase in $i$ on the Demand for Money

Precautionary motive

Income-risk (preference for securities)  Capital-risk (preference for money)

If IR does not totally dominate the speculative motive has “something to bite on” (Kahn 1954, 87)

(IR > CR)       (CR > IR)

Cases 2, 3, 4, 5   Case 1               Indifference        Case 4   Cases 1, 2, 3, 5
($\Delta L < 0$)  ($\Delta L > 0$)            ($\Delta L = 0$)        ($\Delta L < 0$)              ($\Delta L > 0$)

The speculative does not play any role in the portfolio arbitrage (Ibid. 84, 85). However, a person may have a slight preference for holding some financial assets because they offer interest (Ibid. 83).

The precautionary motive reinforces the speculative motive toward higher toward bullishness ($\Delta L << 0$)

The net effect depends on the degree of conviction of the individual regarding his or her expectations. If complete, the precautionary motive plays no role (a bear only holds securities and a bull only holds money as wealth). (Ibid. 83, 87). If complete uncertainty or low confidence, the precautionary motive dominates (Ibid. 84).

The precautionary motive reinforces the speculative motive toward bearishness ($\Delta L >> 0$)

One can see that the net effect of a variation of the rate of interest on the demand for money as a store of wealth (including both speculative and part of the precautionary motive) depends on the “interplay” of the precautionary motive and the speculative motive. The total effect can then be opposite to the traditional case. This straightforward relation is true only in one case and is completely reversed in another case.
This consideration is reinforced if one takes into account the strength of relative income-risk sensitivity (IR/CR) and the role of the degree of conviction. The lower IR/CR, the higher the capacity of the speculative motive to be operative for an individual is. The higher the degree of conviction, the lower the role of the precautionary motive (risk-sensitiveness decreases), and so the higher the responsiveness of money demand to the interest rate if the gentleman-side do not completely dominate (otherwise the speculative motive does not operate\(^9\)). Therefore, one can see that even if expectations are held constant, there is no straightforward relation between the rate of interest and the demand for money at the individual level.

2.4.2. Macroeconomic Implications

At the macrolevel, when the variety of opinions and the proportion of gentlemen relative to fund managers are taken into account, in addition to the degree of conviction and the strength of relative income sensitivity (e.g. some fund managers may be more capital-risk sensitive than others: hedge funds compared to pension funds), the straightforward relation is even more doubtful. Indeed, the normal rate of interest becomes endogenous as the prevailing convention about the future changes. However, even if the normal rate is given, here again there is an indeterminacy because, even if the market is mainly bullish (bearish), the demand for money can still go up (down) following an increase in the rate of interest. Assuming that there are mainly gentlemen, Figure 3 shows the different possible cases:

\[
\begin{align*}
\Delta L &< 0 \\
\Delta L &> 0 \\
\end{align*}
\]

On the contrary, if the market is mainly composed of fund managers, the following configurations in Figure 4 are possible:

\[
\begin{align*}
\Delta L &< 0 \\
\Delta L &> 0 \\
\end{align*}
\]

\(^9\) This situation is hardly probable as people are always concerned about their solvency, even pure rentiers.
Thus, bulls can contribute to the depressive effect on the market price if the expected rise in price is not high enough, and if the majority of the financial community is composed of fund managers. Accordingly, bears can contribute to the boom in the financial market if the financial community is mainly composed of gentlemen. In total, bears and bulls do not need to balance each other: in some periods they can complement each other.

In both type of markets, when bulls and bears compensate each other, the ultimate effect on the willingness to hoard is undetermined even if finite. The ultimate effect will depend on the degree of confidence with which bears and bulls hold their expectations and their relative income-risk sensitivity, but the volatility of the market will be higher when the market is composed mainly of fund managers.

The aggregate demand for money has, thus, an elasticity that depends on the degree of conviction of market participants, the degree of heterogeneity in the market (the financial power of bears relative to the financial power of bulls), the strength of relative income-risk sensitivity (as it operates on the precautionary motives of each individual), as well as the strength of expectations reflected by the level of $E(\Delta i)$ (i.e. the level of $i$ relative to the level of $i_n$). If financial actors have a strong income-risk sensitivity, most of them are buyers when interest rates increase except in one case for which it depends on the strength of expectations and the relative financial power of bulls and bears. If there is a vast majority of fund managers in the market, an increase in the rate of interest may have a negative impact (the ultimate effect depending on the relative income-risk sensitivity of fund managers).

Given expectations, one cannot have, therefore, clear-cut results for the impact of the long-term rate on the demand for money. However, one can also see that as a strength because the preceding allows knowing in which case a particular elasticity is more probable to be true. If the financial actors are mainly fund managers, the speculative liquidity function may be positively related to the long-term interest rate.

