Do supply curves slope up? The empirical relevance of the Sraffian critique of neoclassical production economics

Adam Ozanne*

The Sraffian critique of neoclassical economics has tended to concentrate on the input rather than the output side of production processes—in particular, criticism of neoclassical capital theory. When it has addressed supply questions, it has been concerned more with individual commodities than aggregate supply. Furthermore, Sraffian economists have been criticised for not investigating the relevance of their hypotheses empirically. This paper extends the Sraffian critique of Marshallian partial analysis to cover aggregate supply and reports the results of a simulation exercise based on an econometric model of UK agriculture that finds evidence of perverse supply response at the aggregate level.

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1. Introduction

The notion of Marshallian upward-sloping output supply curves and downward-sloping input demand curves is based on two assumptions; first, the *extens parallel* assumption that it is possible for one price to change while all other prices remain fixed; and second, the assumption that commodities can be identified as either outputs or inputs. However, the separation of outputs and inputs into distinct commodity groups becomes blurred when a commodity produced by one activity or in one time period is used as a factor of production in another activity or time period. Distinguishing between intermediate and finished products helps to resolve this ambiguity; however, intermediate products are usually defined as partly finished goods that form inputs to the production process of another firm or industry rather than the same industry or enterprise. It is therefore useful to identify certain commodities that are essentially own-account intermediate products used by the same enterprise or industry and to consider the implications of the existence of such commodities for both economic theory and empirical modelling exercises.

Such commodities, known as 'produced means of production' or 'produced inputs', have been an important feature of the Sraffian critique of neoclassical economics. The concept was developed by Sraffa (1925, 1926, 1960) and played an important role in the capital re-switching controversies of the 1960s and 1970s, reviews of which can be found in the literature.

*University of Manchester. I would like to thank two anonymous referees for their helpful comments and suggestions.

However, Sraffian economists have made no attempt to verify whether or not their theoretical results are empirically relevant; indeed, this has been a major criticism of their work by neoclassical economists. Thus, although the Cambridge School made much of Samuelson's final capitulation in the capital re-switching controversy (Samuelson, 1966), it is quite clear that Samuelson himself still prefers the 'neoclassical parable' to the 'simple re-switching paradigm' (Samuelson, 1980, pp. 575–577; 1990). Similarly, Blaug has caricatured the economics of the Cambridge School as 'theory without measurement' (Blaug, 1974, pp. 80–81), and stated that until re-switching and capital-reversing 'are shown to be empirically important rather than just logically possible, economists are ill-advised to throw away their textbooks' (ibid, p. 43).

In addition to eschewing empirical work, Sraffians have tended to concentrate on the input rather than the output side of production technologies, and have been concerned with individual commodities rather than aggregate responses, sharing as they do the Austrian suspicion of aggregates (see, for example, the references to the capital re-switching controversies above and the more recent contributions from Steedman, 1985, 1988B and 1994, in this Journal). Yet the output side affords some interesting possibilities, for, although most economists accept that supply curves slope upwards, and therefore that output quantities and prices should move in the same direction in response to exogenous demand shocks, there is in fact considerable evidence to suggest otherwise (Houthakker, 1979; Rotemberg and Saloner, 1986; Bils, 1987; Blanchard, 1987; De Long and Summers, 1991). Shea (1993), reviewing this evidence, notes two common explanations for such 'perversity' results. First, the existence of internal or external economies of scale or countercyclical price–cost margins may lead to flat or downward-sloping short-run supply curves. Second, even where supply curves are upward sloping, cost shocks may induce negative co-movements of output and prices. These explanations, however, neglect the possibility that such instances of perversity may reflect the existence of produced means of production.

The Sraffian critique of neoclassical economics may be particularly relevant to the agricultural sector, where seed, feedgrain, silage and breeding livestock can all be regarded as important produced means of production. In the UK between 1975 and 1985, for example, the quantity of wheat used for animal feed and seed was equivalent to 48% of domestic production, while the corresponding figure for barley was 62% (MAFF, 1982, 1987). It may be conjectured that, given such high percentages, the conventional neoclassical distinction between inputs and outputs may be misleading—in which case, if produced means of production are not taken account of, conventional neoclassical predictions may break down.

