3.1 Introduction

1. In 'A Reconsideration of the Theory of Value' (1934) John Hicks and R. G. D. Allen revise Alfred Marshall's theory of consumer behaviour on ordinalist lines. In each place where a measurable concept of utility is used in Marshall's theory, they substitute in its place an ordinal concept of utility; for the cardinally measurable utility function, they substitute an ordinal utility function or a scale of preferences (which is represented by an indifference map), and for the 'law of diminishing marginal utility', they substitute the 'law of increasing (which later became known as "diminishing") marginal rate of substitution'. In the opinion of Hicks and Allen, the revision is necessitated by Pareto's proof of the immeasurability of utility. They observe that although Pareto (1909, p. 168 and passim) introduced indifference curves into consumer theory, he failed to re-formulate his consumer theory solely in ordinalist terms (see Hicks and Allen, 1934, pp. 196–7).

2. The importance of understanding the Hicks–Allen theory in a study of the Samuelson Programme is that Samuelson's decision to create a new theory can be attributed to a dissatisfaction with ordinal utility theory. In Samuelson's opinion, ordinal utility theory fails to become an observational theory, one which is to be free from the philosophical and psychological controversies in which utility theory had been enmeshed throughout its history (1938a, pp. 61–2):

Hence, despite the fact that the notion of utility has been repudiated or ignored by modern theory, it is clear that much of even the most modern analysis shows vestigial traces of the
utility concept. Thus, to any person not acquainted with the
history of the subject, the exposition of the theory of consumer’s
behaviour in the formulation of Hicks and Allen would seem
indirect. The introduction and meaning of the marginal rate of
substitution as an entity independent of any psychological,
introspective implications would be, to say the least, ambiguous,
and would seem an artificial convention in the explanation of
price behaviour.

It is thus fitting to begin a study of the Samuelson Programme with
an interpretation of ordinal utility theory, for, if we are to evaluate
whether or not revealed preference theory succeeds in avoiding the
pitfalls of ordinal utility theory, we must have an understanding of
ordinal utility theory which is developed independently of
Samuelson’s interpretation of that theory. And a study of the
Hicks–Allen theory would be sufficient for our purposes since some
of Samuelson’s objections against ordinal utility theory are directed
at the Hicks–Allen theory.¹

3. Thus we present a rational reconstruction of the Hicks–Allen
Programme. Its purpose is threefold. First, it sets up the background
to the Samuelson Programme. Second, it illustrates our method of
understanding a theory through an application to a well-known
theory. Third, it draws attention to a number of theoretical difficulties
in the Hicks–Allen Programme that warrant further investigation.

3.2 The problem-situation of the Hicks–Allen theory

1. An understanding of the Hicks–Allen theory of consumer
behaviour requires an appreciation of the problem to which it is
a proposed solution. The problem for Hicks and Allen is to revise
Marshall’s theory of consumer behaviour without the use of an
immeasurable concept of utility.

2. In Marshall’s theory a consumer is motivated by the pursuit
of utility which can be acquired through the consumption of goods.
The amount of utility that can be derived from the consumption of
any single good is subject to the law of satiable wants, i.e. the law of
diminishing marginal utility (1920, III, III, 1):

the additional benefit which a person derives from a given
increase of his stock of a thing diminishes with every increase
in the stock that he already has.
In addition, in a given time with given material resources, the marginal utility of money to a consumer is constant.

3. According to Marshall a consumer chooses a particular combination of goods with his income and at a set of prices that he cannot affect such that the marginal utility of each good divided by its price is the same for all goods. Contrariwise, if the ratio of marginal utility to price is not the same for all goods bought, total utility can be increased by purchasing more of those goods which have a higher ratio of marginal utility to their price. Since the law of diminishing marginal utility applies to all goods, the marginal utility of those goods, the purchase of which has increased, will decrease. The adjustment of expenditure will cease when the ratio of marginal utility to price is the same for all goods (1920, III, V, 2):

And in a money-economy, good management is shown by so adjusting the margins of suspense on each line of expenditure that the marginal utility of a shilling's worth of goods on each line shall be the same.

4. From Marshall's theory the 'law of demand' can be derived, i.e. the proposition that the quantity demanded of a good is inversely related to its price, ceteris paribus. Assume that a consumer is maximizing utility. Then the price of each good bought is equal to the ratio of its marginal utility to the marginal utility of money—i.e. a constant. If the price of one good increases while all other prices do not change, then the marginal utility of that good must increase if utility is to be at its maximum. Given the law of diminishing marginal utility, marginal utility will increase if less of that good is purchased. Thus, if the price of a good increases (decreases), less (more) of that good will be demanded.

