FIRST ESSAY

THE THEORY OF VALUE OF DAVID RICARDO

An attempt at a rigorous analysis

No study made by man can be said to be true knowledge unless it has been mathematically demonstrated. 
Leonardo da Vinci

I assert that the amount of true science to be found in any knowledge in the natural sciences is no greater than the amount of mathematics to be found in it. 
E. Kant

The mathematical method is invariably applicable as a tool whatever the subject under investigation. 
H. C. Carey

Mathematics is the prototype of deductive science; as its complication increases, every deduction becomes mathematical and it is precisely the complexity of deductive reasoning which makes essential the language of mathematical signs. 
E. Wundt

Anyone who accepts abstract analysis ought also to accept all its natural logical tools (including the mathematical method). 
L. Slonimsky

Mathematics should be used where it is impossible to arrive at the truth without its assistance. Were there to be such antipathy to mathematical analysis in other branches of knowledge as there is in political economy, we should have remained in total ignorance of the most important laws of nature. 
J. H. von Thünen

Writers (economists) would appear to have formed a false idea concerning the way in which mathematical analysis is applied to the theory of wealth. They have imagined that the use of symbols and formulae can have no other purpose apart from numerical operations... However, those who have a knowledge of mathematics know that mathematical analysis is not concerned solely with figures, that it also serves to establish relationships between quantities which cannot be numerically defined, between functions whose law cannot be expressed by algebraic symbols... It is natural to use mathematical notations whenever the need is to establish relationships between quantities. Even were mathematical notations not to be strictly necessary, it would obviously be unscientific to reject them solely because they are not equally comprehensible to every reader, given that they can facilitate the exposition, make it more accurate, lead to more detailed explanations and avoid the digressions of vague argument... 
A. Cournot
1. INTRODUCTION: THE THEORY OF 'PRODUCTION COSTS' BEFORE RICARDO

The simplest formula expressing the relationship between price and production cost is

\[ \text{Price } \geq \text{ production costs.} \] (1)

This formula is not a result of a scientific analysis of the phenomena of economic life, but a simple statement of the self-evident fact that production cannot continue (at least for any appreciable length of time) if the price of the product does not cover the costs incurred.

It is strange, therefore, to ascribe the discovery of this truth to any given economist.\(^1\) What had to be done to pass from this fact to a complete theory of production costs in economics was, first, to state the laws defining the magnitude of that surplus which is incorporated in a price over and above the costs incurred; second, to analyse actual costs incurred in production by the entrepreneur. The first problem was not satisfactorily solved even by Smith: he, of course, defines profit in terms of the relationship between demand and supply of capital, i.e. by a feature dependent on market conditions. Very little had been done before Smith also for the analysis of real production costs in the narrow sense, \textit{not including profit}.

Note that we completely disregard, as having nothing in common with science, all the unsubstantiated assertions concerning laws of value proclaimed by various 'thinkers' without any more foundation than the 'authority' of their propounders. They include, for example, the 'theory' which states that value is determined by the amount of labour expended on the production of the product (Franklin and Petty)\(^2\) or by the amount of labour and land (Cantillon, Locke and others).\(^3\)


\(^2\) See The \textit{Works of Benjamin Franklin}, edited by J. Sparks, 1856 ['Trade in general being nothing else but the exchange of labor for labor, the value of all things is . . . most justly measured by labor'], Vol. 2, p. 267. Petty, however, attempts to make his assertion less arbitrary by stipulating \textit{ceteris paribus}: 'If a man can bring to London an ounce of Silver out of the Earth in Peru in the same time that he can produce a bushel of Corn, then one is the natural price of the other; now if by reason of new and more easie Mines a man can get two ounces of Silver as easily as formerly he did one, then Corn will be as cheap at ten shillings the bushel, as it was before at five shillings \textit{ceteris paribus}'; W. Petty, \textit{A treatise of taxes and contributions}, London, 1662, Ch. 5, pp. 50-1 of the 1899 edition. The approach is undoubtedly scientific, but even so it limits the very meaning and sphere of application of the law as stated.

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We find the most detailed analysis of production costs in the works of Smith's immediate forerunner, Steuart. According to Steuart's theory (Principles of Political Economy, 1767, Book 2) the actual value of a thing is made up of the following elements: 'The value of the workman's subsistence and necessary expense both for supplying his personal wants and providing the instruments belonging to his profession' and 'the value of the materials, that is the first matter employed by the workman'.

According to Steuart, these three elements combined define the lower limit, below which the market price of the product cannot fall. What we see here is essentially a simple, detailed list of the expenditure which the capitalist producer incurs (as before, profit is related to market conditions, i.e. to the supply and demand of the given commodity); no traces of scientific analysis are, as yet, to be noted in this 'theory' of production costs. The only exception is the subsistence wage theory. Even before Adam Smith's work appeared, the theory had become established in economic science that wages tended toward the means of subsistence. It is even possible to find a fairly detailed development of the concept of the 'means of subsistence' (Cantillon, Petty and Turgot).1

However, it was only in Adam Smith's work that an explanation was given of the mechanism of the process by which wages are constantly maintained at the level of the means of subsistence. To sum up what has been said, we may express the state of the theory concerning the relationship between value and production costs at a time immediately preceding the appearance of Adam Smith's work by the following formula:

\[
\text{Price} = \text{outlay on wages} \times \left(\text{the number of working days} \times \text{daily subsistence of the worker in terms of the product} \times \text{the price of the produce consumed by the workers}\right) \\
+ \text{outlay in the replacement of tools and materials} \times \text{the quantity of tools and materials consumed in production} \\
+ \text{the total of profit} \\
+ \text{rent} \ (= \text{the sum paid for 'the assistance of natural forces').}
\]

(2)

The quantities set in italics are the unknowns.

Naturally, at this stage of development the theory of production costs

1 See Cantillon, 1755, who is quoted by Smith himself.

W. Petty defines the value of the average daily pay by what the worker needs to live, work and reproduce himself (The Political Anatomy of Ireland, London, 1691, p. 64, p. 181 of 1899 edition). Turgot states 'Workers are continuously obliged to lower the price one against the other. For all kinds of labour it must happen and does in fact happen that the worker's wage is limited to what is necessary for his subsistence' (A. R. J. Turgot, Réflexions sur la formation et la distribution des richesses, 1770, § vi, p. 10 of the 1844 edition).
fully merits the reproach so often levelled at the theory of production costs in general (consequently also in its fully developed form), that it defines price from prices, that it defines one unknown from other unknowns.\(^1\) The problem facing Adam Smith was not an easy one, and it is therefore not surprising that his solution of it was far from complete. It was only in the writings of his successor, Ricardo, that the theory of production costs was completed. Nevertheless, Smith did contribute a very great deal to the correct solution of the problem. Above all, we find in Adam Smith a correct formulation of the problem to be solved which is undoubtedly very important for its correct solution.

Smith states that ‘the relative or exchangeable value of goods’ is determined by ‘rules which men naturally observe in exchanging them either for money or for one another’.\(^2\) This first eliminated any question of the intrinsic value of commodities: the object of research should be merely the relative value of commodities, their ratio of exchange (the term is borrowed from Jevons\(^3\)) to avoid confusion arising from the use of the word value in two senses: exchange value and use value; use of the term ‘ratio of exchange’ eliminates the need for any qualification concerning the different meanings of the word ‘value’, such as is made by Smith\(^4\) and Ricardo.

Smith then proceeds to an analysis of the concept of production costs or, to be more precise, to an analysis of those elements from which they are made up for the capitalist entrepreneur. In his theory of wages Adam Smith merely develops and provides greater basis for the hypothesis

\(^1\) E.g. Sieber, 1885, p. 109. Sieber quotes the words of Kamorzhinsky: ‘It may be objected against theories of production costs that they explain the price of a good not from such elements as would be independent of price, but from other prices, because production costs are calculated from the price of all the goods needed for production’. Sieber adds to this: ‘The formulation of the question of production costs given by us is a clear expression of the discontent which arises in the minds of some, unfortunately very few of the newest economists when discussing terms which only seemingly contain a known and definite meaning. . . .’

\(^2\) A. Smith, An enquiry into the nature and causes of the wealth of nations (1776), Book i, Ch. 4, p. 42, of the 1814 edition.

\(^3\) Jevons uses the term ‘the ratio of exchange’; Zalesky translates this in his thesis, quite unsuccessfully in our opinion, by the words obmen ékonom [exchange relation]. See Zalesky Uchenie o tsennosti [Theory of value], Kazan, 1883, Book ii, p. 122.

\(^4\) ‘The word value’, states Smith, ‘has two different meanings and sometimes expresses the utility of some particular object, and sometimes the power of purchasing other goods which the possession of that object conveys. The one may be called “value in use”; the other, “value in exchange”’ (Book i, Ch. 4, p. 42 of the 1814 edition).

Although Adam Smith explained the concept of exchange value excellently he did not venture to give an equally precise definition of ‘use value’ or ‘usefulness’. The first completely correct definition of the concept of ‘usefulness’ is found in F. Galliani, an Italian economist of the last century: ‘I call utility the attitude of an object to procure us happiness’, Della moneta (1750), Ch. 2, p. 59.
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already stated by preceding economists that real wages have a tendency to coincide with the essential means of subsistence of the worker.¹

The main changes made by Smith to the formula of production costs relate to the second and third terms of the second part of equation (2). Smith was the first to point out that the second term, the value of the tools and materials used in production, could invariably be broken down, in its turn, into wages, profit and rent (by ‘profit’ and ‘rent’ we shall invariably understand the sum of profit and the sum of rent in money) so that all production costs may be reduced to the three elements: wages, profit and rent. These three parts, states Smith,

'seem either immediately or ultimately to make up the whole price of corn. A fourth part, it may perhaps be thought, is necessary for replacing the stock of the farmer, or for compensating the wear and tear of his labouring cattle, and other instruments of husbandry. But it must be considered that the price of any instrument of husbandry, such as a labouring horse, is itself made up of the same three parts; the rent of the land upon which he is reared, the labour of tending and rearing him, and the profits of the farmer who advances both the rent of this land, and the wages of this labour. Though the price of the corn, therefore, may pay the price as well as the maintenance of the horse, the whole price still resolves itself either immediately or ultimately into the same three parts of rent, labour and profit.'