This paper builds on previous work by Metcalfe and Steedman (1972), Gram (1985) and, particularly, Salvadori and Steedman (1984, 1985) to discuss the theoretical implications of the existence of produced inputs with and without aggregation of commodities. It is shown that the existence of produced inputs in the production process undermines the ceteris paribus assumption that is the basis of Marshall's system of intersecting supply and demand curves; in particular, it is shown that supply responses may be perverse. Finally, the results of a simulation exercise based on an econometric model of UK agriculture are reported, which provide an insight into the empirical relevance of the theoretical arguments and, therefore, of at least part of the Sraffian critique of neoclassical economics.
2. Duality theory and produced means of production

The theory of the firm assumes that perfect competition prevails and economic decision-makers are price-taking profit maximisers. Decisions regarding the level of production, output mix and choice of technique are described in terms of a constrained optimisation problem:

$$\max_Y \pi = \sum_{i=1}^{n} P_i Y_i$$
subject to $F(Y) = 0$ \hspace{1cm} (1)

where $Y = Y_1, \ldots, Y_n$ is the netput vector, $P = P_1, \ldots, P_n$ is the corresponding price vector, and $F(Y)$ is the transformation function describing the production technology.\(^1\)

Solution of the first-order conditions yields the set of output supply and input demand functions:

$$Y_i^* = Y_i^*(P) \quad (i=1, \ldots, n)$$ \hspace{1cm} (2)

which, when substituted back into (1), give the profit function:

$$\pi^* = \pi^*(P) = \sum_{i=1}^{n} P_i Y_i^*(P)$$ \hspace{1cm} (3)

Thus, for any well-behaved transformation function, there is a profit function that must be: (a) non-negative, (b) non-decreasing in the netput prices, (c) convex and continuous, and (d) positively linear homogeneous in the netput prices (McFadden, 1978).

Convexity of the profit function implies that the diagonal elements of its Hessian matrix are non-negative. Making use of Hotelling's Lemma, differentiation of expression (3) with respect to own prices leads to:

$$\frac{\partial^2 \pi^*}{\partial P_i^2} \frac{\partial Y_i^*}{\partial P_i} \geq 0 \quad (i=1, \ldots, n)$$ \hspace{1cm} (4)

That is, output supply curves must be upward sloping and input demand curves must be downward sloping; from the perspective of duality theory, the Laws of Output Supply and Factor Demand are embedded in the convexity property of the profit function.

These laws can be derived without requiring specific restrictions or assumptions about the structure of a production technology; the only assumption required is that producers maximise profits. Consider, for example, two different sets of netput prices. Let the first set of prices be $P_i^0 \ (i=1, \ldots, n)$ and the second set be $P_i^1 \ (i=1, \ldots, n)$. Then let the set of profit maximising output supply and input demand quantities corresponding to the first price configuration be $Y_i^0 \ (i=1, \ldots, n)$ and the optimal set corresponding to the second price configuration be $Y_i^1 \ (i=1, \ldots, n)$. Profit maximisation implies that:

\(^1\)The 'netput' concept is used for mathematical convenience. Netputs are positive for an output and negative for an input. Note that, despite the use of the netput concept, outputs and inputs are at this point in the analysis being treated as distinct commodity groups, as envisaged in convention theory; issues relating to the existence of produced means of production raised below are as yet absent. Dixit (1974, p. 135) discusses the properties of a well-behaved transformation function. All inputs are treated as being variable for convenience, since the following analysis is not affected if any are fixed or quasi-fixed.
\[ \sum_{i=1}^{n} p_i^1 y_i^1 \geq \sum_{i=1}^{n} p_i^0 y_i^0 \]  \hspace{2cm} (5)

and:

\[ \sum_{i=1}^{n} p_i^0 y_i^0 \geq \sum_{i=1}^{n} p_i^0 y_i^1 \]  \hspace{2cm} (6)

Addition of expressions (5) and (6) leads to the fundamental inequality of profit maximisation:

\[ \sum_{i=1}^{n} (p_i^1 - p_i^0) (y_i^1 - y_i^0) \geq 0 \]  \hspace{2cm} (7)

If only one price, say \( p_0 \), changes we obtain, ceteris paribus:

\[ (p_i^1 - p_i^0)(y_i^1 - y_i^0) \geq 0 \]  \hspace{2cm} (8)

which is the analogue of expression (4).

