CHAPTER II

THE LAW OF CONSUMER'S DEMAND

1. We have now, from the conditions of equilibrium and the basic assumption of regularity, set out in the preceding chapter, to deduce laws of market conduct—to find out what can be said about the way the consumer will react when prices change. Discussion of equilibrium conditions is always a means to an end; we seek information about the conditions governing quantities bought at given prices in order that we may use them to discover how the quantities bought will be changed when prices change.

This stage of our investigation corresponds to the stage in Marshall's theory where he deduces the downward slope of the demand curve from the law of diminishing marginal utility. The particular way in which Marshall carries out that deduction is worth noting. He assumes that the marginal utility of money is constant. Therefore, the ratio between the marginal utility of a commodity and its price is a constant ratio. If the price falls, the marginal utility must be reduced too. But, by the law of diminishing marginal utility, this implies an increase in the amount demanded. A fall in price therefore increases the amount demanded. This is the argument we have to reconsider.

What is meant by the marginal utility of money being constant? Making our translation, it would appear to mean that changes in the consumer's supply of money (that is, with respect to the problem in hand, his income) will not affect the marginal rate of substitution between money and any particular commodity X. (For the marginal rate of substitution equals the ratio of the marginal utilities of X and money.) Therefore, if his income increases, and the price of X remains constant, the price of X will still equal the marginal rate of substitution, without any change in the amount of X bought. The demand for X is therefore independent of income. His demand for any commodity is independent of his income.

1 This, of course, abolishes any distinction between the diminishing marginal utility of a commodity and the diminishing marginal rate of substitution of that commodity for money. Consequently, it explains why Marshall was satisfied with diminishing marginal utility.
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It will appear in what follows that this is actually what the constancy of the marginal utility of money did mean for Marshall; not that he really supposed that people's demands for commodities do not depend upon their incomes, but that in his theory of demand and price he generally neglected the income side. We shall find that he had quite good reasons for doing so, that the constancy of the marginal utility of money is in fact an ingenious simplification, which is quite harmless for most of the applications Marshall gave it himself. But it is not harmless for all applications; it is not always a good thing to be vague about the effects of changes in income on demand. There are distinct advantages to be gained from having a theory of value in which the relations of demand, price, and income are all made quite clear.

2. Let us now revert to the indifference diagram, and begin by investigating the effects of changes in income. We shall go on to investigate the effects of price-changes later, but price-changes will be easier to deal with if we examine the effects of income-changes first. Let us therefore continue to suppose, as in the last chapter, that the prices of X and Y are given, but now suppose the consumer's income to vary.

We have seen before that if his income is OL (measured in terms of X) or OM (measured in terms of Y), the point of equilibrium will be at P, where LM touches an indifference curve (Fig. 5). If now his income increases, LM will move to the right, but the new line L'M' will still be parallel to LM, so long as the prices of X and Y are unchanged. (For, then, \( OM'/OL' = OM/OL \), the unchanged price-ratio.) The new point of equilibrium will be at P', where L'M' touches an indifference curve.

As income continues to increase, L'M' continues to move to the right, and the point P' traces out a curve, which we may call the income-consumption curve.\(^1\) It shows the way in which consumption varies, when income increases and prices remain unchanged. Through any point P on the diagram an income-consumption curve could be drawn; thus there will be an income-consumption curve corresponding to each possible system of prices.

What can be said about the form of the income-consumption curve? Mere experience in drawing diagrams is enough to convince

\(^1\) In 'A Reconsideration of the Theory of Value' I called this the expenditure curve. It was clearly a bad name.
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one that it will ordinarily slope upwards and to the right; but that, it is not enough to show that it will necessarily behave in this way.

In fact, there is only one necessary restriction on its shape. An income-consumption curve cannot intersect any particular indifference curve more than once. (For if it did so, that would mean that the indifference curve had two parallel tangents—which is impossible, if the indifference curves are always convex to the origin.) Consequently, while there is most 'room' for the income-consumption curves to slope upwards and to the right,

it is also possible for them to creep round to the left or downwards (PC₁ or PC₂ in Fig. 6) without ever cutting an indifference curve more than once.