5. In Marshall's theory of consumer behaviour, utility is conceived as a quantitatively measurable concept. The use of this concept is declared illegitimate by Hicks and Allen because Pareto proved that utility is immeasurable from observations of behaviour: '[Pareto proved] that the facts of observable conduct make a scale of preferences capable of theoretical construction... but they do not enable us to proceed from the scale of preference to a particular utility function' (Hicks and Allen, 1934, p. 52). The acceptance of this proof compels them to revise Marshall's theory because, in their opinion, concepts must be observational, i.e. correspond to observable phenomena. The task of revising subjective value theory, of
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which Marshall’s theory is one version, is left undone by Pareto, who failed to reformulate his theory of consumer behaviour in light of his proof: ‘[The task of the present paper] is the more pedestrian one of examining what adjustments in the statement of the marginal theory of value are made necessary by Pareto’s discovery’ (Hicks and Allen, 1934, p. 54).

6. Apart from the assumption that utility is measurable, Hicks and Allen do not voice any other objections against Marshall’s theory. It seems that they find the rest of Marshall’s problem-situation unobjectionable.

7. The theoretical aims of Hicks and Allen are those of Marshall: the explanation of consumer behaviour and of the law of demand. Marshall’s general analytical framework is individualistic. His explanation of consumer behaviour forms the basis of an explanation of market demand and market price. Hicks and Allen retain this individualism, which requires that all social phenomena must be explained in terms of individual behaviour. This is evident in Hicks (1939), in which the explanation of individual behaviour is a necessary step in the explanation of market demand (p. 34, emphasis added):

we have been concerned with the behaviour of a single individual. But economics is not, in the end, much interested in the behaviour of single individuals. Its concern is with the behaviour of groups. A study of individual demand is only a means to the study of market demand.

Furthermore, the explanation of market demand is integrated into a general theory of price which, it should be pointed out, follows in the tradition of Walras and Pareto rather than of Marshall. The importance of the constraint of individualism to the Hicks–Allen problem-situation is that any proposal to alter the theory will have to be examined on its implications for the general theory of price.

8. Marshall’s individualism is psychologistic. Individual behaviour is explained in terms of the individual’s psychology; social phenomena are therefore explained in terms of individuals’ psychology. The preponderance of such terms as ‘desires’, ‘motives’, ‘aspirations’, ‘human nature’ and ‘utility’ in Marshall’s explanations of human actions attests to this (1920, I, II, 1):

[Economics] concerns itself chiefly with those desires, aspirations and other affections of human nature, the outward
manifestations of which appear as incentives to action in such a form that the force or quantity of the incentives can be estimated and measured with some approach to accuracy . . .

However, Marshall's adherence to psychologism is qualified; he countenanced that an individual's tastes may be influenced by his behaviour (1920, III, III, 2).

9. Likewise, the Hicks–Allen theory is psychological. The behaviour of a consumer is explained in terms of his preferences, tastes, desires, wants, etc., which all refer to the individual's state of mind. Hicks's and Allen's objection to cardinal utility is not that it is a psychological concept but that it does not correlate with observable phenomena (Hicks, 1939, p. 18):

'Given wants' can be quite adequately defined as a given scale of preferences; we need only suppose that the consumer has a preference for one collection of goods rather than another, not that there is ever any sense in saying that he desires the one collection 5 per cent more than the other, or anything like that.

10. The fundamental importance of psychological concepts to the Hicks–Allen analysis is apparent. Changes in consumer behaviour are attributed to changes in the conditions external to the consumer under study. Thus (Hicks, 1956, p. 5):

The human individual only comes into plain economics as an entity which reacts in certain ways to certain stimuli; all that the Plain Economist needs to be interested in are the laws of his reactions.

The link between external changes and changes in the consumer's behaviour is preferences (tastes), which, for Hicks and Allen, are assumed to be given and unchanging. The strict independence of preferences from conditions external to the consumer is emphasized in part II of Value and Capital, in which Hicks comments on the applicability of indifference analysis, the formal structure of his theory of consumer behaviour, to other areas (Hicks, 1939, p. 55):

The objects bought and sold need not be consumers' goods, or they need not all be consumers' goods; the necessary condition is only that they should be objects of desire, which can be bought and sold, and which can be arranged in an order of preference (an indifference system) which is itself independent of prices.
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11. In summary, the problem-situation of Hicks and Allen is to revise Marshall's theory of consumer behaviour. The explanation of consumer behaviour and the law of demand are to be given within the framework of psychologicist individualism but without the use of an immeasurable concept of utility.

3.3 The Hicks–Allen solution

1. The Pareto proof of the immeasurability of utility is important in two ways for the Hicks–Allen Programme. First, it provided a sufficient reason for rejecting Marshall's theory, and thereby delimited the class of possible replacements. Second, it suggested the solution to the problem of revising Marshall's theory. In the course of setting out his proof, Pareto showed that a scale of preferences can be constructed from the facts of observable conduct. However, he never exploited this idea to its fullest. In the hands of Hicks and Allen, the concept of a scale of preferences becomes the basis for a new theory of consumer behaviour, the solution to the problem of revising Marshall's theory.