It has been pointed out by Steedman (1985) that, although the fundamental inequality is a direct result of constrained profit maximisation, to reach expression (8) the additional assumption has to be made that one price can be changed relative to all other prices whilst holding all the relative values of these latter prices fixed. Thus, expression (8) is not a partial equilibrium statement that may be invalid when output and input prices are linked, as they are when one or more outputs are also used as produced inputs.

To make this point clearer, follow Salvadori and Steedman (1984, 1985) and consider a simple Ricardo-type corn model. Let \( Y_t \) represent total cereal production and \( Y_s \) represent seed. Make the further assumption that, because it is a produced input, the price of seed is related to the price of cereals. In fact, to make the analysis tractable, assume that \( P_s \) is a linear homogeneous function of \( P_t \):

\[ P_s = k(P_t) = kP_t \]  \hspace{2cm} (9)

where \( k \) is a scalar. Salvadori and Steedman call such an economy a 'proportional-price' economy.\(^1\) The parameter \( k \) may be viewed as a transmission coefficient that takes into account the extra costs in terms of storage and transport incurred through use of the produced input. It can also be viewed as an interest factor since, ignoring inflation, we can write \( k = 1 + r \), where \( r \) is the interest rate.

In considering the supply response of total cereals, it will be observed that \( P_t \) cannot now change without \( P_s \), the price of seed, also changing relative to other prices. As a result, the fundamental inequality of profit maximisation can be reduced no further than:

\[ (p_t^1 - p_t^0)(y_t^1 - y_t^0) + k(y_s^1 - y_s^0) \geq 0 \]  \hspace{2cm} (10)

\[ (p_s^1 - p_s^0)(z_t^1 - z_t^0) \geq 0 \]  \hspace{2cm} (11)

\(^1\) There are obvious similarities between this analysis and that underlying the composite commodity theorem, due to Hicks (1939), which asserts that a group of commodities may be aggregated if their prices move in parallel. The difference here is that it is argued that the standard economic laws apply only to the aggregate commodity.
where \( Z = Y_t + kY_n \) is neither an output nor input (in the conventional dichotomy), nor, unless the transmission coefficient is unity, a net output. Here, \( Z \) is viewed as net output equivalent, in this case net grain equivalent, reflecting the fact that it represents total grain output net of seed use adjusted by a transmission factor that takes account of the extra cost involved in its use. Note that since \( Y_n < 0 \) by definition, \( Z < Y_t \).

It can be seen from expression (11) that this net grain equivalent is a non-decreasing function of the price of grain, \( P_t \), but that no similar statement can be made about the price responses of either total grain output, \( Y_t \), or seed, \( Y_n \). It is also clear that the own-price responses of all other netputs are normal, because changing their prices incur 'feedback' effects via the produced input.

As a result of the introduction of the produced input, the profit function in expression (3) can now be written as \( \pi(P_1, \ldots, P_{n-1}, k) \). Since it is convex in prices \( P_1, \ldots, P_{n-1} \) and the transmission coefficient, \( k \), repeated partial differentiation and application of Hotelling's Lemma now leads to the following inequalities:

\[
\frac{\partial^2 \pi}{\partial P_t^2} = \frac{\partial (Y_t + kY_n)}{\partial P_t} = \frac{\partial Z}{\partial P_t} 
geq 0 \tag{12}
\]

\[
\frac{\partial^2 \pi}{\partial P_i^2} = \frac{\partial Y_i}{\partial P_i} \geq 0 \quad (i=2, \ldots, (n-1)) \tag{13}
\]

\[
\frac{\partial^2 \pi}{\partial k^2} = P_t \frac{\partial Y_n}{\partial k} \geq 0 \tag{14}
\]

These expressions are restatements of the results obtained directly from the fundamental inequality of profit maximisation. In particular, (12) is a restatement of expressions (10) and (11). Expression (14) provides additional information: an increase in the rate of interest, reflecting perhaps an increase in the additional costs incurred in using a produced input, leads to a decrease in the use of the produced input (remembering that netput \( Y_n \) being an input in the conventional dichotomy, is negative). The other own-price responses represented by expression (13) are as expected.