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10 Two bulls who expect the same change in $i$ but who have different degrees of conviction and different relative risk-income sensitivity, will have different commitments in the financial markets.
3. Conclusion

The preceding aimed at showing that a “smooth” downward sloping curve for the speculative demand function is a rough simplification as Kahn already concluded (Kahn 1954, 90). Even in a stable financial market there is no smooth relationship between \( i \) and \( L \). Assuming so, leaves aside the importance of the market configuration and the role of a given state of expectation. One could argue that the traditional curve is drawn for a given state of expectation so that expectations are included in the analysis. However, the impact of this given state of expectation on the economy is not analyzed for itself. The given state of expectation is just assumed to allow drawing nice curves without looking at the implication of this assumption. Post Keynesians have often been criticized for emphasizing the role of uncertainty and expectations. These criticisms have been done mainly to defend the mathematical framework and led to qualify Post Keynesians as nihilist or unproductive because, by taking into account uncertainty, the curves would become unstable. However, this criticism misses the point. Post Keynesians do not need to assume that curves are unstable to criticize this framework of analysis. What they say is that even if expectations are given it is necessary to look at their implication for the analysis of economic activity.

This has several implications in terms of theory and policy. In terms of theory three basically conclusion can be reached. First, Hicks’s conclusion that the Keynesian system is a “special case,” for which the demand for money is horizontal, is clearly incorrect. This version of the liquidity trap, even if a theoretical possibility, was not seen by Keynes as practically relevant. What is relevant is the relation between current interest rate and normal interest rate, and the way this affects expectations (Kregel 2003). Second, more generally, the Marshallian analysis of the demand for money can only capture a simplistic version of the liquidity preference theory, as presented in chapters 13 and 15 of the General Theory. A better version of the liquidity preference theory is contained in chapter 17. Third, Fisher’s condition of indifference, \( i = r + E(\pi) \), leaves aside the importance of capital gain/loss and so may not protect the liquidity and solvency (and so purchasing power) of a bond holder: increase in interest rates may not be good for bond holders. Finally, this has implication in terms of policy, because it shows that under some financial market configurations, moving interest rates frequently creates more instability than stability (“fine tuning”); this is notably the case when the financial markets are mainly composed of funds managers.

REFERENCES


APPENDIX A

DURATION, CONVEXITY

WE KNOW THAT:

\[ V \equiv \frac{C}{(1+i)} + \frac{C}{(1+i)^2} + \cdots + \frac{C+M}{(1+i)^T} \]

which is a convex relationship between the fair price and the interest rate. Taking the first derivative, we have:

\[ \frac{dV}{di} = -\frac{C}{(1+i)^2} + \frac{-2C}{(1+i)^3} + \cdots + \frac{-T(C+M)}{(1+i)^{T+1}} \]

By factorizing by \(-1/(1+i)\) we have:

\[ \frac{dV}{di} = -\frac{1}{1+i} \left[ \frac{C}{(1+i)} + \frac{2C}{(1+i)^2} + \cdots + \frac{T(C+M)}{(1+i)^T} \right] \]

Let us note \(D\) the following:

\[ D \equiv \frac{C}{(1+i)} + \frac{2C}{(1+i)^2} + \cdots + \frac{T(C+M)}{(1+i)^T} \]

Then, it follows that:

\[ \frac{dV}{di} = -\frac{1}{1+i} \times V \times D \]

Therefore:

\[ \frac{dV}{V} = -\frac{D}{1+i} \times di \quad \Leftrightarrow \quad D = -\frac{dV}{V} / \frac{1+i}{i} = e_{vi} \times \frac{1+i}{i} \]

Thus, the duration is measure of the fair-price sensitivity to a change in market yield. The next step is then to calculate the sum \(D\) the definition of duration. We know that:

\[ D = \frac{1}{V} \sum_{t=1}^{T} \frac{tC}{(1+i)^t} + \frac{M}{V} \frac{T}{(1+i)^T} \]

Say that:

\[ S = \sum_{t=1}^{T} \frac{tC}{(1+i)^t} \Rightarrow S(1+i) = \sum_{t=1}^{T} \frac{tC}{(1+i)^{t-1}} \]

Subtracting \(S\) to \(S(1+i)\) one has:

\[ S(1+i) - S = \sum_{t=0}^{T-1} \frac{C}{(1+i)^t} - \frac{T}{(1+i)^T} \]

Then, by applying the same procedure to \(\sum_{t=0}^{T-1} \frac{C}{(1+i)^t}\), we have:

\[ S = \frac{C}{i} \left[ \frac{1-(1+i)^{-T}}{1-(1+i)^{-1}} - \frac{T}{(1+i)^T} \right] \]