This hypothesis is subsequently extended by Smith to all other products (Smith, Book 1, Ch. 6, p. 81 of 1814 Edition).

Smith himself notes instances when one (and sometimes even two) of these three basic elements of price are absent, so that the price of the product is reduced in the last analysis to only two elements, wages and profits (Ibid.).

In view of the inconsistency of Smith’s views on rent, we shall subsequently consider only the latter case. Since, as is explained by Ricardo, the cause for the appearance of rent is that different portions of the same commodity sold in the same market (and consequently commanding the same price) are produced with different costs, in order to exclude rent from price, we have to make the conventional assumption that all units of a given commodity are produced with equal costs (and as a corollary of this, that all portions of the capital employed in the given production are equally productive). Formula (2) then becomes:

\[ X_A = (n_A a X_o + n_2 a X_o + \ldots + n_n a X_o) + (y_A + y_1 + y_2 + \ldots + y_n) \] (3)

¹ Smith himself cites Cantillon on this question (Smith, Book 1, Ch. 8, p. 110 of 1814 edition); the theory of the ‘iron’ law of wages reached its final development, of course, in the writings of David Ricardo, and we shall therefore defer closer examination of this question until we analyse Ricardo’s theory of value.
I. The theory of ‘production costs’ before Ricardo

where $X_A$ is the price of product $A$; $n_A$, $n_1$, $n_2$, ..., $n_m$ are the number of working days expended in production; $a$ is the amount of a product, e.g. corn, consumed by a worker in a day (in order to simplify the formula we assume that a worker consumes one product, e.g. corn, which is of course a simplification that Ricardo also makes in his analysis; we shall see subsequently that nothing is altered in our analysis if we accept that the workers consume several products); $X_a$ is the price of product $a$; $y_A$, $y_1$, $y_2$, ..., $y_m$ are the profits incorporated in the price of product $A$; these include both the profit obtained by the producer of product $A$ himself, and the profit of the producers of the tools and materials consumed in the production of product $A$. Or if

$$\begin{cases} 
    n_A + n_1 + n_2 + \cdots + n_m = N_A \\
    y_A + y_1 + y_2 + \cdots + y_m = Y_A
\end{cases} \quad (4)$$

then we obtain

$$X_A = N_A ax_a + Y_A, \quad (5)$$

where $N_A$ is the total sum of the labour directly or indirectly expended in the production of product $A$, and $Y_A$ is the total sum of the profit received by all the producers involved directly or indirectly (i.e. by the production of materials and tools) in the production of commodity $A$.

Therefore, the total price of product $A$ is, in the absence of rent, made up of only two elements: wages and profit. Smith repeatedly objected to this hypothesis; these objections have once again been advanced comparatively recently, as an argument against the labour theory of value, by economists of the ‘Austrian school’, supporters of the theory of marginal utility.

What these objections amount to is that because capital is essential in all branches of production in the modern economy, it is impossible to eliminate the element of capital when calculating production costs. For the production of capital it is once again capital which is invariably needed. It is asked how it is possible to calculate the amount of labour expended for the production of a given economic good from the very beginning of history, when man managed without capital, down to the present time. There is no doubt that at present capital is invariably produced by capital; it is also correct that it is an impossible task to calculate the amount of labour expended in a given product from the time of the creation of the first capital by labour alone. However, there is no need for such a calculation: the sum of the labour expended on the production of a given product may be determined without such historical digressions.

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Let us denote by $N$ the total amount of labour directly and indirectly expended on the production of a unit of commodity $A$; let the amount of labour directly consumed in production be $n_a$; let several kinds of 'technical capital' $K_1, K_2, \ldots, K_m$ be involved in production; let there be consumed in production $1/m_1$ of the capital $K_1$, $1/m_2$ of the capital $K_2, \ldots, 1/m_M$ of the capital $K_M$; further, let the amount of labour directly and indirectly expended on the production of the capital $K_1$ be $N_1$, that expended on production of the capital $K_2$ be $N_2, \ldots$, that expended on production of the capital $K_M$ be $N_M$, in which case the total sum of the labour expended on the production of a unit of commodity $A$ will be:

$$N_A = n_a + \frac{1}{m_1} N_1 + \frac{1}{m_2} N_2 + \cdots + \frac{1}{m_M} N_M. \quad (6)$$

Since $n_A$ and $m_1, m_2, \ldots, m_M$ are here quantities given by the technical conditions of production of the product $A$, $N_A, N_1, N_2, \ldots, N_M$ are unknowns.

Other capital goods, some of which are included in this series and others not, are involved in their turn in the production of capital goods $K_1, K_2, \ldots, K_M$, to which the quantities of labour $N_1, N_2, \ldots, N_M$ of this equation correspond. Let the number of all the different capital goods involved both directly and indirectly in the production of the product $A$ be $M$ (the number is always finite).\(^1\)

For the amount of labour needed for the production of any capital $K_i$ out of the $M$ capital goods it is obviously possible to compile an equation completely similar to equation (6); quantities $N$ corresponding to the capital goods involved in the production of the capital $K_i$ will be incorporated in the second part of such an equation, and since $M$ is a finite number, we shall obtain $M$ equations with $M$ unknowns ($N_1, N_2, N_3, \ldots, N_M$); adding in equation (6), we obtain a system of $(M+1)$ equations with $(M+1)$ unknowns ($N_A, N_1, N_2, N_3, \ldots, N_M$) which is always adequate for the determination of $N$, giving the required sum of labour expended on the production of the product $A$. Therefore, without any digressions into the prehistoric times of the first inception of technical capital, we can always find the total sum of the labour directly and indirectly expended on the production of any product under present-day production conditions, both of this product itself and of those capital goods involved in its production. As we have seen, the fact that all capital under present-day conditions is itself produced with the assistance of other capital in no way hinders a precise solution of the problem.

It should not, however, be thought that the whole system of our $(M+1)$ equations is indispensable for the determination of the total

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\(^1\) This is because, despite the diversity and complexity of present-day technology, even the number of all possible qualitatively different capital goods is always a finite quantity.
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labour expended on the production of any product \( I \); all the unknowns incorporated in the expression of this sum may frequently be excluded from the smallest number of equations. For example, let the capital good \( K_1 \) be involved in production of the product; the capital goods \( K_2 \) and \( K_3 \) in production of capital good \( K_1 \); \( K_1 \) and \( K_2 \) in production of \( K_3 \) and so on; in that case, using the same notations as before, we shall have a system of four equations with four unknowns, from which \( N_1 \) is determined by successive substitution:

\[
\begin{align*}
N_1 &= n_1 + \frac{1}{m_1} \cdot N_1 \\
N_2 &= n_2 + \frac{1}{m_2} \cdot N_2 + \frac{1}{m_3} \cdot N_3 \\
N_3 &= n_3 + \frac{1}{m_4} \cdot N_1 + \frac{1}{m_5} \cdot N_3 \\
N_3 &= n_3 + \frac{1}{m_6} \cdot N_1 + \frac{1}{m_7} \cdot N_2
\end{align*}
\]

It is, of course, possible to imagine even simpler cases.\(^1\)

Thus the production costs formula may always be reduced to the expression:

\[ X_A = N_{AA}X_a + Y_A \]  

\(^*\) Ed. note. The system of equations given by Dmitriev does not lend itself to solution by substitution: commodities 1 and 3 require commodity 2 and vice versa; commodity 1 requires commodity 3 and vice versa, i.e. there are feedbacks in the determination of the price system by substitution. However, Dmitriev is right in thinking that the solution of the system is simplified by the existence of a number of zero-technical coefficients (for instance, when the input-output matrix is triangular, or quasi-triangular) and that it can sometimes be solved by substitution, although in the following footnote he rejects the idea of progressive 'layers' of production.

\(^1\) We are absolutely unable to agree with the opinion of Tugan-Baranovsky who, while quite correctly opposing von Wieser's objection to the labour theory of value, states that 'In passing from one branch of industry to another manufacturing goods of increasingly higher orders relative to our product ... we ultimately arrive at branches of industry which manufacture their own constant capital (in the terminology of Marx)' (Vurdieckého Vestič, October 1890, p. 223). Such a completely arbitrary assumption deprives the solution of the problem of the generality which is required. Nor can we accept, either in form or in content, the 'mathematical' solution of the problem which he proposes at the end of the paper; the conclusion at which he arrives may be obtained only thanks to the completely arbitrary and unreal assumption that the denominator of an infinite descending progression remains constantly the same. Furthermore, it is impossible to equate incommensurate quantities.
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If we take the appropriate formula for any product, \( B, C, \ldots \)

\[
\begin{align*}
X_B &= N_B a X_a + Y_B \\
X_C &= N_C a X_a + Y_C
\end{align*}
\]

(9)

\[
\begin{align*}
X_{AB} &= \frac{X_A}{X_B} = \frac{N_A a X_a + Y_A}{N_B a X_a + Y_B} \\
X_{AC} &= \frac{X_A}{X_C} = \frac{N_A a X_a + Y_A}{N_C a X_a + Y_C}
\end{align*}
\]

(10)

and so on, where \( X_{AB} \) will denote the value of product \( A \) in terms of \( B \), i.e., the number of units of the product \( B \) given in the market for a unit of the product \( A \). For \( X_{AB} \) to be known, the quantities \( Y_A \) and \( Y_B \) must be given; Adam Smith's second important contribution to development of the theory of value was the analysis of these quantities. Smith first notes that the quantity \( Y \) is always related to the sum of the capital expended in production and to the time during which it is in circulation (in the production concerned). If, therefore, we denote the capital by \( Z \) and the time by \( T \) and assume that all the other quantities on which the amount of profit may depend are constant, we shall have:

\[ Y = F(Z, T). \]

(11)

If we denote the sum of the profit attained in the given production \( A \) by a unit of capital (expressed in the same unit of value as the sum of the profit) in a time unit\(^2\) by \( r_A \) (which we shall refer to as 'the rate of profit in the production of \( A \)'), the sum of the profits attained in the same production by \( Z \) units of capital in unit time will be \( Z r_A \), if we take into consideration our assumption made above (with the methodological aim of excluding the phenomenon of rent from our analysis) that all capital goods expended in production are equally productive. Adding this profit in unit time to the initial capital \( Z \), we obtain \( Z + Zr_A = Z(1 + r_A) \); if this

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1 We adopt the condition \( X_{ab} = X_a/X_b \) without special proof as being sufficiently evident; for a detailed proof see L. Walras, *Éléments d'économie politique pure*, Lausanne, 1874, Lesson 11, pp. 153-63 of the English translation of the 1926 edition, by W. Jaffé, *Elements of pure economics*, London, 1934. Walras shows by mathematical analysis that 'We do not have perfect, or general market equilibrium unless the price of any two commodities in terms of the other is equal to the ratio of the prices of these two commodities in terms of any third commodity' (p. 157 of the English edition). See also Smith, *Wealth of Nations*, Book I, Chs. 6 and 10.