Inequality (12) can be expressed in elasticity form as:

\[
P_t \frac{\partial Y_t}{Y_t} + \left( \frac{P_n Y_n}{P_t Y_t} \right) \left( \frac{P_t \partial Y_n}{Y_n \partial P_t} \right) \geq 0 \tag{15}
\]

which holds as long as the relationship between \( P_n \) and \( P_t \) is linear homogeneous. Expression (15) indicates that the feedback effect is likely to be more important where the value of produced inputs is large in relation to total production. Obviously, if none is used, \( Y_n = 0 \) and the possibility of perversity is eliminated. As the ratio \( P_n Y_n / P_t Y_t \) — i.e. the value of the produced input as a proportion of total production — tends to zero, the possibility becomes more and more negligible, but as it increases the likelihood of perverse supply response for total production becomes greater.

3. Aggregation without produced inputs

The existence of produced means of production also has implications for separability and aggregate supply response. First, consider technologies that have no produced inputs.
A transformation function is homothetically separable in the commodity subset $Y_1, \ldots, Y_m$ if it can be partitioned such that:

$$F(Y_1, \ldots, Y_m) = F(g(Y_1, \ldots, Y_m), Y_{m+1}, \ldots, Y_n) = 0$$  \hfill (16)

where $g(Y_1, \ldots, Y_m) = G[h(Y_1, \ldots, Y_m)]$, $G[h]$ is a monotonic function, $h(Y_1, \ldots, Y_m)$ is linearly homogeneous and $h(0) = 0$. Given such a partition, $Y = h(Y_1, \ldots, Y_m)$ can be viewed as an aggregate commodity and the single-stage constrained profit maximisation problem represented by (1) is equivalent to the following two-stage problem.$^1$

First stage.$^2$

$$\text{Max} \quad \hat{r} = \sum_{i=1}^{m} P_i Y_i \quad \text{subject to} \quad Y = h(Y_1, \ldots, Y_m)$$  \hfill (17)

Solving (17) yields the first-stage solutions:

$$Y_i^* = Y_i^*(P_1, \ldots, P_m, Y) \quad (i = 1, \ldots, m)$$  \hfill (18)

and:

$$\hat{r}^* = \sum_{i=1}^{m} P_i Y_i^*(P_1, \ldots, P_m, Y) = \hat{r}^*(P_1, \ldots, P_m, Y) = P Y$$

where $P = \hat{r}^*(P_1, \ldots, P_m)$ represents the price index for the aggregate commodity, $Y = h(Y_1, \ldots, Y_m)$, which can be factored out in the above expression because $g(Y_1, \ldots, Y_m)$ is assumed to be homothetic (see Gorman, 1959).

Second stage.$^3$

$$\text{Max} \quad \sum_{m+1}^{n} Y_{m+1} \quad \text{subject to} \quad F(G(Y), Y_{m+1}, \ldots, Y_n) = 0$$  \hfill (19)

Solving (19) yields the second-stage solutions:

$$Y^* = Y^*(P, P_{m+1}, \ldots, P_n)$$  \hfill (20)

$^1$ Weak separability, which was identified independently by Leontief (1947) and Sono (1961), requires that the marginal rate of substitution, $MRS_{ij}$, between two members of a partition is independent of any commodity outside the partition, i.e.:

$$\frac{\partial}{\partial Y_k} (MRS_{ij}) = \frac{\partial}{\partial Y_k} \left( \frac{F_i}{F_j} \right) = 0 \quad (i, j \in 1, \ldots, m; k \in (m+1), \ldots, n)$$

Strode (1957, 1959) showed that weak separability is sufficient for the first stage. Gorman (1959) showed that homothetic separability, which involves the additional requirement that the function $g(Y_1, \ldots, Y_m)$ is homothetic, is necessary to obtain the second stage. For a detailed discussion of separability, see Geary and Morishima (1973).

$^2$ The first stage may be viewed as a revenue maximisation problem in which the composition of a given level of aggregate $Y$ is determined.

$^3$ The second stage may be viewed as a profit maximisation problem in which the aggregate price $P$ is fixed and given by the solution to the first stage, while the aggregate quantity, $Y$, is allowed to vary.
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\[ Y_i^* = Y_i^*(P, P_{m+1}, \ldots, P_n) \quad (i = m+1, \ldots, n) \]  

(21)

The first-stage and second-stage solutions, and expressions (18), (20) and (21), may be combined to give:

\[ Y_i = Y_i(P_1, \ldots, P_m, Y^* (n^*(P_1, \ldots, P_m), P_{m+1}, \ldots, P_n)) \quad (i = 1, \ldots, m) \]  