And clearly that is as it should be. Curves such as PC₁ do occur. They are found whenever the commodity X is an 'inferior' good, largely consumed at low levels of income, but replaced, or partially replaced, by goods of higher quality when income rises. Margarine is obviously a case in point; its inferiority is well attested by statistical investigation.¹ But it can hardly be doubted that there are a great many others. Most of the poorer qualities of goods offered for sale are probably, in our sense, inferior goods.²

¹ Cf. Allen and Bowley, Family Expenditure, pp. 36, 41.
² It is a curious illustration of the muddle into which the theory of value was liable to fall, so long as the principle of diminishing marginal utility was not wholly abandoned, that that principle can easily be interpreted in a way which
Although the diagrammatic apparatus we have just been using is onlyvalid for the case of two goods (X and Y), it is evident that a similar argument must hold however many are the goods among which income is being distributed. If income increases, and the increased income is spent, then there must be increased consumption in some directions, perhaps most directions or even all; but it is perfectly possible that there will be a limited number of goods whose consumption will be actually diminished. This is a very negative result and obviously needs no further elaboration.

\[\text{Fig. 6.}\]

3. Let us now pass on to consider the effects of a change in price. Here again we begin with the case of two goods. Income is now to be taken as fixed, and the price of Y as fixed; but the price of X is variable. The possibilities of consumption now open are represented on the diagram (Fig. 7) by straight lines joining M (OM is income measured in terms of Y, and is therefore fixed) to points on OX which vary as the price of X varies. Each price would exclude inferior goods from any place in economics. This interpretation was actually put forward by Pareto at one period in the development of his ideas (Manuale di economia politica, pp. 502–3; but cf. the later French edition, pp. 373–4). Instead of relying solely upon the true principle of diminishing marginal rate of substitution (that the rate will diminish when X is substituted for Y along an indifference curve), he put forward also what we may now justly regard as a false principle—that the marginal rate of substitution of X for Y will diminish when the supply of Y is reduced without any increase in the supply of X. If this were always true, it would exclude the possibility of X being an inferior good. Therefore this principle of Pareto's cannot be always satisfied.
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of $X$ will determine a line $LM$ ($OL$ increasing as the price falls); and the point of equilibrium corresponding to each price will be given by the point at which the line $LM$ touches an indifference curve. The curve $MPQ$ joining these points may be called a price-consumption curve. It shows the way in which consumption varies, when the price of $X$ varies and other things remain equal.

Starting off from a particular position of $LM$, we have thus two sets of straight lines, and corresponding points of contact. We have the lines parallel to $LM$, whose points of contact trace out the income-consumption curve. We have the lines passing through $M$, whose points of contact trace out the price-consumption curve. Any particular indifference curve must be touched by one line from each of these sets. Take an indifference curve $I_p$, which is higher than the indifference curve $I_1$, touched by $LM$. The curve $I_2$ is touched by a line parallel to $LM$ at $P'$, by a line through $M$ at $Q$. Now it is at once obvious from the diagram (it follows from the convexity of the indifference curve) that $Q$ must lie to the right of $P'$. This property must hold for all indifference curves which are higher than the original curve; and it therefore follows that...
as we go up on to higher indifference curves the price-consumption curve through $P$ must always lie to the right of the income-consumption curve through $P$ (Fig. 8).

This proposition, which looks like a mere piece of geometry, turns out to have much economic significance, and to be indeed quite fundamental to a large part of the theory of value. Let us try to see its implications.

When the price of $X$ falls, the consumer moves along the price-consumption curve from $P$ to $Q$. We now see that this movement from $P$ to $Q$ is equivalent to a movement from $P$ to $P'$ along the income-consumption curve, and a movement from $P'$ to $Q$ along an indifference curve. We shall find it very instructive to think of the effect of price on demand as falling into these two separate parts.

A fall in the price of a commodity does actually affect the demand for that commodity in two different ways. On the one hand, it makes the consumer better off, it raises his 'real income', and its effect along this channel is similar to that of an increase in income. On the other hand, it changes relative prices; and therefore, apart from the change in real income, there will be a
tendency to substitute the commodity whose price has fallen for other commodities. The total effect on demand is the sum of these two tendencies.