2 The 'period of the production concerned' may be taken as the unit in the interests of simplicity.
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sum is left in production, after a further unit of time (assuming that the conditions of production remain unaltered) we have: \( Z(1 + r_a)(1 + r_a) = Z(1 + r_a)^2 \), and repeating \( T \) times, we shall have \( Z(1 + r_a)^T \), from which the sum of the profit from \( Z \) units of capital in \( T \) units of time will be:

\[
Y_A = Z(1 + r_a)^T - Z = Z[(1 + r_a)^T - 1].
\]  
(12)

(See Smith, Book i, Ch. 9, p. 160–1 of the 1814 edition.)

In setting this expression of profit as a function of the sum of capital and of time in our formulas of production costs, we obtain for the simplest case in which \( N_A \) working days are expended on the production of a unit of the product without the participation of technical capital:

\[
Y_A = N_a a X_a[(1 + r_a)^{T_A} - 1]
\]  
(13)

where \( T_A \) will denote the time having elapsed between expenditure of the capital \( N_a a X_a \) (it is assumed in the interests of simplicity that the whole sum is expended simultaneously) and sale of the product.

In addition to the labour directly expended in the production of \( A \), let there also be expended some capital \( K_1 \), and let this capital itself be produced by \( n_1 \) days of labour with the assistance of the capital \( K_2 \), and let us assume in the interest of simplicity that this capital \( K_2 \) is itself produced without the participation of new technical capital by \( n_2 \) days of labour (this methodological approach is, of course, regularly used by Ricardo in his researches on value with the object of simplifying formulas and thus making them more suitable for analysis). Let the capitals \( K_1 \) and \( K_2 \) involved in the productions \( A \) and \( K_1 \) be completely used up in production without residue (such an assumption will undoubtedly simplify the formulas more than the equally arbitrary assumption made by Ricardo that capital goods are perpetual).

Suppose the time expended in production of the capital \( K_2 \) be \( T_{K_2} \), in which case, assuming in the interests of simplicity that the whole sum expended on production of the capital \( K_2 \), which equals \( n_a a X_a \), is expended simultaneously, we shall have as an expression of the price of the capital \( K_2 \):

\[
X_{K_2} = n_a a X_a (1 + r_{K_2})^{T_{K_2}}
\]  
(14)

where \( r_{K_2} \) is the 'rate of profit' in the production of \( K_2 \). Further, let the time expended on production of the capital \( K_1 \) be \( T_{K_1} \), in which case, deliberating in the same way as before (and making the same arbitrary assumptions), we shall have for \( X_{K_1} \):

\[
X_{K_1} = n_a a X_a (1 + r_{K_1})^{T_{K_1}} + n_a a X_a (1 + r_{K_2})^{T_{K_2}}(1 + r_{K_1})^{T_{K_1}}.
\]  
(15)
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If, finally, the time expended on manufacture of the product $A$ is equal to $T_A$, we shall have for $X_A$

$$X_A = n_AaX_a(1 + r_A)^{T_A} + n_1aX_a(1 + r_{K1})^{T_{K1}}(1 + r_A)^{T_A} +$$

$$+ n_2aX_a(1 + r_{K2})^{T_{K2}}(1 + r_{K1})^{T_{K1}}(1 + r_A)^{T_A}. \quad (16)$$

If we compare this expression of price with the former expression

$$X_A = N_AaX_a + (y_A + y_1 + y_2 + \ldots + y_m) \quad (17)$$

we see that instead of the unknowns $y_{A1}, y_{K1}, \ldots$, which denoted the sums of the profits of the different entrepreneurs directly or indirectly involved in production of product $A$, we now have another series of unknowns $r_A$, $r_{K1}, r_{K2}, \ldots$ denoting the rate of profit in the different branches of industry involved in the production of $A$. Therefore, the number of unknowns still remains the same ($T_A$, $T_{K1}$, $T_{K2}, \ldots$ incorporated in the new expression of price are known quantities dependent on the technical conditions of production of $A$, $K_1$, $K_2 \ldots$ and so on). The importance of the transformations made by us to the production costs formula is revealed only in connection with another hypothesis of prime importance established by Adam Smith, namely the hypothesis that the "rate of profit" tends to be equalised in all branches of industry. By virtue of this hypothesis we shall have $r_A = r_{K1} = r_{K2} \ldots = r$, where $r$ is taken to mean the general level towards which the rate of profit of individual branches of industry tends.\(^1\)

Adam Smith arrives at this hypothesis deductively from the basic premise that every man aspires to the greatest advantage (Smith, Book I, Ch. 10, p. 162 of 1814 edition). Smith reasons as follows: if profit in some branch of industry $A$ is higher than in others, this will compel industrialists from other branches to convert to production of the product $A$; as a consequence of this production will expand, the supply of the product $A$ will increase and the price of the product which is, *caeteris paribus*, inversely proportional to the supply, will fall. But since production costs will remain the same, the profit on capital, which is the difference between the price and the production costs, will fall; if; nevertheless, it were still to be above the general level, this would cause a new conversion of producers from other branches and a new reduction of the price until, finally, profit reached the general level; there could be no further reduction of profit, since this would destroy the motive (exceptional profit) for the conversion of producers from other branches.

At this point we shall not make a critical analysis of this theory, which is completely accepted by Ricardo; we shall demonstrate its incorrectness and arbitrary nature in the Second Essay when analysing the "theory

\(^1\) We naturally assume that $r, r_{K1}, r_{K2}, \ldots$ are all adjusted to a common unit of time and unit of value expended.
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of competition' (all that we have done here has been to indicate the arbitrary assumptions by setting them in italics). The whole of Smith's reasoning in this case is based on the arbitrary assumption that the amount of a given good may be increased without limit by the application of labour and capital and that its production is under the influence of free competition, and that therefore the law of the equality of the 'profit rate' in different branches of production applies only to goods which satisfy this arbitrary assumption.

If we make the corresponding transformations to the expressions $X_A, X_B, \ldots$ we shall have

$$X_A = n_A aX_a(1+r)^{t_A} + n_2 aX_a(1+r)^{t_{A1} + t_{A2}} + n_3 aX_a(1+r)^{t_{A1} + t_{A2} + t_{A3}} + \ldots$$

(18)

for any product $A$, where the terms $t_{A1}, t_{A2}, \ldots$ denote the periods of time expended on the production of capital goods of the first, second and higher orders involved in manufacture of product $A$, and

$$X_B = m_B aX_a(1+r)^{t_B} + m_2 aX_a(1+r)^{t_B + t_{B1}} + m_3 aX_a(1+r)^{t_B + t_{B1} + t_{B2}} + \ldots$$

(19)

for any product $B$ [where $m_B, m_1$ and $m_2$ are labour outlays occurred respectively $t_B, t_{B1}, t_{B2}$ time periods earlier].

We take the ratio of $X_A$ to $X_B$ and obtain:

$$X_{AB} = \frac{n_A aX_a(1+r)^{t_A} + n_2 aX_a(1+r)^{t_{A1} + t_{A2}} + n_3 aX_a(1+r)^{t_{A1} + t_{A2} + t_{A3}} + \ldots}{m_B aX_a(1+r)^{t_B} + m_2 aX_a(1+r)^{t_B + t_{B1}} + m_3 aX_a(1+r)^{t_B + t_{B1} + t_{B2}} + \ldots}$$

(20)

If $r$ is given, $X_{AB}$ will also be a quite definite quantity and, consequently, the problem of the exchange proportion will be solved (since all the other exchange proportions may be similarly determined for a given product $A$: $X_{AD}, X_{AE}, X_{AF}$ etc.).

However, Adam Smith did not proceed any further in his analysis of production costs. The honour for a complete solution of the problem belongs to his great successor Ricardo. Smith himself related the magnitude of $r$ to the abundance of the supply of capital. He says: 'The increase of stock, which raises wages, tends to lower profit. When the stocks of many rich merchants are turned into the same trade, their mutual competition naturally tends to lower its profits; and when there is a like increase of stock in all the different trades carried on in the same society, the same competition must produce the same effect in them all' (Smith, Book I, Ch. 9, p. 143 of the 1814 edition).

If we denote the supply of capital by $D$, then $r = \phi(D)$ and $d\phi(D)/dD < 0$, i.e. as the supply of capital increases the profit rate declines. The actual form of the function $\phi$ is taken by Smith to be empirically given,
despite the fact that the relationship between $r$ and $D$ undoubtedly already belongs to the sphere of economic analysis (as is shown by the writings of Ricardo); Smith does not give us any such analysis, although we find a completely accurate statement of the cause of the reduction in profit when capital goods increase independent of competition (see Smith, Book I, Ch. 9, p. 151 of the 1814 edition).