2. In the Hicks–Allen theory, a scale of preferences (an ordinal utility function) depicts the individual's tastes. First, it is assumed that a consumer's preferences are defined over all possible bundles of goods. Given any two bundles, he prefers one to the other or else he is indifferent. The locus of those bundles which have the same rank define an indifference surface; the indifference surfaces together make up an indifference map. Second, he prefers more goods to less goods. Given any two bundles, he prefers one to the other if it contains more of at least one good and no less of all other goods. Third, his preferences are consistent in the sense that if he prefers one bundle to a second he does not prefer the second to the first, and that if he prefers one bundle to a second, the second to a third bundle, he prefers the first bundle to the third. Fourth, his preferences exhibit a diminishing marginal rate of substitution throughout. This property is known as the 'law of diminishing marginal rate of substitution': the more good \( x_1 \) that is substituted for good \( x_2 \), such that the consumer maintains the same level of satisfaction, the less will be the amount of \( x_2 \) which will be given up for successive amounts of \( x_1 \).

3. In the Hicks–Allen theory, the actions that a consumer may take to fulfil his desires are limited by certain material conditions
external to him. These conditions are represented by the budget constraint, which determines the bundles that he may purchase with his money income at a set of prices which he cannot influence. In Shackle’s words (1967, p. 80):

Price generalizes the obstacles, giving to each participant of the market, in effect, a knowledge of the whole field of possibilities and enabling him to profit by the desires and consequent potential conduct of every one else.

4. In the Hicks–Allen theory, the explanation of consumer behaviour is effected by bringing together the description of the individual’s tastes, the indifference map, with the description of the material circumstances confronting the individual, the budget constraint. The link between the two parts is established by the assumption that the individual seeks to maximize his satisfaction or utility, which is now conceived as an ordinal concept. Thus the action of the consumer, qua consumer, is seen as the outcome of the confrontation of tastes and the obstacles to their fulfilment. The explanation of consumer behaviour, of why a consumer bought what he did, is that, given his income and market prices, he bought the bundle which was the highest on his scale of preferences; all other bundles were not bought because they were either too expensive or were lower on the scale of preferences.

5. The explanation of the law of demand is more complicated. Since changes in the actions of a consumer, qua consumer, are attributed to changes in the material circumstances, a change in the price of a good will result in a change in the quantity of that good demanded. Consider, for example, the effect on the quantity demanded of good $x_1$ of a decrease in the price of that good, $p_1$. The quantity demanded of good $x_1$ will change for two reasons. First, as a consequence of the law of diminishing marginal rate of substitution, more of good $x_1$ will be bought because it is now relatively cheaper. This is known as the ‘substitution effect’. Second, the quantity demanded of good $x_1$ will change because the real income of the consumer has increased. This is known as the ‘income effect’. Whether there will be an increase or a decrease in the quantity of that good demanded is unknown in the Hicks–Allen theory. Thus the net result of the two effects together is not discernible in the Hicks–Allen theory; but, because the cases in which the income effect is negative and outweighs the substitution effect are examples
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of Giffen goods, which are empirically rare occurrences, the quantity demanded of good \( x_i \) will increase (decrease) when the price of \( x_i \) decreases (increases) (Hicks, 1939, p. 35):

Thus, as we might expect, the simple law of demand – the downward slope of the demand curve – turns out to be almost infallible in its working. Exceptions to it are rare and unimportant.

6. The Hicks–Allen theory can be expressed in a mathematical form in which the behaviour of the consumer is described as the outcome of a constrained maximization problem:

\[
\max U = f(x_1, \ldots, x_n) + \lambda(I - \Sigma p_i x_i). \tag{3.1}
\]

The first-order conditions are:

\[
U_i = f_i - \lambda p_i = 0, \quad i = 1, \ldots, n \tag{3.2}
\]
\[
U_\lambda = I - \Sigma p_i x_i = 0
\]

and the second-order conditions are such that the bordered Hessian determinants alternate in sign, beginning with positive:

\[
|\bar{A}_3| > 0; \quad |\bar{A}_4| < 0; \quad |\bar{A}_5| > 0; \ldots \tag{3.3}
\]

where

\[
|\bar{A}| = \begin{vmatrix}
0 & -p_1 & -p_2 & \cdots & -p_n \\
-p_1 & U_{11} & U_{12} & \cdots & U_{1n} \\
-p_2 & U_{21} & & \cdots & \\
\vdots & \ddots & \ddots & \ddots & \\
-p_n & U_{n1} & & & U_{nn}
\end{vmatrix} \tag{3.4}
\]

From (3.2) we can, by solving for \( x_i, \ldots, x_n \), derive the demand functions:

\[
x_i = h_i(p_1, \ldots, p_n, I), \quad i = 1, \ldots, n. \tag{3.5}
\]

7. By total differentiation of (3.2) and solving for \( dx_1, \ldots, dx_n, \) \( d\lambda \), we can express the effect on quantity demanded of \( x_i \) due to a change in \( p_i \) as

\[
(\delta x_i/\delta p_i) = (\delta x_i/\delta p_i)_{U=U_0} - x_i(\delta x_i/\delta I). \tag{3.6}
\]

(3.6) will have a positive value if and only if

\[
(\delta x_i/\delta I) < 0 \tag{3.7}
\]

and

\[
|x_i(\delta x_i/\delta I)| > |(\delta x_i/\delta p_i)_{U=U_0}|. \tag{3.8}
\]
Because (3.7) and (3.8) are considered to be conditions rarely satisfied, it is assumed that (3.6) is almost always negative.