The relative importance of these tendencies can be further shown to depend upon the proportions in which the consumer was dividing his expenditure between this commodity \((X)\) and other goods. For the extent to which he is made better off by a fall in the price of \(X\) will depend upon the amount of \(X\) which he was initially buying; if that amount was large relatively to his income, he will be made much better off, and the first effect (the Income Effect, we may call it) will be very important; but if the amount was small, the gain is small, and the income effect is likely to be swamped by the Substitution Effect.

It is this last point which is the justification of Marshall’s ‘constant marginal utility’. It will be observed that our two effects stand on a different footing as regards the certainty of their operation. It follows from the principle of diminishing marginal rate of substitution that the substitution effect is absolutely certain—it must always work in favour of an increase in the demand for a commodity when the price of that commodity falls. But the income effect is not so reliable; ordinarily it will work the same way, but it will work in the opposite way in the case of inferior goods. It is therefore a consideration of great importance that this unreliable income effect will be of relatively little importance in all those cases where the commodity in question plays a fairly small part in the consumer’s budget; for it is only in these cases (fortunately, they are most important cases) that we have a quite unequivocal law of demand. It is only in these cases that we can be quite sure that a fall in price will necessarily lead to a rise in the amount demanded.

Marshall concentrated his attention upon these cases; and therefore he neglected the income effect. He did this by means of his assumption that the marginal utility of money could be treated as constant, which meant that he neglected the effect on demand of the changes in real income which result from changes in price. For many purposes this was a quite justifiable simplification, and it certainly did simplify his theory enormously. It is indeed one of those simplifications of genius, of which there are several instances in Marshall. Economists will continue to use these simplifications, though their path is made safer when they know
exactly what it is that they are neglecting. We shall find, as we proceed, that there are other problems, not much considered by Marshall, that are made definitely easier when we are clear in our minds about the income effect.

4. The geometrical argument of the preceding section appears to apply only to the case when the consumer divides his expenditure between two commodities and no more; but it is not actually as limited as that. For suppose we regard $X$ and $Y$, not as bread and potatoes, or tea and margarine (physical commodities in that sense), but as bread (some physical commodity) for one, and general purchasing power (Marshall's 'money') for the other. The choice of the consumer is a choice between spending his money on bread or keeping it available for expenditure on other things. If he decides not to spend it on bread, he will subsequently convert it into some other form by buying some other commodity or commodities with it. But even if $Y$ were potatoes, it might still be converted into other forms, some of the potatoes being roast, some being boiled. These possibilities do not prevent us from drawing up a determinate indifference system for bread and potatoes. Similarly, so long as the terms on which money can be converted into other commodities are given, there is no reason why we should not draw up a determinate indifference system between any commodity $X$ and money (that is to say, purchasing power in general). The distribution of purchasing power among other commodities is exactly similar to the distribution of a commodity among various uses, which may take place even if there is only one other commodity in a physical sense.

This principle is of quite general application. A collection of physical things can always be treated as if they were divisible into units of a single commodity so long as their relative prices can be assumed to be unchanged, in the particular problem in hand. So long as the prices of other consumption goods are assumed to be given, they can be lumped together into one commodity 'money' or 'purchasing power in general'. Similarly, in other applications, if changes in relative wages are to be neglected,

1 It is, in fact, a consequence of the principle, noted at the end of the last chapter, that the marginal rate of substitution must diminish, for substitutions in any direction. (See Appendix, § 8 (4) and § 10.)
it is quite legitimate to assume all labour homogeneous. There will be other applications still to notice as we go on.¹

For the present, we shall only use this principle to assure ourselves that the classification of the effects of price on demand into income effects and substitution effects, and the law that the substitution effect, at least, always tends to increase demand when price falls, are valid, however the consumer is spending his income.

5. In all our discussions so far, we have been concerned with the behaviour of a single individual. But economics is not, in the end, much interested in the behaviour of single individuals. Its concern is with the behaviour of groups. A study of individual demand is only a means to the study of market demand. Fortunately, with our present methods we can make the transition very easily.