(22)

\[ Y_k = Y_k(n^*(P_1, \ldots, P_m), P_{m+1}, \ldots, P_n) \quad (k = (m+1), \ldots, n) \]  

(23)

The signs of the individual and aggregate own-price responses can now be investigated using the same procedure as for the disaggregated model. From the first stage, we obtain the fundamental inequality:

\[ \sum_{i=1}^{m} (P_i^1 - P_i^0) (Y_i^1 - Y_i^0) \geq 0 \quad \text{for} \quad Y = h(Y_1, \ldots, Y_m) = \text{constant} \]  

(24)

Assuming the prices of the aggregated commodities, \( P_i (i = 1, \ldots, m) \) can be changed independently of one another, this implies that:

\[ (P_i^1 - P_i^0) (Y_i^1 - Y_i^0) \geq 0 \quad (i = 1, \ldots, m; Y = f(Y_1, \ldots, Y_m) = \text{constant}) \]  

(25)

Similarly, for the second stage, we obtain:

\[ (P^1 - P^0) (Y^1 - Y^0) + \sum_{i=m+1}^{n} (P_i^1 - P_i^0) (Y_i^1 - Y_i^0) \geq 0 \]  

(26)

Again, assuming the prices can be changed independently of each other (i.e. that there are no produced inputs), expression (26) implies that:

\[ (P^1 - P^0) (Y^1 - Y^0) \geq 0 \]  

(27)

\[ (P_i^1 - P_i^0) (Y_i^1 - Y_i^0) \geq 0 \quad (i = (m+1), \ldots, n) \]  

(28)

If the production technology can be represented by a twice-continuously differentiable profit function, the first-stage inequalities given in expression (25) imply that the own-price elasticities conditional on \( Y \) for the aggregated commodities, \( i = 1, \ldots, m \), must be non-negative:

\[ \left. \frac{P_i \partial Y_i}{Y_i \partial P_i} \right|_{Y} \geq 0 \quad (i = 1, \ldots, m) \]  

(29)

Similarly, the second-stage inequalities given by (27) and (28) can be expressed in elasticity form as, respectively:

\[ \frac{P \partial Y}{Y \partial P} \geq 0 \]  

(30)

\[ \left. \frac{P_i \partial Y_i}{Y_i \partial P_i} \right|_{Y} \geq 0 \quad (i = m + 1, \ldots, n) \]  

(31)
Inequalities (28) and (31) affirm the laws of output supply and input demand for the individual commodities \((i=(m+1), \ldots, n)\) which have not been aggregated. Inequalities (27) and (30) indicate that the same basic laws apply to the aggregate commodity as well. Homothetic separability therefore ensures consistent aggregation, which means that it is possible to compute indices for aggregate quantities and prices that may be used in applied production analysis in the same way quantities and prices of individual outputs and inputs are used.

The comparative statics for the aggregated commodities \((i=1, \ldots, m)\) may be further investigated by partially differentiating expression (22) and writing in elasticity from to obtain the gross own-price elasticities for the aggregated commodities:

\[
\frac{P_i \partial Y_i}{Y_i \partial P_i} = \frac{P_i \partial Y_i}{Y_i \partial P_i} + \left( \frac{Y_i \partial Y_i}{Y_i \partial Y_i} \right) \left( \frac{P_i \partial Y_i}{Y_i \partial Y_i} \right) \left( \frac{P_i \partial \hat{k}}{Y_i \partial \hat{k} P_i} \right) 
\]

\((i=1, \ldots, m)\) \tag{32}

Since the aggregator function \(h(Y_1, \ldots, Y_m)\) is linear homogeneous:

\[
\frac{Y_i \partial Y_i}{Y_i \partial Y_i} = 1 
\]

\((i=1, \ldots, m)\) \tag{33}

Furthermore, application of the Envelope Theorem to the first-stage solution gives:

\[
\frac{\partial h^*(P_1, \ldots, P_m, Y)}{\partial P_i} = \frac{\partial h^*(P_1, \ldots, P_m, Y)}{\partial P_i} = Y_i(P_1, \ldots, P_m, Y) 
\]

\((i=1, \ldots, m)\)

so that:

\[
\frac{P_i \partial \hat{k}}{Y_i \partial \hat{k} P_i} = \frac{PY}{P_i} 
\]