3.4 A problem-shift in the Hicks–Allen Programme

1. Part I of Hicks's *Value and Capital* (1939) is generally recognized to contain a restatement of the problem of revising Marshall's theory of consumer behaviour and the ordinalist solution of Hicks and Allen, as well as an integration of ordinal utility theory into a general theory of price determination. However, it has gone unnoticed in the literature that Hicks's restatement of the problem is substantially different from the original formulation of Hicks and Allen (1934). This point escaped the attention of such influential commentators as Schumpeter (1954, pt IV, appendix to ch. 7), Houthakker (1961) and Shackle (1967, ch. 8), as well as reviewers of *Value and Capital* (see Harrod, 1939; Haley, 1939; Boulding, 1939; Machlup, 1940; and Morgenstern, 1941). This change, or problem-shift, as we shall argue greatly diminishes the importance that has been attached to the Hicks–Allen Programme.

2. It should be remembered that, for Hicks and Allen, the acceptance of Pareto's proof of the immeasurability of utility required the rejection of Marshall's theory. It was in this context that the problem of how to revise Marshall's theory arose (Hicks and Allen, 1934, p. 55, emphasis added):

   What has now to be done is to take in turn a number of the main concepts which have been evolved by the subjective theory; to examine which of them are affected by the immeasurability of utility; and of those which have to be abandoned, to enquire what, if anything, can be put in their place. It is hoped in this way to assist in the construction of a theory of value in which all concepts that pretend to quantitative exactitude, can be rigidly and exactly defined.

The major ingredient of their solution is the concept of a scale of preferences, or ordinal utility. The change from a cardinal to an ordinal concept of utility, in Hicks's and Allen's opinion, 'rested on a positive demonstration that the facts of observable conduct make a scale of preferences capable of theoretical construction...but they do not enable us to proceed from the scale of preference to a particular utility function' (1934, p. 52; emphasis added).
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3. In Hicks (1939) the rationale for revising Marshall's theory is changed. The revision is not defended on the grounds that an ordinal utility rather than a cardinal utility function can be constructed from the facts of observable conduct, but on the grounds that only an ordinal concept of utility is necessary for the explanation of consumer behaviour (p. 18, emphasis added):

Pareto's discovery only opens a door, which we can enter or not as we feel inclined. But from the technical economic point of view there are strong reasons for supposing we ought to enter it. The quantitative concept of utility is not necessary in order to explain market phenomena. Therefore, on the principle of Occam's razor, it is better to do without it.

4. The citing of 'Occam's razor' reveals a change in Hicks's attitude to the question whether utility, cardinal or ordinal, is an observational concept. This is evident from the fact that the reference to Pareto's discovery in the above passage is not to his proof of immeasurability of utility but to his innovative use of indifference maps (Hicks, 1939, p. 16, emphasis added):

We come now to the really remarkable thing about indifference curves – the discovery which shunted Pareto's theory on to a different line from Marshall's, and opened a way to new results of wide economic significance.

Moreover, nowhere in Value and Capital does Hicks assert that the 'facts of observable conduct' enable, theoretically, a scale of preferences (an ordinal utility function) to be constructed. Pareto's proof of the immeasurability of utility, which is Hicks's and Allen's raison d'être for revising Marshall's theory, goes unmentioned in Value and Capital.

5. In arguing that only an ordinal concept of utility is necessary to explain consumer behaviour, Hicks drops two important situational constraints from the Hicks–Allen problem of revising Marshall's theory, namely that concepts should correspond to observable phenomena and that utility is not measurable. It is not suggested here that Hicks replaced these two constraints by their converse. The point is simply that the issue of measurability of utility and the wider issue that concepts should be observational are irrelevant and immaterial to Hicks's reformulation of the problem. Although Hicks himself prefers an ordinal concept of
utility, he does adopt a more tolerant attitude towards those who subscribe to a cardinal concept (1939, p. 18):

Now of course this does not mean that if any one has any other ground for supposing that there exists some suitable quantitative measure of utility, or satisfaction, or desiredness, there is anything in the above argument to set against it. If one is a utilitarian in philosophy, one has a perfect right to be a utilitarian in one’s economics. But if one is not (and few people are utilitarians nowadays), one also has the right to an economics free of utilitarian assumptions.