Therefore:
The duration of a consol is obtained by assuming that $T \to \infty$ and knowing that $V = Cl_i$ so that:

$$D = \frac{1+i}{i}$$

The duration of a zero-coupon bond is obtained by knowing that $C = 0$ and $V = \frac{M}{(1+i)^T}$ so that by replacing these values in $D$ one has:

$$D = T$$

The convexity of the bond is calculated by taking the second derivative of the fair price relative to the rate of interest and by factorizing by $1/(1+i)^2$ so that:

$$\frac{d^2V}{di^2} = \frac{1}{(1+i)^2} \left[ \frac{1\times2C}{(1+i)} + \frac{2\times3C}{(1+i)^2} + \cdots + \frac{T(T+1)(C+M)}{(1+i)^T} \right]$$

Let us call $Cx$ the following:

$$Cx = \frac{1\times2C}{(1+i)} + \frac{2\times3C}{(1+i)^2} + \cdots + \frac{T(T+1)(C+M)}{(1+i)^T}$$

Then, it implies that:

$$\frac{d\left(\frac{dV}{di}\right)}{di} = \frac{Cx}{(1+i)^2} \times V$$

Taking the integral, we have:

$$\int \frac{dV}{V} = Cx \cdot \int \frac{1}{(1+i)^2} di = -\frac{Cx}{1+i}$$

The change in price due to convexity can be added to the change in price induced by the duration to have a better approximation of a price change when changes in $i$ are large.

$$\Delta V/V \approx dV/V + [dV/V]_{Cx}$$

The calculation of $Cx$ can be done by having $S \equiv \sum_{t=1}^{T} \frac{t(t+1)}{(1+i)^T}$, taking the difference

$$S(1+i) - S = \sum_{t=1}^{T} \frac{2t}{(1+i)^{t+1}} - \frac{T(T-1)}{(1+i)^T}$$

In the end, we have:

$$S = \frac{2}{i} \left( \frac{1-(1+i)^{-T}}{1-(1+i)^{-1}} \right) - \frac{T(T+1)}{i(1+i)^{T-1}}$$

Therefore:

$$Cx = \frac{1}{V \cdot (1+i)^2} \left[ C \cdot S + \frac{T(T+1)M}{(1+i)^T} \right]$$
APPENDIX B
IMMUNIZATION PROCEDURE WITH CONSOLS

The fair price of a consol is \( V = \frac{C}{i} \) and so
\[
dV/di = -C/i^2
\]
After, having held a consol for \( h \) years and placed all the coupons at the same rate one will get:
\[
RI = c \cdot \left[ \frac{(1+i)^h-1}{i} \right] - hc
\]
By taking the first derivative relative to \( i \), we have:
\[
\frac{dRI}{di} = \frac{C}{i^2} (1 + ih(1+i)^{h-1} - (1+i)^h)
\]
Therefore, the change in the reinvestment income will be superior or inferior to the change in the price depending on the sign of \( ih(1 + i)^{h-1} - (1 + i)^h \). It is straightforward to show that, this difference is positive (negative) if \( i \) is superior (inferior) to \( 1/(h-1) \).
\[
ih(1 + i)^{h-1} - (1 + i)^h \geq 0 \iff i \geq 1/(h-1)
\]
Therefore, say that someone targets a rate \( \tilde{i} \), then, one needs to have a holding period equal to the duration, given the targeted rate:
\[
h = (1 + \tilde{i})/\tilde{i}
\]
Therefore,
\[
\frac{dRI}{di} = \frac{C}{\tilde{i}^2} (1 + i\tilde{i}(1+i)^{h-1} - (1+i)^h)
\]
Thus, if one holds a consol until duration, one is sure to secure at least a yield \( \tilde{i} \) because if \( i \) decreases relative to \( \tilde{i} \), \(-dRI < dV = C/i^2\); the capital gain will more than compensate the loss of income. Inversely, if \( i \) increases relative to \( \tilde{i} \), then \( dRI > dV \); the income gain after \( h \) years more than compensates for the capital losses. Therefore, if \( h = D_p \), there is relationship between the market yield rate and the difference between the actual yield rate and targeted yield rate for a coupon bond:
\[
threeaxis{axis}{\begin{scope}[thick]
\addplot[domain=0:12] {x^2};
\end{scope}}
\]

The convexity of the relation depends on the sensitivity of the price and reinvestment income to change in interest rate. The lower the coupon rate, the flatter the relationship and for zero-coupon bonds, the relationship is a flat relationship: \( \tilde{i} = \tilde{i} \) whatever the market yield. The higher the maturity, the higher the sensitivity of price to market change and so the higher the convexity and duration.