2. RICARDO’S THEORY OF VALUE

The most important point in Ricardo’s theory is undoubtedly his theory of the conditions defining the ‘average’ profit rate to which, according to Smith’s theory, profit tends in the individual branches of industry. As we have seen, this question was left unanswered in the writings of Smith, if we disregard his references to the relationship between the demand and supply of capital. We have to ask ourselves whether it was actually solved by Ricardo. Strange as it may seem, in view of the remarkable clarity of Ricardo’s writings, negative answers to this question are still to be found in economic literature. It will suffice to mention the critical writings on the question of profit of E. Böhm-Bawerk (Kapital und Kapitalzins, Vol. 1, Geschichte und Kritik der Kapitalzins Theorien, pp. 101-11) and Zaleski (Uchenie o proiskhozhdeni pribyli na kapital [The theory of the origin of profit on capital], Kazan, 1898, Vol. II, p. 52); such views are particularly surprising when they are expressed by economists who have used the precise mathematical method for their analysis.¹

Thus, in his most interesting analysis of Ricardo’s theory of value² (which contains a model analysis of the theory of rent), Yu. Zhukovsky states, after having expounded Ricardo’s theory of profit,

‘But all this defines only the relative magnitude of the profit on capital or the order of its reduction. Ricardo however does not provide any answer to the question of how to determine the initial magnitude of the profit from which wages are subsequently deducted, the initial magnitude of the percentage on capital, and we would note that from this aspect the question still remains unanswered’ (p. 345).

‘... Ricardo gave no answer at all on how to determine this absolute initial magnitude of the percentage and whether this initial magnitude of the percentage, from which wages have to be subtracted, remains constant when the product becomes dearer; from this aspect this quantity remains completely undefined’ (pp. 356-7).

¹ We should also include here in part the general criticisms which Thünen makes of the theory of production costs; see p. 58, footnote 2.
2. Ricardo's theory of value

'The only theory to which we may point as defining the initial level of the profit which may be taken by the capitalist consists in the level or excess of capital goods, and this level should be dependent on the ratio between the supply and demand for capital or r/s' (p. 357).

'Ricardo assumes that capital goods may flow freely like a fluid under the influence of gravity from one place and point to another and may tend toward equalisation of profits and to a general level of them, the height of which is determined by nothing other than their greater or lesser excess'

'If we denote the volume of the capital goods by (a), the space over which they are poured by (b), and the height by (h), we shall have the condition: \( hb = a \); from which \( h = a/b \)' (p. 342).

We would be in complete agreement with all these remarks by Zhukovsky had they been made in relation to Smith's theory, but to assert that we find no other definition for the general level of profit in Ricardo's work than the formula \( x = r/s \) is to fail to understand the very basis of Ricardo's theory.

Walras states the same criticism even more clearly.

'Let \( P \) be the aggregate price received for the products of an enterprise; let \( S, I \) and \( F \) be respectively the wages, interest charges and rent laid out by the entrepreneurs, in the course of production, to pay for the services of personal faculties, capital and land. Let us recall now that, according to the English School, the selling price of products is determined by their costs of production, that is to say, it is equal to the cost of the productive services employed. Thus we have the equation

\[
P = S + I + F
\]

and \( P \) is determined for us. It remains only to determine \( S \), \( I \) and \( F \).

Surely, if it is not the price of the products that determines the price of productive services, but the price of productive services that determines the price of the products, [we must be told what determines the price of the services]. This is precisely what the English economists try to do. To this end, they construct a theory of rent according to which rent is not included in the expenses of production,

\[
P = S + I.
\]

Having done this, they determine \( S \) directly by the theory of wages. Then, finally, they tell us that "the amount of interest or profit is the excess of the aggregate price received for the products over the wages expended on their production"; in other words, that it is determined by the equation

\[
I = P - S.
\]

It is clear now that the English economists are completely baffled by the problem of price determination; for it is impossible for \( I \) to deter-
The theory of value of David Ricardo

mine $P$ at the same time that $P$ determines $I$. In the language of mathematics one equation cannot be used to determine two unknowns. This objection is raised without any reference to our position on the manner in which the English School eliminates rent before setting out to determine wages (Walras, Elements, 1874, Lesson 40, § 368, pp. 424–5 of the English edition).

The subsequent analysis will show us how justified these reproaches are.

The last formulas derived by us on p. 49, expressing the farthest point reached by Adam Smith in his analysis of the connection between the price of a product and its production costs relate, as we have noted, only to commodities: (1) whose quantity may be increased without limit by the application of labour and capital, (2) separate portions of which are produced with identical production costs (in order to exclude rent), (3) whose production and sale take place under the influence of 'unlimited competition'. These are precisely the forms that are also the starting point for Ricardo's analysis;

'In speaking, then, of commodities, of their exchangeable value, and of the laws which regulate their relative prices, we mean always such commodities only as can be increased in quantity by the exertion of human industry, and on the production of which competition operates without restraint.'

Before turning to an examination of the conditions determining 'the general rate of profit' $r$, Ricardo pauses to analyse instances in which the sought-after quantities $X_{AB}, X_{AC}, X_{AD}$ and so on (i.e. the value of any product $A$ in terms of the value of the product $B, C$ etc.) may be determined independently of the magnitude of $r$. Let the products $A, B$ and so on be produced solely by 'current labour', without the use of capital goods (i.e. of tools and materials which are themselves a result of the expenditure of labour). Let $N_A$ days of labour have been expended on the production of a unit of product $A$, $N_B$ days of labour on the production of product $B$ and so on; further, let the time needed for manufacture and delivery to the market be $t_A$ for product $A$, $t_B$ for the product $B$ and so on (Ricardo invariably assumes that a product is sold immediately on delivery to the market). If in each period a worker consumes a units of corn (which Ricardo assumes is the only consumption of workers) and the price of corn is $X_a$, the price of $N_A$ days of labour is $N_A a X_a$ and the price of $N_B$ days of labour is $N_B a X_a$. If we now assume that the 'profit rate' in industries

1 D. Ricardo: On the principles of political economy and taxation, London, 1817, Ch. 1, 'On Value'; throughout this chapter Ricardo assumes that individual units of the same product are obtained at the same cost, but he does not specifically state this assumption; the conditions under which rent may arise are not introduced until the second chapter.
2. Ricardo's theory of value

\( A, B \) and so on is \( r \), and assume for simplicity that the capital used to hire workers is all expended simultaneously at the beginning of production, we shall have the following expression for \( X_{AB} \):

\[
X_{AB} = \frac{N_A a X_a (1 + r)^{t_A}}{N_B a X_a (1 + r)^{t_B}}
\]

(21)

If \( t_A = t_B \) in this expression, we shall have for \( X_{AB} \), after the appropriate simplifications:

\[
X_{AB} = \frac{X_A}{X_B} = \frac{N_A}{N_B}
\]

(22)

i.e. the relative value of products \( A \) and \( B \) equals the ratio of the amount of labour expended on the production of a unit of product \( A \) to the amount of labour expended on the production of a unit of product \( B \); the cost of a unit of product \( A \) is related to the cost of a unit of product \( B \) as the amount of labour expended on the production of a unit of product \( A \) is related to the amount of labour expended on the production of a unit of product \( B \).

Suppose the same number of workers are occupied from the beginning to the end in the production of \( A \) and in the production of \( B \); this number will be \( N_A/t_A \) for \( A \) and \( N_B/t_B \) for \( B \); let the payment to these workers be advanced not for the whole time until production of the product is completed, but only for a unit of time (e.g. for one day); in this case the expenditure at the beginning of each unit of time will be expressed by \( N_A a X_a / t_A \) for product \( A \) and \( N_B a X_a / t_B \) for product \( B \); if the profit from a unit of capital over a unit of time remains equal to \( r \), we shall have for \( X_{AB} \):

\[
X_{AB} = \frac{N_A a X_a (1 + r)^{t_A} / t_A + N_A a X_a (1 + r)^{t_A-1} / t_A + \cdots + N_A a X_a (1 + r) / t_A}{N_B a X_a (1 + r)^{t_B} / t_B + N_B a X_a (1 + r)^{t_B-1} / t_B + \cdots + N_B a X_a (1 + r) / t_B} =
\]

\[
= \frac{N_A a X_a [1 + (1 + r)^{t_A} + (1 + r)^{t_A-1} + \cdots + (1 + r)]}{N_B a X_a [1 + (1 + r)^{t_B} + (1 + r)^{t_B-1} + \cdots + (1 + r)]} \tag{23}
\]

and if we set \( t_A = t_B \) in this expression, we once again have \( X_{AB} = N_A / N_B \).

In addition to the directly expended or 'current' labour, let there now be additionally expended in the production of \( A \) a certain amount of capital; this capital good is itself the product of a certain amount of current labour assisted by a certain amount of new capital good; ascending ever higher and higher to 'production goods of higher orders' (the Produkttüter höherer Ordnung of the theoreticians of marginal utility), let us finally arrive at a capital good (or capital goods) produced solely
by current labour. In that case, as we have shown when describing
Smith’s theory, the total sum of the production costs of a unit of product
A (in the Ricardoian sense, i.e. including profit) will be given by

\[ X_A = n_a aX_a(1+r)^{t_1} + n_1 aX_a(1+r)^{t_1+t_2} + n_2 aX_a(1+r)^{t_1+t_2+t_3} + \cdots + n_m aX_a(1+r)^{t_1+t_2+\cdots+t_m} \]  

(24)

where \( n_A, n_1, n_2 \) and so on are the amounts of current labour expended
on the production of product A and of the capital goods \( K_1, K_2, K_3, \ldots \)
used in the production of product A; \( t_1, t_2, \ldots \) and on denote the
‘production period’ of the product A and of capital goods \( K_1, K_2, K_3, \ldots \).
If, in the interests of brevity, we employ the notations \( t_A, t_1 = t_{A1},
\( t_1+t_2 = t_{A2} \) and so on, we have:

\[ X_A = n_a aX_a(1+r)^{t_A} + n_1 aX_a(1+r)^{t_A+t_1} + n_2 aX_a(1+r)^{t_A+t_1+t_2} + \cdots + n_m aX_a(1+r)^{t_A+t_1+\cdots+t_m} \]

(25)

where

\[ t_{Am} > t_{A(m-1)} > t_{A(m-2)} > \cdots > t_{A2} > t_{A1} > t_0 \]

correspondingly for increasingly long periods of time separating the
times at which the amounts of labour \( n_1, n_2, \ldots, n_A \) are expended
from the time at which the finished product A is delivered to the market.