6. Unfortunately, it is this tolerance that undermines the significance of the Hicks–Allen problem. Since it is now Hicks’s opinion that the belief in measurability is not unreasonable, as it was considered in Hicks and Allen (1934), it cannot be argued that there is the same sense of urgency in revising Marshall’s theory. Consequently, the significance of the Hicks–Allen Programme in the theory of consumer behaviour needs to be reconsidered.

7. It must now be asked why Hicks changes his attitude to Pareto’s proof, and drops, in the process, the two situational constraints to which the Hicks–Allen theory owes its importance. Hicks himself does not mention, and neither does he allude to, the fact that emphasis has shifted from Pareto’s proof to the latter’s innovative use of indifference maps. The task for us here is to set out the logic of a continual acceptance in the Hicks–Allen Programme of Pareto’s claim that a scale of preferences, but not a cardinal utility function, can be constructed from the facts of observable conduct.

8. We conjecture that Hicks abandons his previous position because he appreciates the difficulties that are associated with any actual attempt to construct indifference maps.

9. Consider, for example, the method of constructing an individual’s indifference map through questioning him. In the context of the Hicks–Allen Programme, this procedure is unacceptable because it does not use facts of observable conduct. If this procedure were acceptable, their objection to measurable utility would vanish, for if it is permissible to question an individual about the scale of his preferences, there can be no valid objection against questioning him about the intensity of his preferences. There remains, however, the question whether a consumer knows his preferences, the scale and/or intensity of them.
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10. The only viable alternative from the Hicks–Allen position is to draw on observations of an individual's market behaviour. But this procedure encounters a serious difficulty. If observations of market behaviour are used to construct a scale of preferences, it turns ordinal utility theory into a circular explanation of consumer behaviour: consumer market behaviour is explained in terms of the individual's scale of preferences, or ordinal utility function, which, in turn, is explained by his market behaviour.⁹

11. There is another difficulty. It may be impossible to discern an individual's preferences from observations of his market behaviour if these preferences are influenced by the material circumstances, i.e. prices and income. This point is evident from Hicks's comment on the applicability of his analysis of consumer behaviour to other fields (1939, p. 55):

The objects bought and sold need not be consumers' goods, or they need not all be consumers' goods; the necessary condition is only that they should be objects of desire, which can be bought and sold, and which can be arranged in an order of preferences (an indifference system) which is itself independent of prices.

3.5 **On the law of diminishing marginal rate of substitution**

1. If the concepts of utility and marginal utility are replaced, then so must the law of diminishing marginal utility be replaced. Hicks (1939) argues that a replacement for this law is necessary to ensure that the point of equilibrium is stable. To serve this end, the law (principle) of diminishing marginal rate of substitution¹⁰ is proposed: the more of good \( x_1 \) that is substituted for good \( x_2 \), such that the consumer maintains his level of satisfaction, the less will be the amount of \( x_2 \) given up for successive amounts of \( x_1 \).

2. In this section we will examine the reason that is given as to why the law of diminishing marginal rate of substitution (LDMRS) is necessary for the Hicks–Allen theory. With reference to the explanation given in Hicks (1939), we shall argue that the theoretical necessity of the LDMRS should be distinguished from the issue of whether the LDMRS is true or not, and that these two issues are confused in the Hicks–Allen theory.

3. The theoretical necessity of the LDMRS is stated in Hicks (1939) as the requirement that the point of equilibrium should be
stable. This is incorrect because Hicks stipulates that a condition for a point to be an equilibrium one is that it must be stable. The necessity of the law for the Hicks–Allen explanation of consumer behaviour can be briefly stated as follows. In order to explain why a particular bundle, \( X^0 \), is bought, it is insufficient to say that \( X^0 \) is the bundle which maximizes the consumer's satisfaction. In more positive terms, it is necessary to explain why all other bundles other than \( X^0 \) were not bought. That is why the budget constraint alone is regarded as an unsatisfactory explanation because it rules out from consideration only those bundles which exceed the value of the budget. An explanation is still needed as to why bundles which cost less than or are equal to the budget are not bought. It is in the context of this problem in the theory of consumer behaviour that the issue of stability arises.

4. Consider, for example, Figure 3.1, in which the preferences of a consumer are represented by indifference curves which contain 'flat' portions. With the given budget, the point of maximum utility is not unique. In other words, the statement that \( X^0 \) was bought because it is the point of maximum utility is an unsatisfactory answer to the question of why \( X^0 \) was bought by the consumer since all bundles between \( X^1 \) and \( X^n \) are points of (equi-) maximum

![Figure 3.1](image-url)
utility. A satisfactory answer would require in addition an explanation of why all bundles other than $X^0$ were not bought. Instead of introducing additional premises to complete the explanation, the Hicks–Allen theory rules out explicitly the possibility of multiple equilibria\textsuperscript{11} through the requirement that the law of diminishing marginal rate of substitution is true.