\((i=1, \ldots, m)\) \tag{34}

Thus, expression (32) can be rewritten as:

\[
\frac{P_i \partial Y_i}{Y_i \partial P_i} = \frac{P_i \partial Y_i}{Y_i \partial P_i} + \left( \frac{P_i \partial Y_i}{Y_i \partial Y_i} \right) \left( \frac{P_i Y_i}{PY} \right) 
\]

\((i=1, \ldots, m)\) \tag{35}

The first term on the right-hand side of equation (35) corresponds to movement along the transformation curve described by the aggregator function, \(Y = h(Y_1, \ldots, Y_m)\) with \(Y\) held fixed, brought about by a change in own-price; i.e. it represents a pure substitution effect. The second term on the right-hand side corresponds to a shift of the transformation curve as a result of the consequent change in the price index of the aggregate commodity, \(P = \hat{k}(P_1, \ldots, P_m)\) relative to other prices \(P_{m+1}, \ldots, P_i\) i.e. it represents an expansion effect.\(^1\)

Since the revenue shares \(P_i Y_i / PY\) must be positive as well, inequalities (29) and (30) ensure that the individual gross own-price elasticities for aggregated commodities must be positive:

\[
\frac{P_i \partial Y_i}{Y_i \partial P_i} \geq 0 
\]

\((i=1, \ldots, m)\) \tag{36}

\(^1\) These substitution and expansion effects are, of course, analogous to the substitution and income effects of consumer theory.
Thus, in addition to their aggregate supply response being positive, the members of the aggregate must also be non-decreasing functions of their own prices. The two-stage optimisation procedure maintains the basic laws of output supply and input demand for all the individual outputs and inputs in the model as well as for any (consistent) aggregates of these commodities.

With regard to the cross-price elasticities, partial differentiation of expression (22), transformation into elasticity form, and use of equation (33) shows that:

$$\frac{P_i \cdot \partial Y_i}{Y_i \cdot \partial P_k} = \frac{P_k \cdot \partial Y_k}{Y_k \cdot \partial P_i} \quad (i = 1, \ldots, m; k = (m+1), \ldots, n) \quad (37)$$

where the gross price elasticities may be written as:

$$e_{ik}(P_1, \ldots, P_n) = \frac{P_k \cdot \partial Y_i}{Y_i \cdot \partial P_k} \quad (i, k = 1, \ldots, n)$$

Expression (37) shows that homothetic separability of the transformation function in accordance with partition (16) implies that the gross price elasticities must satisfy the equality restrictions:

$$e_{ik}(P_1, \ldots, P_n) = e_{jk}(P_1, \ldots, P_n) \cdot \quad (i, j = 1, \ldots, m; k = (m+1), \ldots, n) \quad (38)$$

Expression (38) confirms a result obtained by Chambers (1988, pp. 152–153). However, when we partially differentiate expression (23), we obtain:

$$e_{ik} = \frac{P_i \cdot \partial Y_k}{Y_k \cdot \partial P_i} = \left( \frac{P \cdot \partial Y_k}{Y_k \cdot \partial P} \right) \left( \frac{P \cdot \partial P}{P \cdot \partial P} \right) \quad (i = 1, \ldots, m; k = (m+1), \ldots, n)$$

Clearly, the equality restrictions given in (38) are not symmetric:

$$e_{ik}(P_1, \ldots, P_n) \neq e_{jk}(P_1, \ldots, P_n) \quad (i, j = 1, \ldots, m; k = (m+1), \ldots, n)$$

However, if we write the elasticities of transformation defined by Diewet (1974, p. 144) as:

$$o_{ik}(P_1, \ldots, P_n) = \frac{e_{ik}(P_1, \ldots, P_n)}{S_i} \quad (i, k = 1, \ldots, n)$$

where $$S_i = P_i Y_i / P Y$$ and $$o_{ik} = o_{ki}$$ then weak homothetic separability does mean that:

$$o_{ik}(P_1, \ldots, P_n) = o_{jk}(P_1, \ldots, P_n) \quad (i, j = 1, \ldots, m; k = (m+1), \ldots, n) \quad (39)$$

Diewet’s elasticities of transformation are analogous to the partial elasticities of substitution first defined by Allen (1938, pp. 503–509) for single-output technologies. Berndt and Christensen (1973) have shown that, in the single-output case, weak homothetic separability is equivalent to certain equality restrictions on the Allen partial
elasticities of substitution. Expression (39) represents a generalisation of these equality restrictions for multi-product technologies, and provides a means by which aggregation possibilities can be investigated via estimation of a multi-product profit function. Essentially, if any of the equality restrictions are found to be invalid, it must be concluded that the relevant commodities are not weakly homothetically separable and the corresponding aggregation/partitioning procedure is not consistent.