If we assume that in our formula terms having the same powers have
already been summed (so that, for example, \( n_a aX_a(1+r)^{t_A} = m_a aX_a \times
(1+r)^{t_A} + m_2 aX_a(1+r)^{t_1} + \cdots + m_m aX_a(1+r)^{t_m} \)), we may also employ it
for cases in which any number of different forms of capital goods are
directly involved in the production of the product A.

This formula serves equally to express the case in which the labour \( n_1,
n_2 \) and so on preliminarily expended is involved in production of A in the
form of machines, tools and ‘auxiliary materials’ created by it, and to
express the case in which the product A itself passes successively through
different stages of processing (in this case \( t_0, t_1, t_2, \ldots \) will denote the
periods of the separate stages of processing; \( n_A, n_1, n_2 \ldots \) will denote the
amounts of labour used in each stage).

The structure of the formula will not be altered if, instead of assuming
that the amounts \( n_a aX_a, n_2 aX_a \) and so on are advanced simultaneously at the
beginning of the corresponding production processes \( A, K_1, \ldots \) and so on, we assume that they are advanced in parts during the production
period; the only difference from equation (25) will be that the term
\( n_a aX_a(1+r)^{t_1} \) will be replaced by a sum:

\[ m_a(1+r)^{t_1} + m_{A1}(1+r)^{t_A} + m_{A2}(1+r)^{t_{A2}} + \cdots + m_{AV}(1+r)^{t_{AV}}, \]  

(26)

where \( m_A + m_{A1} + m_{A2} + \cdots + m_{AV} = n_A \) and \( t_A > t_{A1} > t_{A2} > \cdots > t_{AV} > 0 \); the term \( n_a aX_a(1+r)^{t_1} \) will be replaced by a sum:

\[ m_1 aX_a(1+r)^{t_A} + m_2 aX_a(1+r)^{t_{A1}} + m_3 aX_a(1+r)^{t_{A2}} + \cdots + m_m aX_a(1+r)^{t_{Am}} \]

(27)
2. Ricardo's theory of value

where $m_1 + m_2 + \ldots + m_w = n_1$, and $t_{A1} > t_{A11} > t_{A12} > \ldots > t_{A}$, and so on. (Compare with the case considered earlier [p. 54].) Consequently a formula of the type

$$X_A = n_A aX_a(1+r)^{t_A} + n_1 aX_{a1}(1+r)^{t_{A1}} + \ldots + n_m aX_{am}(1+r)^{t_{Am}}$$  \hspace{5em} (25)

in which $n_A$, $n_1$, $n_2$, $\ldots$, $t_A$, $t_{A1}$, $t_{A2}$, $\ldots$ can be taken to stand for any magnitudes, will also serve us in this case to express the connection between the price of the product and its production costs.

Accordingly, having selected a formula for the production costs of any product $B$, we obtain

$$X_{AB} = \frac{n_A(1+r)^{t_A} + n_1(1+r)^{t_{A1}} + \ldots + n_m(1+r)^{t_{Am}}}{m_B(1+r)^{t_B} + m_1(1+r)^{t_{B1}} + \ldots + m_p(1+r)^{t_{Bp}}}$$  \hspace{5em} (28)

as an expression of the exchange ratio $X_{AB}$, i.e. as an expression of the value of product $A$ in terms of $B$.

Let the number of terms having different indices be equal both in the numerator and in the denominator, so that $m = p$ and let:

$$t_A = t_B; \quad t_{A1} = t_{B1}; \quad \ldots; \quad t_{Am} = t_{Bp}.$$  \hspace{5em} (29)

In that case

$$X_{AB} = \frac{n_A(1+r)^{t_A} + n_1(1+r)^{t_{A1}} + \ldots + n_m(1+r)^{t_{Am}}}{m_B(1+r)^{t_B} + m_1(1+r)^{t_{B1}} + \ldots + m_m(1+r)^{t_{Bm}}}.$$  \hspace{5em} (30)

Let us further have:

$$\frac{n_A}{m_A} = \frac{n_1}{m_1} = \frac{n_2}{m_2} = \ldots = \frac{n_m}{m_m} = R.$$  \hspace{5em} (31)

We shall then have $n_A = m_B R_1$, $\ldots$, $n_m = m_m R$ and

$$X_{AB} = \frac{R[m_B (1+r)^{t_B} + m_1 (1+r)^{t_{B1}} + \ldots + m_m (1+r)^{t_{Bm}}]}{m_B (1+r)^{t_B} + m_1 (1+r)^{t_{B1}} + \ldots + m_m (1+r)^{t_{Bm}}} = R = \frac{n_A + n_1 + n_2 + \ldots + n_m}{m_A + m_1 + m_2 + \ldots + m_m}.$$  \hspace{5em} (32)

i.e. in this case the value of product $A$ in terms of $B$ once again will not depend on the level of $r$, but only on the amount of labour expended on the production of products $A$ and $B$.

The expression usually given to assumptions (29) and (31) made to obtain this result is that 'the capital goods used up in the branches $A$ and $B$ are of identical organic composition'. We should take this vague expression to mean (1) that the turnover periods of the different portions of the
capital expended in the production of $B$ and of that expended in the production of $A$ are the same, i.e. that it is impossible to find in the production of $A$ a turnover period which could not be found in the production of $B$ and vice versa; (2) that the ratios of the portions of capital with correspondingly equal turnover periods in both branches of production are equal. Any attempt to give a briefier definition of the conditions under which value is equal simply to the ratio of the amount of labour expended on the production of a unit of both products renders the definition less general than is required and makes it necessary to supplement it by a number of qualifications and special rules, which is the approach followed by Ricardo.\(^1\) Now let $t_A$ not be equal to $t_B$ in the simplest formula (equation (21) on p. 53) expressing the exchange value of the commodities $A$ and $B$ produced by the same current labour as a function of their production costs. Let, for example, $t_A > t_B$, in which case we have $(1 + r)^{t_A} > (1 + r)^{t_B}$ and in consequence of this:

$$\frac{N_A}{N_B}x_A(1 + r)^{t_A} > \frac{N_A}{N_B}$$

and

$$X_{AB} > \frac{N_A}{N_B},$$

i.e. the value of $A$ in terms of $B$ will be greater than the ratio of their ‘labour values’ (the amounts of labour used in their production).

It may readily be appreciated from the formula that when $t_A$ and $t_B$ are invariable this difference will be greater, the greater the magnitude of $r$. Thus, in this case, the magnitude of $X_{AB}$ is a function not only of $N_A$ and $N_B$, but also of the level of $r$ and, consequently, cannot be determined independently of it. The same will hold for any $X_{MN}$:

$$X_{MN} = \frac{n_m(1 + r)^{t_m} + n_1(1 + r)^{t_{m1}} + \cdots + n_k(1 + r)^{t_{mk}}}{m_m(1 + r)^{t_m} + m_1(1 + r)^{t_{m1}} + \cdots + m_k(1 + r)^{t_{mk}}}$$

since (as has been shown above) the second part of the equality cannot be expanded into

$$R = \frac{m_m(1 + r)^{t_m} + m_1(1 + r)^{t_{m1}} + \cdots + m_k(1 + r)^{t_{mk}}}{n_m(1 + r)^{t_m} + n_1(1 + r)^{t_{m1}} + \cdots + n_k(1 + r)^{t_{mk}}}$$

\(^1\) In referring to the conditions by virtue of which products are exchanged in the market other than in proportion to the amounts of labour expended on their production, Ricardo first considers the division of capital in different proportions into fixed and ‘circulating’ capital, and then adds those cases in which both commodities are produced by ‘current’ labour, but more time is needed for the production of one than for the production of the other. ‘This case’, states Ricardo, ‘appears to differ from the last, but is, in fact, the same’; finally, all this is qualified by the statement: ‘It is hardly necessary to say, that commodities which have the same quantity of labour bestowed on their production, will differ in exchangeable value, if they cannot be brought to market in the same time’ (Ricardo, Principles, Ch. 1, Section iv, p. 37 of the Sraffa edition).
2. Ricardo’s theory of value

where \( R \) is a quantity that is independent of \( r \). Ricardo examines various special cases that are relevant here (see Principles, Ch. 1). Note that to convert from our formulas to the Ricardian examples we first have to introduce the condition of the ‘durability’ of the capital goods \( K_1, K_2, \ldots \), in which case we obtain equation (25) (a simplification of (24))

\[
X_A = n_1 a X_1 (1 + r)^{t_{1a}} + n_1 a X_1 (1 + r)^{t_{1a}} + \cdots + n_m a X_m (1 + r)^{t_{ma}}. \tag{25}
\]

Thus, in all cases when the organic composition of the capital goods used in the production of \( A \) and \( B \) is not the same, the exchange ratio \( X_{AB} \) of the products \( A \) and \( B \) cannot be determined independently of the level of \( r \); in this case

\[
X_{AB} = \phi(n_{A_{1}}, n_{A_{2}}, \ldots ; m_{B_{1}}, m_{B_{2}}, \ldots ; t_{A_{1}}, t_{A_{2}}, \ldots ; t_{B_{1}}, t_{B_{2}}, \ldots ; r)
\]

(37)

from which, if \( n_{A_{1}}, n_{A_{2}}, \ldots, m_{B_{1}}, m_{B_{2}}, \ldots, t_{A_{1}}, t_{A_{2}}, \ldots, t_{B_{1}}, t_{B_{2}}, \ldots \) have been adopted as quantities dependent on the technical conditions of production of the products \( A \) and \( B \), we obtain \( X_{AB} = \phi(r) \), where \( X_{AB} \) will be a determined quantity when \( r \) is given.