5. Therefore, by requiring that the marginal rate of substitution be diminishing everywhere, any point of maximum utility is unique for some given budget. In terms of the logic of explanation, it is a logically satisfactory explanation to say that a consumer bought what he did because it is the bundle which gives him the maximum utility in the given material circumstances.

6. While it is not difficult to appreciate the importance of the law for the explanation of consumer behaviour, the question of whether the law is true or not still remains. For even if the law enables Hicks to give a logically satisfactory explanation of consumer behaviour, the empirical falsity of the law implies that the explanation is false. In Marshall's theory, the law of diminishing marginal utility, which is the analogous requirement for a complete explanation, was held to be a universal truth of human nature; its truth was considered to be intuitively obvious. Although Hicks makes no such appeal to intuition for the law of diminishing marginal rate of substitution, he nevertheless feels that the law requires justification.

7. Hicks seeks the justification by reflecting on the purpose that the law is to serve in the theory. However, the purpose of the law is a separate issue from that of the truth or falsity of the law. By confusing two separate issues, he makes the issue of truth subsidiary to that of theoretical necessity.

8. The justification for the truth of the law is given in two parts. First, Hicks explains why there are some cases of a diminishing marginal rate of substitution, to paraphrase Hicks, why the law is sometimes true, i.e. why some marginal rates of substitution are diminishing. Then he explains why the law is true, i.e. why all marginal rates of substitution are diminishing. Hicks regards the first explanation as the prelude to the second.

9. The law is sometimes true, according to Hicks, because for a point to be an equilibrium point, the marginal rate of substitution must be diminishing at that point. The next question then is 'Do equilibrium points exist?' Hicks says 'Yes.' His reason is that 'some points of possible equilibrium do exist on the indifference maps of
nearly everyone (that is to say, they do decide to buy such-and-such quantities of commodities, and do not stay hesitating indefinitely like Buridan's ass) (1939, p. 22). In other words, equilibrium points exist because people do buy. But how can Hicks equate the existence of equilibrium points with the fact that people do buy?

10. The link between the existence of equilibrium and the fact that people do buy is the implicit assumption that people always buy the bundle that maximizes their satisfaction, given the material circumstances of price and income. Thus to assert the existence of equilibrium points is equivalent to the assertion that the Hicks–Allen theory of consumer behaviour is true.

11. Hicks correctly points out that the law must be more than sometimes true if it is to be of any use in economic theory. The interest is not only in the explanation of a single act of consumption but in all acts of consumption: 'It is clear, therefore, that for any point to be a possible rate of equilibrium at appropriate prices the marginal rate of substitution at that point must be diminishing' (Hicks, 1939, p. 22). When the market conditions, i.e. prices and/or income, change, Hicks wants to be able to explain why the consumer buys a different bundle (1939, p. 23):

When market conditions change, the consumer moves from one point of equilibrium to another point of equilibrium; at each of these positions the condition of diminishing marginal rate of substitution holds, or he could not take up such a position at all.

It is clear, then, that the law is needed to ensure the completeness of the explanation of consumer behaviour. Moreover, the law is needed in order 'to deduce from it laws of market conduct – laws, that is, which deal with the reaction of the consumer to changes in market conditions' (Hicks, 1939, p. 23). These laws explain the direction and magnitude of change in the quantity bought when the material circumstances confronting the consumer change.

12. Hicks says that when market conditions change, the consumer moves from one position of equilibrium to another, i.e. he buys a different bundle when the prices and income change. But why should the consumer buy a different bundle when market conditions change? Hicks does not say. Because Hicks requires that for a bundle to be bought, i.e. to be an equilibrium point, the marginal rate of substitution must be diminishing; a new bundle will necessarily be bought when market conditions change.
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13. For the marginal rate of substitution to be diminishing everywhere, Hicks assumes that between any two positions of equilibrium, all positions must exhibit a diminishing marginal rate of substitution, i.e. there are no ‘kinks’ in the curves between two equilibrium points. If there are such perversities, Hicks observes that ‘curious consequences follow, such that there will be some systems of prices at which the consumer will be unable to choose between two different ways of spending his income’ (1939, p. 23; emphasis added). This suggests that the assumption guarantees that a consumer can act decisively in accordance with the theory. If a consumer wants to act in accordance with the principle that he should maximize his utility subject to material circumstances, this assumption of regularity ensures that there will be one and only one bundle which will maximize his utility in any given material circumstances. However, the theory is not concerned with whether a consumer can act in accordance with the theory but whether he does act in accordance with it – that his actions can be explained by the theory.