4. Aggregation with produced inputs

Before looking at the proportional-price case where \( P_n = kP_1 \), consider the effect of the more general relation \( P_n = f(P_1) \) on the two-stage aggregation procedure. Because the optimal aggregate price derived from the first stage problem, \( P = \hat{P}(P_1, \ldots, P_m) \), and the seed price, \( P_n = f(P_1) \), are both functions of the grain price, \( P_1 \), we cannot now claim that \( P_n \) and \( P \) are independent. Thus, expression (26) can be reduced no further than:

\[
(P^1 - P)^{Y^1 - Y^0} + (P_n^1 - P_n^0)(Y_n^1 - Y_n^0) \geq 0
\]

Expression (40) indicates that, just as it was not possible to claim that the supply response of total grain in the disaggregated model must be non-negative, we cannot now claim that the aggregate supply response must be non-negative.

The inequalities given in (10) and (40) are analogous statements for the single-stage and two-stage models respectively. If the production technology can be described by a twice continuously differentiable profit function, expression (10) can be rewritten in elasticity form as expression (15). Similarly, if \( P_n \) is a linear homogeneous function of \( P_1 \) such that \( P_n = f(P_1) = kP_n \), inequality (40) can be written in elasticity form as:

\[
\frac{P}{Y} \frac{\partial Y}{\partial P} + \left( \frac{P}{Y} \frac{\partial Y}{\partial P} \right) \left( \frac{P_n}{Y} \frac{\partial Y}{\partial P} \right) \geq 0
\]

The aggregate own-price elasticity need not necessarily be positive given the presence of a produced input and the assumed aggregation procedure.

The effect of the produced input on the own-price elasticities of the commodities that are elements of the aggregate commodity can be examined by combining the effects of the two stages. Partial differentiation of (22), transformation into elasticity form and application of equations (33) and (34) yield the gross own-price elasticities:

\[
\frac{P_1}{Y} \frac{\partial Y}{\partial P} + \left( \frac{P_1}{Y} \frac{\partial Y}{\partial P} \right) \left( \frac{P_1}{Y} \frac{\partial Y}{\partial P} \right) \geq 0
\]

\[
(i = 2, \ldots, m)
\]

Since the signs of:

\[
\frac{P}{Y} \frac{\partial Y}{\partial P} \quad \text{and} \quad \frac{P_1}{Y} \frac{\partial Y}{\partial P_i}
\]

are indeterminate, the signs of both the gross own-price elasticities given in (42) and (43) are also indeterminate.
Do supply curves slope up?

Expression (42) reflects the conclusion drawn from the single-stage model regarding the supply response of commodity 1, cereals inclusive of seed production. However, it is now apparent that, as a result of the chosen aggregation procedure, the signs of the own-price elasticities for the other members of the aggregate commodity are also indeterminate.

Furthermore, partial differentiation of expressions (22) and (23) with respect to cross prices shows that, because of the relationship between $P_1$ and $P_n$, the equality restrictions given in expressions (38) and (39) are not satisfied in the presence of the produced input. This implies that commodities 1, ... $m$ are not homothetically separable from commodities ($m+1), .. n$, indicating that the conditions for consistent aggregation have been violated.

It will be recalled from the original single-stage model presented above that, although total grain, $Y_1$, and seed, $Y_m$, could not be said individually to obey the laws of output supply and input demand, a composite commodity, $Z = Y_1 + kY_m$, called the net output equivalent, would be expected to do so. This suggests that the further breakdown of the law of supply with regard to the aggregate output, $Y = h(Y_1, .., Y_m)$, has arisen from an inconsistent aggregation procedure and could be rectified by adopting a different, consistent, procedure. If aggregation is required, net output equivalents should be found first; these may then be combined with other outputs to obtain indices corresponding to consistent aggregates. However, this procedure assumes a proportional price economy (i.e., linear homogeneous relationships of the kind $P_n = f(P_1) = kP_1$) and requires knowledge of the transmission coefficients. If this is not the case, either because the relationships are not linear homogeneous (i.e., $P_n = f(P_1) = kP_1$) or because the transmission coefficients cannot be estimated, consistent aggregation may not be possible.