How may the required magnitude of \( r \) be determined? Is it possible to use our ‘production cost equations’ (see p. 55) for its determination? Let us write them in the form:

\[
\begin{align*}
X_A &= aX_1(n_{1} (1 + r)^{t_{1a}} + n_{1} (1 + r)^{t_{1a}} + \cdots + n_{m} (1 + r)^{t_{ma}}) \\
X_B &= aX_1[m_{1} (1 + r)^{t_{1a}} + m_{1} (1 + r)^{t_{1a}} + \cdots + m_{n} (1 + r)^{t_{na}}].
\end{align*} \tag{38}
\]

If we assume, as previously, that the quantities \( n_{A_{1}}, n_{A_{2}}, \ldots, m_{B_{1}}, m_{B_{2}}, \ldots, t_{A_{1}}, t_{A_{2}}, \ldots, t_{B_{1}}, t_{B_{2}}, \ldots \) are constants, it is possible to write the expression in square brackets in the form \( f_A(r), f_B(r) \ldots \) where \( f_A(r) > 0, f_B(r) > 0, \ldots \) (i.e. when \( r \) increases, \( f_A(r), f_B(r), \ldots \) is also increased, and vice versa). We then have from the equations for \( A \) and \( B \)

\[
\begin{align*}
\frac{X_A}{aX_1} &= f_A(r) ; & \frac{X_B}{aX_1} &= f_B(r), \text{ and so on.} \tag{39}
\end{align*}
\]

It is evident from these equations that when \( X_A/aX_1, X_B/aX_1, \ldots \) increase, \( r \) also increases by virtue of \( df_A(r)/dr > 0 ; df_B(r)/dr > 0 \). Consequently, when \( X_A, X_B, \ldots \) remain unchanged, \( r \) will be greater the less is \( aX_1 \) and vice versa; i.e. a reciprocal relationship will exist between the profit rate and the level of wages.\(^1\)

\(^1\) Too much importance is often attached to this Ricardian hypothesis. Ricardo’s main contribution to the theory of profit does not lie here, but in his establishment of the laws governing the absolute level of profit.
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Nevertheless, this analysis does not give us the magnitude of \( r \); in the equations

\[
\begin{align*}
X_A &= aX_a \cdot f_A(r) \\
X_B &= aX_a \cdot f_B(r) \\
&\vdots
\end{align*}
\] (40)

and in the equations derived from them:

\[
\begin{align*}
X_{AB} &= \frac{f_A(r)}{f_B(r)}; & X_{AC} &= \frac{f_A(r)}{f_B(r)}; & \ldots
\end{align*}
\] (41)

\( r \) will be a determined quantity if the magnitude \( X_{AB}, X_{AC}, \ldots \) are given. However, without proceeding from production conditions, we have no other equations for the determination of \( X_{AB} \) and \( X_{AC} \) apart from the same equations (41), and the same equation cannot serve for the determination of two unknowns. Thus, we are apparently trapped in a logical circle: profit must be known in order to determine value, but profit itself is dependent on value. There would appear to be no way out of this circle other than to relate value or profit to conditions lying outside the sphere of production. As we have seen, Adam Smith took this way out, when he related the profit rate to the demand and supply of capital. To proceed in this manner is, however, to acknowledge the untenability of the theory of production costs itself.\(^1\)

Ricardo's immortal contribution was his brilliant solution of this seemingly insoluble problem. Let us take a series of 'production cost equations' expressing the connection between price and production cost for the commodities \( A, B, C, \ldots \)

\[
\begin{align*}
X_A &= aX_a[n_A(1+r)^{1a} + n_1(1+r)^{2a1} + n_2(1+r)^{2a2} + \ldots + n_m(1+r)^{2am}] \\
X_B &= aX_a[m_B(1+r)^{1b} + m_1(1+r)^{2b1} + m_2(1+r)^{2b2} + \ldots + m_p(1+r)^{2bp}] \\
&\vdots
\end{align*}
\] (42)

Each new equation incorporates the unknowns \( X_a \) and \( r \) as in the preceding equations, and additionally a further new unknown \( X \) with the appropriate index. Therefore, if the number of equations is \( n \), then the number of unknowns is \((n+2)\). However, no more than \( n \) unknowns may be eliminated with \( n \) equations. When we arrive at the product \( N \) taken by us as the unit of value (e.g. silver), we shall have:

\[
1 = aX_a[P_N(1+r)^{1n} + P_1(1+r)^{2n1} + P_2(1+r)^{2n2} + \ldots + P_p(1+r)^{2np}]. \quad (43)
\]

\(^1\) Compare Thünen's criticisms of Smith (J. H. von Thünen Le solaire naturel et son rapport au taux d'intérêt, Paris, 1857).
This equation does not contain a new unknown. If we add to it the $n$
former equations, we shall have $(n + 1)$ equations with $(n + 2)$ unknowns.
The number of equations will still be inadequate, and the question of the
level of the profit rate will apparently remain unsolved.

It is to Ricardo's credit that he was the first to note that there is one
production equation by means of which we may determine the magnitude
of $r$ directly (i.e. without having recourse for assistance to the other equa-
tions). This equation gives us the production conditions of the product $a$
to which in the final analysis the expenditure in all the products $A, B, C,$
... is reduced. Let us take the 'production costs' equation for this
product $a$ compiled in the same way as we compiled the equations for
the other products:

$$X_a = aX_a[N_a(1 + r)^t_a + N_1(1 + r)^t_{a1} + \cdots + N_q(1 + r)^t_{aq}]$$

(44)

from which:

$$a[N_a(1 + r)^t_a + N_1(1 + r)^t_{a1} + \cdots + N_q(1 + r)^t_{aq}] - 1 = 0$$

(45)

and if we determine $r$ from this equation, we have:

$$r = F(N_a, N_1, N_2, \ldots, N_q; t_a, t_{a1}, t_{a2}, \ldots, t_{aq}; a).$$

(46)

But since $N_a, N_1, \ldots; t_a, t_{a1}, \ldots$ and $a$ are given quantities dependent on
the technical conditions of production of the product $a$ (i.e. the product
forming the essential means of existence of the worker), $r$ is also a given
magnitude, i.e. is independent of the economic circumstances.

If we now insert the magnitude of $r$ found by us in the production cost
equations (42) and so on (on the basis of the law of the equality of profit
rates in different branches of industry), we shall obtain $X_A, X_B, \ldots$
and correspondingly $X_{ab}, X_{ac}, \ldots$ as functions of the same given quantities
$N, n, m, \ldots$ (with the appropriate indices), $t's \ldots$ (with the appropriate
indices) and of the quantity $a$. Before proceeding to a general analysis of
the expression found by us for the magnitude of $r$, let us consider whether
the solution of the problem remains the same if, instead of taking one
product ($a$) consumed by the workers (e.g. corn, as is done by Ricardo)
we take several such products in accordance with reality.¹

Let $\alpha, \beta, \gamma, \ldots$ be products consumed by the workers. Let the daily
consumption of a single worker be $a$ for the product $\alpha$, $b$ for the product $\beta$

¹ Ricardo himself makes a stipulation along these lines (after he has established his law
of profit). He states 'The effects produced on profits would have been the same, or nearly
the same, if there had been any rise in the price of these other necessaries, besides food, on
which the wages of labour are expended' (Ricardo, Principles, Ch. 6, p. 113 of the Staaffa
edition).
and $c$ for the product $y$; if we now take the production costs equation for

\[(a),\text{ we obtain:}\]

\[X_a = N_a(aX_a + bX_b + cX_y \ldots)\]

\[(1 + r)^t_a + N_a (aX_a + bX_b + cX_y \ldots)(1 + r)^{t_{a1}} + \ldots \quad (47)\]

Clearly this equation does not make it possible to determine $r$ directly as in the preceding case, but if we add to this equation the production costs equation for $b, \gamma, \ldots$, we obtain the equation system:

\[
\begin{align*}
X_b &= N_b(aX_a + bX_b + cX_y \ldots) \\
(1 + r)^t_b + N_b (aX_a + bX_b + cX_y \ldots)(1 + r)^{t_{b1}} + \ldots \\
X_y &= N_y(aX_a + bX_b + cX_y \ldots) \\
(1 + r)^t_y + N_y (aX_a + bX_b + cX_y \ldots)(1 + r)^{t_{y1}} + \ldots
\end{align*}
\]

\[\ldots\quad (48)\]

Let us multiply both parts of equation (47) by $a$, both parts of the following by $b$, of the next by $c$ and so on and then add all our equations term by term. We shall obtain:

\[
(aX_a + bX_b + cX_y \ldots) = aN_a(aX_a + bX_b + cX_y \ldots) (1 + r)^t_a
\]

\[
+ aN_{a1}(aX_a + bX_b + cX_y \ldots)(1 + r)^{t_{a1}} + \ldots
\]

\[
+ bN_b(aX_a + bX_b + cX_y \ldots) (1 + r)^t_b
\]

\[
+ bN_{b1}(aX_a + bX_b + cX_y \ldots)(1 + r)^{t_{b1}} + \ldots
\]

\[
+ cN_y(aX_a + bX_b + cX_y \ldots) (1 + r)^t_y
\]

\[
+ cN_{y1}(aX_a + bX_b + cX_y \ldots)(1 + r)^{t_{y1}} + \ldots \quad (49)
\]

Having divided both sides of the equation by $(aX_a + bX_b + cX_y \ldots)$ we shall obtain:

\[
1 = aN_a(1 + r)^t_a + aN_{a1}(1 + r)^{t_{a1}} + \ldots + bN_b(1 + r)^t_b + bN_{b1}(1 + r)^{t_{b1}} + \ldots + cN_y(1 + r)^t_y + cN_{y1}(1 + r)^{t_{y1}} + \ldots \quad (50)
\]