14. To appreciate why Hicks assumes the truth of the law, we must consider why Hicks is interested in the intermediate points between equilibrium points. Hicks is concerned with intermediate points because he wants to make explicable whatever bundle is bought by a consumer. If the marginal rate of substitution is not diminishing at the intermediate points, there is the possibility of multiple equilibria, i.e. there is more than one bundle which maximizes utility subject to the given material conditions (Hicks and Allen, 1934, p. 58):

The assumption that the principle of increasing [diminishing] marginal rate of substitution is universally true, thus means simply that any point, throughout the region we are considering, might be a point of equilibrium with appropriate prices.

15. Therefore, ‘kinks’ are ruled out by assumption, not because their existence implies that a consumer is unable to choose between two or more bundles but because Hicks wants to explain why a consumer bought a particular bundle rather than some other. Hicks wishes us to believe that if there are ‘kinks’ the consumer is not only unable to choose but will not choose. Clearly, the task for Hicks and for the theorist is to explain what the consumer chooses.12

16. In the beginning of this section, we drew attention to the fact that the logical necessity of the law is a separate question from the
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empirical truth of the law. It is evident that Hicks confuses these two questions. For Hicks, the law is true because it is necessary for the theory. This implies that for Hicks the truth of the theory is beyond question. Although Hicks does say that the law's 'accordance with experience seems definitely good' (1939, p. 24), consider how he would react to counter-evidence to this law (Hicks and Allen, 1934, p. 58, emphasis added):

There must be some points at which it is true, or we could get no equilibrium at all. To assume it true universally is a serious assumption but one which seems justifiable until significant facts are adduced which make it necessary for us to pay careful attention to exceptions.

While significant facts should prompt Hicks to reconsider both the law and the facts, Hicks will countenance reconsideration only of the facts. This flagrant disregard for the truth of the law underscores the fundamental importance of the law for the Hicks–Allen theory of consumer behaviour.

3.6 On the law of demand in the Hicks–Allen theory

1. In the history of the theory of consumer behaviour, the law of demand occupies a special place. At one time a theory of consumer behaviour was judged solely on its ability to explain the law. In the 1970s, the opinion of specialists is divided. However, throughout his writings on consumer theory, Hicks is unequivocal on his position (1956, p. 189):

The prime concern of demand theory is with the Law of Demand. It is from the standpoint of its effect in elucidating the law of demand that our theory may best be summarized, and that it may claim to be judged.

2. Chapter 2 of Hicks's Value and Capital (1939) is entitled 'The Law of Consumer's Demand'. This underscores the significance of the law to the Hicks–Allen Programme of revising Marshall's theory. Because the law of demand is derivable in Marshall's theory, its derivation became a major theoretical task for the Hicks–Allen theory. We are therefore in agreement with Shacke's assessment (1967, p. 86) of the two theories:

To give this 'law of demand' some reasoned position in our
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store of ideas, to find other propositions which can be exhibited as the premises from which it flows or as consequences which flow from it, to build under it an explanatory argument; this sort of purpose is, for Marshall and Hicks alike, the central if not almost the sole concern of the theory of consumer's demand.

3. The fact that the law of demand is not derivable from the Hicks-Allen theory was seen by Hicks and Allen as a minor deficiency in their theory. They argued that upward-sloping demand curves, the exceptions to the law, are rare because they are examples of the pathological phenomenon of Giffen goods, which, in their opinion, are unlikely empirical occurrences. In the words of Hicks, ‘the simple law of demand - the downward slope of the demand curve - turns out to be almost infallible in its workings’ (1939, p. 35). In this section, we shall criticize this argument on the grounds that it is both ad hoc and insufficient to resolve the theoretical difficulty, namely the logical possibility of upward-sloping demand curves. In addition, we shall set out the alternative strategies that Hicks and Allen could have taken to deal with the theoretical difficulty.

4. It is important to note that Hicks and Allen thought it was necessary to deal with exceptions to the law. This view is not widely held. To many economists the logical possibility of upward-sloping demand curves in the Hicks-Allen theory remains a curiosity. Stigler is representative of this large body of opinion when he chides economists for devoting too much attention to this issue:

   For more than half a century economists have recognized the possibility of a positively sloping demand curve. They have desired a real example, probably to reassure themselves of the need for discussing the possibility and almost invariably they have used Marshall’s Giffen paradox as this example (1947, p. 152).

5. The possibility of deriving an upward-sloping demand curve in the Hicks-Allen theory is due to the unknown sign of the income term of the Slutsky equation for the change in quantity bought with respect to the change in its own price. That the substitution term is negative is a consequence of the law of diminishing marginal rate of substitution but the theory places no restriction on the sign of the income term. Thus there arises in the Hicks-Allen theory the unintended logical possibility of upward-sloping demand curves, the counter-examples to the law of demand.
6. Exceptions to the law of demand are identified by Hicks and Allen as examples of the Giffen case, and since they considered this phenomenon to be rare, they concluded that the law is almost infallible in its working:

it is only possible at low levels of income, when a large proportion of expenditure is devoted to this 'inferior' commodity, and when, among the small number of other objects consumed, there are none that are at all easily substitutable for the first. As the standard of living rises, and expenditure becomes increasingly diversified it is a situation which becomes increasingly improbable (1934, pp. 68–9).