Market demand has almost exactly the same properties as individual demand. This can be seen at once if we reflect that it is the actual change in the amount demanded (brought about by a small change in price) which we can divide into two parts, due respectively to the income effect and the substitution effect. The change in the demand of a group is the sum of changes in individual demands; it is therefore also divisible into two parts, one corresponding to the sum of the individual income effects, the other to the sum of the individual substitution effects. Similar propositions to those which held about the individual effects hold about the group effects.

(1) Since all the individual substitution effects go in favour of increased consumption of the commodity whose price has fallen, the group substitution effect must do so also.

(2) Individual income effects are not quite reliable in direction; therefore group income effects cannot be quite reliable either. A good may, of course, be inferior for some members of a group, and not be inferior for the group as a whole; the negative income effects of this section being offset by positive income effects from the rest of the group.

(3) The group income effect will usually be negligible if the

¹ Beyond this, it does not seem necessary to worry about the definition of a 'commodity'. What collections of things we regard as composing a commodity must be allowed to vary with the problem in hand.
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6. We are therefore in a position to sum up about the law of demand. The demand curve for a commodity must slope downwards, more being consumed when the price falls, in all cases when the commodity is not an inferior good. Even if it is an inferior good, so that the income effect is negative, the demand curve will still behave in an orthodox manner so long as the proportion of income spent upon the commodity is small, so that the income effect is small. Even if neither of these conditions is satisfied, so that the commodity is an inferior good which plays an important part in the budgets of its consumers, it still does not necessarily follow that a fall in price will diminish the amount demanded. For even a large negative income effect may be outweighed by a large substitution effect.

It is apparent what very stringent conditions need to be fulfilled before there can be any exception to the law of demand. Consumers are only likely to spend a large proportion of their incomes upon what is for them an inferior good if their standard of living is very low. The famous Giffen case, quoted by Marshall, exactly fits these requirements. At a low level of income, consumers may satisfy the greater part of their need for food by one staple foodstuff (bread in the Giffen case), which will be replaced by a more varied diet if income rises. If the price of this staple falls, they have a quite considerable surplus available for expenditure, and they may spend this surplus upon more interesting foods, which then take the place of the staple, and reduce the demand for it. In such a case as this, the negative income effect may be strong enough to outweigh the substitution effect. But it is evident how rare such cases must be.

Thus, as we might expect, the simple law of demand—the downward slope of the demand curve—turns out to be almost infallible in its working. Exceptions to it are rare and unimportant. It is not in this direction that our present technique has anything new to offer.

7. But as soon as we pass beyond this standard case, we do begin to get some effective clarification.

Principles, p. 132.
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So far we have assumed the consumer’s income to be fixed in terms of money. What happens if this is not so, if he comes to the market not only as a buyer but also as a seller? Suppose he comes with a fixed stock of some commodity $X$, of which he is prepared to hold back some for his own consumption, if price-conditions are favourable to that course of action.

It is clear that so long as the price of $X$ remains fixed, our previous arguments are unaffected. We may suppose, if we like, that he exchanges his whole stock into money at the fixed price, when he will find himself in exactly the same position as our consumer whose income was fixed in terms of money. He can then buy back some of his $X$ if he wants to.

But what happens if the price of $X$ varies? The substitution effect will be the same as before. A fall in the price of $X$ will encourage substitution of $X$ for other goods; this must favour increased demand for $X$, that is to say, diminished supply. But the income effect will not be the same as before. A fall in the price of $X$ will make a seller of $X$ worse off; this will diminish his demand (increase his supply) unless $X$ is for him an inferior good.

The significant difference between the position of the seller and that of the buyer thus comes out at once. In the case of the buyer income effect and substitution effect work in the same direction—save in the exceptional case of inferior goods. In the case of the seller, they only work in the same direction in that exceptional case. Ordinarily they work in opposite directions.

The position is made more awkward by the fact that sellers’ income effects can much more rarely be neglected. Sellers usually derive large parts of their incomes from some particular thing which they sell. We shall therefore expect to find many cases in which the income effect is just as powerful as the substitution effect, or is dominant. We must conclude that a fall in the price of $X$ may either diminish its supply or increase it.