Consequently:

\[
r = F(N_a, N_{a1}, \ldots; N_b, N_{b1}, \ldots; N_y, N_{y1}, \ldots; \quad (51)
\]

\[
a, b, c, \ldots; t_a, t_{a1}, \ldots; t_b, t_{b1}, \ldots; \]

\[
t_y, t_{y1}, \ldots; \ldots)
\]

Therefore, our system of equations (47), (48) of the "production costs" of products consumed by the workers still yields $r$ as a function of the same given quantities.\footnote{If the number of products consumed by the workers is $n$, we shall have $n$ equations in which we have $(n+1)$ unknowns; $(X_a, X_b, X_y, \ldots)$ yielding $n$ unknowns, to which the unknown $r$ is added.} Consequently we may establish that the level of
the profit rate \( r \) is determined by the production costs of products consumed by the workers. (There is no need for us to repeat the qualification made by Ricardo ‘on that land or with that capital which yields no rent’, since it has already earlier been proposed to exclude rent from our investigation.) Production costs in this case should be understood as ‘only costs in the “objective sense”’ (similar to what was denoted by Rodbertus by the term \( \text{Kosten des Gutes} \) [cost of the good] in contrast with \( \text{Auslagen des Unternehmers} \) [expenditure of the entrepreneurs] or \( \text{Kosten des Betriebs} \) [cost of running the enterprise], namely the quantity of goods used in production, and the period of reproduction (i.e. the time between the moment or moments of expenditure of ‘production goods’ and the time at which the ready product appears on the market).

If we assume \( a, b, c, \ldots \) to be constants (which, if the ‘iron law of wages’ prevails, amounts to the assumption that the minimum of means of subsistence of the worker is invariable), we obtain \( r \) exclusively as a function of the quantities of labour and time \( N_{a1}, N_{b1}, \ldots; N_{\beta}, N_{\gamma}; \ldots; t_{a1}, t_{b1}, \ldots; \) corresponding to industries, \( \alpha, \beta, \gamma, \ldots \) making products consumed by the workers. Once these quantities are given, the magnitude of \( r \), i.e. the profit rate, is a fully defined quantity.

Consequently, Ricardo succeeded in finding a solution to the problem. Our formulas of ‘production costs’ have now taken the general form:

\[
\begin{align*}
X_A &= F(n_{a1}, n_{a2}, \ldots; t_{a1}, t_{a2}, \ldots; N_{a1}, N_{a2}, \ldots) \\
X_B &= F(m_{b1}, m_{b2}, \ldots; t_{b1}, t_{b2}, \ldots; N_{b1}, N_{b2}, \ldots) \\
\end{align*}
\]

where the element of ‘price’ does not appear at all in the second part of the equation. To level at Ricardo’s theory the hackneyed reproach that it ‘defines price in terms of price’ is to manifest a complete lack of understanding of the writings of this very great theoretical economist.

The starting point for Ricardo’s analysis was provided by the present-day capitalist system based on the use of hired human labour; it would, however, be extremely erroneous to imagine that the conclusions at which he arrived have a bearing only on the present time. Zhukovsky has quite correctly understood and explained in his book the importance of Ricardo’s theoretical conclusions. Although Ricardo’s theory of rent may serve as an

\[ \text{Ricardo's theory of value} \]

\[ \text{Ricardo, Principles, Ch. 6, p. 126 of the Sraffa edition.} \]

\[ \text{See C. Rodbertus, Zur Erkenntnis unserer staatswirtschaftlichen Zustände, 1842, pp. 25–6.} \]

\[ \text{History of 19th Century Political Theories (Vol. 1, pp. 388–9). Zhukovsky states that} \]

\[ \text{Ricardo deals with the question of distribution only in the sense of the division of separate} \]

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example: the movement of individuals from more fertile to less fertile areas of land is taken as the starting point, but the theory retains its significance even if the opposite assumptions are made. Ricardo subsequently clarifies the laws of rent only in land rent and rent from mines, but this does not prevent the laws established by him from being of general significance for all cases to which the conditions of the origin of rent stated by him apply. (See, in this respect, Zhukovsky, History..., p. 318.) In our equations for 'production costs' (see equation (25), p. 54), let us set the magnitudes $a_1, a_1', a_2', a_3, a_3', a_4, a_4', a_5, a_5'$ to be respectively $A, A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_9, A_{10}$; in this case $A, A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_9, A_{10}$ will denote the quantity of some good $a$ to the expenditure of which we may, in the final analysis, reduce the production costs of the products $A, B, \ldots$. Having effected such a transformation in the 'production costs' equation of the product $a$, we have:

$$X_a = A_aX_a(1+r)^{t_a} + A_{a1}X_a(1+r)^{t_{a1}} + \cdots$$

(53)

Shortening as necessary and solving the equation with respect to $r$, we shall have:

$$r = F(A_a, A_{a1}, \ldots, t_a, t_{a1}, \ldots).$$

(54)

Since the periods of production $t_a, t_{a1}, \ldots$ are always finite, it follows that when

$$A_a + A_{a1} + A_{a2} + \cdots < 1,$$

(55)

we have $r > 0$.

Equation (54) does not contain the quantities $n_a, n_{a1}, \ldots$, i.e. the quantity of labour used in the production of the product $a$, and it yields $r$ as a function of the production period and the quantity of the good $a$ expended in production.

Equation (54) shows that whenever a known quantity of some product $a$ has been used up in the production of $a$ and we can obtain a larger quantity of the same product within some finite period of time as a result of the production process, the profit rate in the given branch of industry will be a fully-determined quantity greater than zero, irrespective of the price of the product $a$. If the production costs of the other goods, $A, B, C, \ldots$ are parts of a product between the three elements of price—rent, past labour and current labour in the sense of elements of price (p. 388).

It is a matter of indifference for Ricardo's theory whether these elements correspond in a given society to individual classes and persons or not, since 'by worker, rentier and capitalist he always understands more or less abstract fictitious persons'. However, 'Not merely does this not do any harm to the formulation of the problem of distribution given by Ricardo but, on the contrary, it is an indication of the theoretical, philosophical nature of this formulation, which ensures that Ricardo's deductions, should they be correct, have the nature of general laws' (p. 389).
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reduced in the final analysis to the same product α, the same profit rate should also be established in these branches under conditions of free mobility from one branch of production to another (irrespective of what the ratios $X_{Aα}, X_{Bα}, \ldots$ will be). The essence of the production process by means of which a 'production good' α yields as a result the products $A, B, C, \ldots$ and new quantities of the same good α is a matter of complete indifference for determination of the rate of profit. Whether the potential energy incorporated in the production good α is released and used in production in the form of human labour, as happens at present, or by means of some other process (not involving the participation of human labour) is a matter of indifference; whenever we have:

$$1 = A_α(1 + r)^{α} + A_{α1}(1 + r)^{α1} + \cdots$$

(56)
on

on condition that

$$A_α + A_{α1} + \cdots < 1,$$

(55)

the profit $r$ will be a fully-defined quantity and greater than zero. For example, suppose some production good β, to which the production costs of all the economic goods $A, B, C, \ldots$ may be ultimately reduced, be used in production by means of the conversion of its potential energy into the work of some living creatures other than man. On the basis of the conditions of production we can have and we shall have all the conditions needed for the occurrence of a profit. In this case the profit rate will be a fully-defined magnitude greater than zero, despite the fact that no unit of human labour was used in production.\footnote{The quantities $A_α, A_{α1}, \ldots$ are determined in a completely similar manner to the determination of production costs when human labour is used: $A_α = N_α b, A_{α1} = N_{α1} b, A_{α2} = N_α b, \ldots$, where $N_α, N_{α1}, N_{α2}, \ldots$ are the quantities of living labour (in any unit), and $b$ is the quantity of product β which must be expended for the production of one unit of labour.}

Finally, it is theoretically possible to imagine a case in which all products are produced exclusively by the work of machines, so that no unit of living labour (whether human or of any other kind) participates in production, and nevertheless an industrial profit may occur in this case under certain conditions; this is a profit which will not differ essentially in any way from the profit obtained by present-day capitalists using hired workers in production.

Suppose that a machine $M$ is able, without the participation of human labour, and using natural forces as a motor, to produce machines of the following orders: $M_1, M_{α}, M_α, \ldots$; let these machines in their turns singly or in combination automatically produce machines of an even higher order $M_1, M_α, M_α, \ldots$ until we ultimately arrive at machines $M_3, M_{β}, M_β, \ldots$ which directly produce the consumer products $A, B, C, \ldots$. 

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In this case the production costs of these products \(A, B, C,\ldots\) may always be reduced in the final analysis to the number (or parts) of machines \(M\) consumed in the production of the products \(A, B, C,\ldots\).

Suppose also that among the machines directly or indirectly produced by the machine \(M\), there is the machine \(M\) itself, i.e., in other words, let the machine \(M\) be capable of reproduction. In this case we shall have

\[
\begin{align*}
X_A &= n'_M x_M (1 + r)^{t_a} + n''_M x_M (1 + r)^{t_{a1}} + \ldots \\
\vdots \\
X_M &= N'_M x_M (1 + r)^{t_M} + N''_M x_M (1 + r)^{t_{M1}} + \ldots
\end{align*}
\]

where \(n'_M, n''_M,\ldots, N'_M, N''_M,\ldots\) will denote the number of machines \(M\) (or parts of the machine \(M\), if \(n'_M, n''_M,\ldots\) are less than unity) used up in the production of units of the products \(A, B, C,\ldots, M,\ldots\). If \(N'_M + N''_M + \ldots < 1\) in the equation for \(M\), then \(r\) will be greater than zero and a fully-defined quantity, provided that the quantities \(N'_M, N''_M,\ldots, t_M, t_{M1},\ldots\) are given.