Similarly, Hicks writes (1939, p. 35):

Consumers are likely to spend a large proportion of their incomes upon what is for them an inferior good if their standard of living is very low. The famous Giffen case, quoted by Marshall, exactly fits these requirements. At a low level of income, consumers may satisfy the greater part of their need for food by one staple foodstuff (bread in the Giffen case), which will be replaced by a more varied diet if income rises. If the price of this staple falls, they have a quite considerable surplus available for expenditure, and they may spend this surplus upon more interesting foods, which then take the place of the staple, and reduce the demand for it. In such a case as this, the negative income effect may be strong enough to outweigh the substitution effect. But it is evident how rare such cases must be.

7. Before we determine whether the argument is sufficient to exclude the possibility of upward-sloping demand curves, we shall first comment on the validity of the method of argument.

8. In citing empirical evidence to resolve a theoretical difficulty, Hicks and Allen are confusing a question of fact with a question of logic (see section 3.5 above). An enquiry into the empirical existence of upward-sloping demand curves is of no relevance to the question, of whether or not the law of demand is derivable from the theory. It is apparent that the appeal to the facts is purely *ad hoc*. For example, Hicks and Allen do not cite any empirical evidence for the proposition that the substitution term is negative. Although this proposition is derivable from the theory, it is an open question
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whether it is empirically true. Our contention that there is a deep-seated confusion of logic with fact is supported by a consideration of the reason given for the truth of the law: 'It follows that in strictness the Law of Demand is a hybrid; it has one leg resting in theory, and one in observation' (Hicks, 1956, p. 59). On this account it would be impossible to cite empirical evidence for or against the law of demand.

9. Let us now consider the validity of the Hicks–Allen argument that upward-sloping demand curves are rare because Giffen goods are rare.

10. We must first determine the necessary and sufficient conditions for an upward-sloping demand curve in the context of the Hicks–Allen theory. The Slutsky equation for the change in quantity bought of a good, \( x_t \), with respect to a change in its price, \( p_t \), is

\[
(\delta x_t/\delta p_t) = (\delta x_t/\delta p_t)_{U=U_0} - x_t(\delta x_t/\delta I) \tag{3.6}
\]

because

\[
x_t > 0, \quad (\delta x_t/\delta p_t)_{U=U_0} < 0. \tag{3.6}^*
\]

(3.6) takes on a positive value if and only if \( x_t \) is an inferior good:

\[
(\delta x_t/\delta I) < 0 \tag{3.7}
\]

and

\[
| x_t(\delta x_t/\delta I) | > | (\delta x_t/\delta p_t)_{U=U_0} |. \tag{3.8}
\]

11. The characteristics of a Giffen good are identified by Hicks and Allen as follows:

(a) the good is inferior;
(b) the standard of living of the individual is low; and
(c) a large proportion of the individual's income is spent on that good.

While (a) is a necessary condition for an upward-sloping demand curve in the Hicks–Allen theory, it is obviously not sufficient. However, (b) and (c) are not equivalent to (3.8). The fact that a good with an upward-sloping demand curve is inferior does not necessarily mean that the individual concerned has a low standard of living and that he spends a large proportion of his income on that good. Within the context of the Hicks–Allen theory, there is no criteria for determining what is a low standard of living or what constitutes a large proportion of an individual's income. Given the conditions for an upward-sloping demand curve in the Hicks–Allen theory, it is clear that what matters is the relative size of the absolute values of
the income and substitution terms. Thus, in the Hicks–Allen theory a Giffen good is not the same thing as a good with an upward-sloping demand curve.\textsuperscript{17}

12. The error in Hicks’s and Allen’s argument can be attributed to their uncritical appropriation of Marshall’s explanation of the Giffen good. Marshall (1920) argued that it is through the effect on the marginal utility of income that a rise in the price of a good upon which an individual spends a large proportion of his income can lead to a rise in the demand for the good.\textsuperscript{18}

13. Thus we conclude that even if we disregard the illegitimacy of Hicks’s and Allen’s use of empirical evidence to resolve a theoretical difficulty, the argument is nevertheless insufficient to exclude the logical possibility of an upward-sloping demand curve.

14. What logical options remain to resolve the theoretical difficulty in the Hicks–Allen theory? First, the assumption that all goods are non-inferior can be added to the theory to exclude the logical possibility of upward-sloping demand curves. Second, the theory can be amended to exclude all inferior goods which give rise to upward-sloping demand curves. Third, the unintended logical consequence of upward-sloping demand curves can be reinterpreted as an intended logical consequence of the theory.\textsuperscript{19} If any one of the three options are taken, consideration must be given to the implications of the change not only for the Hicks–Allen theory but also for the theory of price, of which the Hicks–Allen theory is an integral part.