The practical importance of such a supply curve is no doubt most evident in the case of the factors of production. Thus a fall in wages may sometimes make the wage-earner work less hard, sometimes harder; for, on the one hand, reduced piece-rates make the effort needed for a marginal unit of output seem less worthwhile, or would do so, if income were unchanged; but on the other, his income is reduced, and the urge to work harder in order to
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make up for the loss in income may counterbalance the first tendency.¹

Such a supply curve will appear, however, whenever there is a possibility of reservation demand; that is to say, whenever the seller would prefer, other things being equal, to give up less, rather than more. The supply of agricultural products from not too specialized farms is thus another good example. Any such supply curve, drawn on a price-quantity diagram, is likely to turn back on itself at some point. We cannot be at all confident that it will be upward-sloping (Fig. 9).

That there existed this asymmetry between supply and demand has long been familiar; it should perhaps be reckoned as one of the discoveries of Walras.² But so long as the reason for the asymmetry was not made clear, it was rather too easy to forget its existence. To have cleared up this matter may be regarded as the first-fruits of our new technique. It is itself a good thing to have cleared up, and, we shall find as we go on, it opens the way to some very convenient analytical methods.

² Walras, Éléments d'économie politique pure (first published 1874), leçons 5–7.
The doctrine of Consumer's Surplus has caused more trouble and controversy than anything else in book iii of Marshall's *Principles*; the results we have just reached throw some light upon it; consequently, although it lies off the main track of our present inquiry, it may usefully be examined here.

Consumer's surplus is the one instance in this field where Marshall was, perhaps, just a shade too ingenious; but he was very ingenious, and we must be careful not to fall into the most common error of writers on this matter, which is to fail to give him the credit for the ingenuity he showed. We are dealing with one of those deceptive doctrines which appear to be a good deal simpler than they are. It can easily be stated in a way which is altogether fallacious; and it is easy to overlook the fact that Marshall did go to some considerable trouble in order not to state it in a fallacious way.

It is thus useful to begin by contrasting Marshall's argument with that of the original inventor of consumer's surplus—Dupuit. Dupuit, writing in 1844, gave a version that has none of Marshall's refinement.¹

¹ Dupuit's work appeared in the *Annales des Ponts et Chaussées*, and was thus
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He held straightforwardly that 'l'économie politique doit prendre pour mesure de l'utilité d'un objet le sacrifice maximum que chaque consommateur serait disposé à faire pour se le procurer' (p. 49), and therefore that the 'utility' secured by being able to purchase on units of a commodity at the price pm is given by the area dpk on the price-quantity demand diagram (p. 63). This without any qualification. Marshall uses the same diagram (Fig. 10) and arrives at the same result; but he makes the significant qualification that the marginal utility of money must be supposed constant.¹

The force of this can be readily shown on the indifference diagram, measuring, as before, the commodity X along one axis and money on the other (Fig. 11). If the consumer's income is OM, and the price of X is indicated by the slope of ML, which touches an indifference curve at P, ON will be the amount of X purchased, and PF the amount of money paid for it. Now P is on a higher indifference curve than M, and what is wanted is a money measure of this gain in 'utility'. Like Dupuit, Marshall takes 'the excess of the price which (the consumer) would be willing to pay rather than go without the thing, over that which he actually does pay'.² The price he actually does pay is measured on our diagram by PF, the price he would be willing to pay by RF, where very inaccessible until M. de Bernardis' elegant reprint entitled De l'utilité et de sa mesure (Turin, 1933), from which I quote.

¹ Marshall, Principles, p. 842,
² Ibid., p. 124.
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\( R \) lies on the same indifference curve as \( M \) (so that if he bought \( ON \) and paid \( RF \) for it, he would be no better off by making the transaction). Consumer's surplus is therefore the length of the line \( RP \).

\( RP \) is a perfectly general representation of consumer's surplus, independent of any assumption about the marginal utility of money. But it is not necessarily equal to the area under the demand curve in Marshall's diagram, unless the marginal utility of money is constant. This can be seen as follows. If the marginal utility of money is constant, the slope of the indifference curve at \( R \) must be the same as the slope of the indifference curve at \( P \), that is to say, the same as the slope of the line \( MP \). A slight movement to the right along the indifference curve \( MR \) will therefore increase \( RF \) by the same amount as a slight movement along \( MP \) will increase \( PF \). But the increment in \( PF \) is the additional amount paid for a small increment in the amount purchased at the price given by \( MP \), an amount measured by the area \( pnn'a' \) in Fig. 10. The length \( RF \) is built up out of a series of such increments, and must therefore be represented on Fig. 10 by the area built up out of increments such as \( pnn'a' \). This is nothing else than \( dpmo \).