5. An empirical example

An example can further show that cases that violate the laws of supply and demand and the requirements for consistent aggregation may not only be theoretically possible but also empirically relevant. This section reports the results of an investigation of the importance of produced means of production in UK agriculture using an agricultural policy model developed at the University of Manchester.

A number of agricultural commodities, such as seed, feedgrain, silage and breeding livestock, may be classified as produced inputs. The policy model captures and simulates interactions between agricultural sub-sectors (cereals, livestock, dairy products, horticulture) through a system of econometrically estimated price equations, each of which contains output or input quantities and institutional prices such as the threshold and intervention prices for wheat and barley. Since output prices in some equations appear as input prices in others, a change in an institutional price variable in one sub-sector can affect other sub-sectors via the relevant prices. Examples of such interactions are the relationships between cereal prices (as conventionally defined outputs) and the prices of feedstuffs used (as conventionally defined inputs) in the livestock sub-sectors. These are relevant because the analysis of production with produced inputs outlined above sprang from such relationships.

The policy model was used to simulate a permanent 20% increase in the intervention and threshold prices for wheat and barley for the period 1978 to 1982. The results of the 'base' run without these changes and the 'modified' run with them for indices of
aggregate agricultural output and total feedstuffs and their respective prices are presented in Table 1.\footnote{For a full description of the policy model see Burton (1992). For a more detailed discussion of the simulation exercise reported in this section see Ozanne (1992, ch. 5; 1993).}

From the left side of the table, it can be seen that the 20% increase in institutional prices leads to a 3.7 ($4.1/112.2$)% increase in the price index for aggregate output by the end of the simulation period. However, the level of aggregate output has decreased by about 2%, suggesting that UK agriculture exhibits perverse supply response at the aggregate level with a short-run, mutatis mutandis, elasticity of approximately $-0.55$.

The explanation for this perverse result may be found by looking at the right side of the table, where it is seen that the index for total feedstuffs increases by about 14%. This increase occurs by the end of the first year and arises from the compound feed price equations embedded in the policy model, which relate the prices of the three main compound feeds used in the UK to the price of barley. Essentially, increases in the cereal policy price variables lead to increases in the market prices of wheat and barley and these lead to increases in the prices of feedstuffs. This in turn leads to decreases in the use of compound feed in the livestock sectors, reflected in the 6.7% decrease in the index number for total feedstuffs, and associated decreases in the output from the livestock sectors. It is these latter decreases which, when combined with changes in the field crop and horticultural sectors, lead to the 2% decrease in aggregate output noted above.

6. Conclusions

It has been shown that if the conventional distinction between outputs and inputs is relaxed to allow for the existence of produced means of production, the laws of output
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supply and input demand break down as a result of the abrogation of the ceteris paribus assumption. Empirical work on a model of UK agriculture supports this view: evidence has been found of perverse aggregate supply response resulting from a feedback effect induced by the use of feedgrain, an output from the cereal sector, as produced input in the livestock sectors, together with the greater responsiveness in the latter.

This result, which arises not so much from the physical presence of produced inputs in the production process as from direct relationships between the prices of commodities hitherto viewed as distinct outputs and inputs, is consistent with the broader, Sraffian critique of neoclassical economics. Sraffian economists, however, have tended to concentrate their criticisms of Marshallian partial analysis on the input rather than the output side of production processes—in particular, criticism of neoclassical capital theory. Furthermore, when they have addressed supply questions, they have been concerned with individual commodities rather than joint commodity production—corn used to produce corn rather than corn used to produce beef and milk as well—or aggregate supply. This paper has extended the analysis to cover these topics.

Finally, it should be noted that the mechanism generating perversity outlined here is not inconsistent with economically rational behaviour. Rather, it arises from the neglect of produced means of production in conventional production theory and the need to take account of the existence of produced inputs when aggregating commodities. Computation of net output equivalents, if practicable, should remove apparent perversities and re-establish the laws of output supply and input demand at whatever level of aggregation analysis is conducted. If, however, such allowance is not made for produced means of production, there is no justification for the belief that perversity does not occur. Given that one of the main criticisms of the Sraffian school is that its hypotheses have seldom been tested empirically, this is thought to be a worthwhile finding.

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