\( RP \) will therefore be represented on Fig. 10 by \( dpk \)—Marshall's consumer's surplus.

This is valid so long as the marginal utility of money is constant—so long as income effects can be neglected. But how legitimate is it in this case to follow Marshall in neglecting income effects? This is not a case in which they can be very safely ignored. Marshall neglects the difference between the slope of the indifference curve at \( P \) and the slope of the indifference curve at \( R \). It is true that this difference is likely to be less important, the less important in the consumer's budget is the commodity we are considering. But the difference may still be important, even if the proportion of income spent upon the commodity is small; it will still be important, if \( RP \) itself is large, if the consumer's surplus is large, so that the loss of the opportunity of buying the commodity is equivalent to a large loss of income.

This is the weakness which remains even in Marshall's version of the consumer's surplus theory; but there is really no reason why it should be allowed to remain. We must remember that the notion of consumer's surplus is not wanted for its own sake; it is wanted as a means of demonstrating a very important proposition, which was supposed to depend upon it. However, in fact that proposition can be demonstrated without begging any questions at all.

As we have seen, the best way of looking at consumer's surplus is to regard it as a means of expressing, in terms of money income, the gain which accrues to the consumer as a result of a fall in price. Or better, it is the compensating variation in income, whose loss would just offset
the fall in price, and leave the consumer no better off than before. Now it can be shown that this compensating variation cannot be less than a certain minimum amount, and will ordinarily be greater than that amount. This is all that is needed.

Suppose the price of oranges is 2d. each; and at this price a person buys 6 oranges. Now suppose that the price falls to 1d., and at the lower price he buys 10 oranges. What is the compensating variation in income? We cannot say exactly, but we can say that it cannot be less than 6d. For suppose again that, at the same time as the price of oranges fell, his income had been reduced by 6d. Then, in the new circumstances, he can, if he chooses, buy the same amount of oranges as before, and the same amounts of all other commodities; what had previously been his most preferred position is still open to him; so he cannot be worse off. But with the change in relative prices, it is probable that he will be able to substitute some quantity of oranges for some quantities of other things, and so make himself better off. But if he can lose 6d. and still remain better off, 6d. must be less than the compensating variation; he would have to lose more than 6d. in order to be just as well off as before.\(^1\)

This is all that is necessary in order to establish the important consequences in the theory of taxation which follow from the consumer's surplus principle. It shows, for example, why (apart from distributional effects) a tax on commodities lays a greater burden on consumers than an income tax. If the price of oranges falls from 2d. to 1d. as the result of a reduction in taxation, then (assuming constant costs) the reduction in tax receipts from our particular consumer is 6d. If this is taken from him by an income tax, he is still left better off, and the government no worse off.

Other deductions which have been drawn from the consumer's surplus principle can presumably be tested out in a similar way.\(^2\)

\(^1\) The compensating variation can thus be proved to be greater than the area \(khph\) on Fig. 10. Can it also be proved to be less than the area \(ksp'h}\)? At first sight, one might think so; but in fact it is not possible to give an equally rigorous proof on this side. This comes out clearly if we use the indifference diagram (Fig. 11). The line exhibiting opportunities of purchase, when the price of oranges falls by 1d., and income is reduced by 10d., no longer passes through the original point of equilibrium \(P\). Thus we have no reliable information about the indifference curve it touches. We are left to infer from our earlier argument that the compensating variation will be less than the larger rectangle, so long as the marginal utility of money can be taken as constant.

\(^2\) In an article which appeared after I had written the above ("The General Welfare in relation to Problems of Taxation and of Railway and Utility Rates", *Econometrica*, July 1938) Professor Hotelling gives a substantially similar argument and applies it to broader problems of economic welfare. It would be interesting to submit all the fundamental part of Professor Pigou's book to this sort of criticism; my impression is that most of it would come out pretty well.