We have therefore seen, proceeding from Ricardo's analysis, that the origin of industrial profit does not stand in any 'special' relationship to the human labour used in production. Profit may equally well occur in other production processes provided that they satisfy the quite definite conditions stated above. Whether or not such modes of production are capable of existing in the present state of technical knowledge is not a subject for political economy.

Let us assume that the production costs of the economic goods, \(A, B, C,\ldots\) may be reduced in the final analysis to the expenditure of the production good \(a\), whose production costs are themselves determined by the formula:

\[
X_a = A_a x_a (1 + r)^{t_a} + A_{a1} x_a (1 + r)^{t_{a1}} + \ldots
\]

Let us assume that by using a different production process (e.g. by using the work of animals in place of human labour) we can reduce the production costs of the goods \(A, B, C,\ldots\) to the expenditure of a production good \(b\), whose production costs are themselves determined by the formula:

\[
X_b = A_b x_b (1 + r)^{t_b} + A_{b1} x_b (1 + r)^{t_{b1}} + \ldots
\]

Let us assume that by using a further production process (e.g. exclusively the work of machines employing natural forces as a motor) we shall have correspondingly:

\[
X_M = A_M x_M (1 + r)^{t_M} + A_{M1} x_M (1 + r)^{t_{M1}} + \ldots
\]

Let \(r\), determined from equation (58), be \(r_a\) and let \(r\) from equation (59)
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be denoted by \( r_\theta \) and so on. Which of the possible production processes will in reality be used? Obviously, the one which yields the greatest value for \( r \) (this follows directly from the hypothesis that an economic subject tends to pursue the greatest advantage). Consequently, for any given production process actually to determine the profit rate, it is still insufficient that it could in general serve as a source of profit, and it is further necessary that it should yield a higher profit rate than all other possible processes. Consequently, for example, if the present state of technology could realise the hypothesis made above of production of all commodities exclusively by machines capable of reproducing themselves, the condition \( A_{M} + A_{M1} + \cdots < 1 \) would still be inadequate for the rate of profit in fact to be defined by the equation

\[
1 = A_{M}(1 + r)^{t_{M}} + A_{M1}(1 + r)^{t_{M1}} + \cdots \tag{61}
\]

from which

\[
r_{M} = f(A_{M}, A_{M1}, \ldots, t_{M}, t_{M1}, \ldots); \tag{62}
\]

it would further be necessary for \( r_{M} \) to be greater than the profit rate \( r_{a} \) established when human labour is used in production. We have previously assumed for simplicity that there is only one series of production costs equations corresponding to each production good \( a, \beta, \gamma, \ldots, \) to the expenditure of which the production costs of each product \( A, B, C, \ldots \) (including the products \( a, \beta, \gamma, \ldots \) themselves) may be reduced:

\[
\begin{align*}
X_{A} &= A_{a}X_{a}(1 + r)^{t_{a}} + A_{a1}X_{a}(1 + r)^{t_{a1}} + \cdots \\
X_{B} &= A_{b}X_{a}(1 + r)^{t_{a}} + A_{b1}X_{a}(1 + r)^{t_{a1}} + \cdots \\
& \vdots \\
X_{a} &= A_{a}X_{a}(1 + r)^{t_{a}} + A_{a1}X_{a}(1 + r)^{t_{a1}} + \cdots 
\end{align*} \tag{63}
\]

In reality, however, there may undoubtedly be several equations systems corresponding to each of them; thus, in addition to the system (63), it is also possible to have any system (64) for the same production good \( a \).

\[
\begin{align*}
X_{A} &= A_{a}X_{a}(1 + r)^{t_{a}} + A_{a1}X_{a}(1 + r)^{t_{a1}} + \cdots \\
X_{B} &= A_{b}X_{a}(1 + r)^{t_{a}} + A_{b1}X_{a}(1 + r)^{t_{a1}} + \cdots \\
& \vdots \\
X_{a} &= A_{a}X_{a}(1 + r)^{t_{a}} + A_{a1}X_{a}(1 + r)^{t_{a1}} + \cdots 
\end{align*} \tag{64}
\]

Thus, for example, let the product \( a \) be a product consumed by the workers and at the same time a product consumed by some other living creatures to whose work it is also possible to reduce the production
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costs of each of the products \( A, B, C, \ldots \), so that \( A_1 = n_1 a, A_{A1} = n_2 a, \ldots, A_B = m_1 a, A_{B1} = m_2 a, \ldots, A'_A = n_1 a', A'_{A1} = n_2 a', \ldots, A'_B = m_1 a' \), \( A'_{B1} = m_2 a' \), where \( n_1, n_2, \ldots, m_1, m_2, \ldots \) are the quantities of human work used in production; \( n_1, n_2, \ldots, m_1, m_2, \ldots \) are the quantities of the work of animals capable of being used instead of people in the productions \( A, B, \ldots \); \( a \) is the amount of the product \( a \) per unit of work consumed by a man; \( a' \) is the amount of the product consumed per unit of work completed by an animal.

The systems (63), (64), and so on will correspond to different production processes by means of which the potential energy of a production good \( a \) is used in the production \( A, B, C, \ldots \). There can be assumed to be any number of such processes corresponding to each of the production goods \( a, \beta, \ldots \).

But in reality one out of all these equation systems will be in force, namely the one which will yield the greatest value for \( r \) (determined from the equation for \( r \)). This is because no one will begin to use modes of production which yield a low-profit rate if it is possible to use a mode determining a higher rate.

Now let us assume that the production costs of the products \( A, B, C, D, \ldots, L \) may be reduced to the expenditure of the product \( D \); let the production good \( D \) be the only good to which the production costs of all the products \( A, \ldots, L \) (including \( D \) itself) may be reduced. The 'production costs' equation for \( D \) from the foregoing gives us a fully determined magnitude of \( r \). From the equation

\[
X_D = A_DX_D(1+r)^{t_A} + A_{D1}X_D(1+r)^{t_{D1}} + \ldots
\]

we have

\[
r_D = f_D(A_D, A_{D1}, \ldots; t_D, t_{D1}, \ldots);
\]

the same profit rate is also established in the productions \( A, B, \ldots, L \).

Now let us assume that the production costs of the remainder of the products \( M, N, \ldots, P \), cannot be reduced to the product \( D \).\(^1\) Let the only product to which they may be reduced be \( N \);\(^2\) in which case the production costs equation of the product \( N \):

\[
X_N = A_NX_N(1+r)^{t_N} + A_{N1}X_N(1+r)^{t_{N1}} + \ldots
\]

still yields a fully-determined quantity for \( r \):

\[
r_N = f_N(A_N, A_{N1}, \ldots; t_N, t_{N1}, \ldots).
\]

\(^1\) For example, let \( D \) be a product consumed by animals; in that case only the production costs of commodities producible by the labour of animals may be reduced to the product \( D \), since if the products \( M, N, \ldots, P \) may be produced only by human labour, their production costs can no longer be expressed in terms of \( D \).

\(^2\) If products \( M, N, \ldots, P \) are products of human labour, \( N \) should be understood as the product consumed by the workers.
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This profit level is also established in the branches of production $M, N, \ldots, P$.

Therefore, we shall have one rate of profit $r_D$ for one part of the production $A, B, \ldots, L$, and another rate $r_N$ for the other part $M, N, \ldots, P$.

Let $r_D > r_N$; in that case producers will begin to forsake the branches $M, N, \ldots, B$ and transfer to the branches $A, B, C, \ldots, L$. The price of the products $A, B, C, \ldots, L$ will begin to fall (owing to the excess of supply over demand), but since the profit rate determined from the equations

$$r_D = f_D(A_D, A_{D1}, \ldots, t_D, t_{D1}, \ldots)$$
$$r_N = f_N(A_N, A_{N1}, \ldots, t_N, t_{N1}, \ldots)$$

(69)

is not dependent on $X_D$ and $X_N$, and consequently is not dependent on $X_{DN} = X_D/X_N$, the profit level $r_D$ will continue to remain above $r_N$ however much capital may be 'poured' from the branches $M, N, \ldots, P$ into the branches $A, B, \ldots, L$. However much value the product $D$ may lose owing to the excess of supply, it will nevertheless be more advantageous to entrepreneurs to lay out capital in the production of $D$ than in production of the product $N$ (or any other product from among $M, N, \ldots, P$), which is very costly, since the entrepreneur will receive a larger sum per unit of value expended in unit time in the production of $D$ than per unit of value in the production of $N$. The reason for this is that a drop in the price of the finished product $D$ will invariably be matched by a proportionate drop in its production costs, since these costs are reduced to the product $D$ itself.

Whatever quantities we insert for $X_D$ in equation $D$ we obtain the same magnitude for the rate of profit $r_D$ (the amount of profit per unit of value expended in unit time). Consequently, there is apparently no natural limit to the movement of capitalists from the branches $M, \ldots, P$ into the branches $A, \ldots, L$ apart from complete cessation of the production of $M, \ldots, P$. Such a conclusion would be correct if, when $r_D > r_N$, economic expectation always led producers to transfer from the branch $N$ to $D$. In fact, the hypothesis that producers tend to transfer from branches with a low rate of profit to branches with a high rate holds only for cases in which all the quantities entering into the economic calculation of the entrepreneurs have a finite value. Since this latter condition does in fact hold in most instances, the foregoing hypothesis is in general found to be correct in practice, but if adopted as the basis of abstract analysis it may lead to false conclusions. Let us assume that an entrepreneur previously expended $N$ units of value (in an arbitrary unit) in production of $A$, so that in a unit of time we have $N(1 + r_A) - N$ units.
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of profit, and in $T$ units of time we have $N(1 + r_A)^T - N$ units of profit; were he to place this amount of value in production $B$, he would have $N(1 + r_B)^T - N$. If $r_B > r_A$, we have in general

$$N(1 + r_B)^T - N > N(1 + r_A)^T - N$$  \(70\)

from which

$$N[(1 + r_B)^T - (1 + r_A)^T] > 0.$$  \(71\)

The situation will, however, be different if one of the factors in the first part of the inequality disappears. For example, let the price of product $B$ fall to nothing owing to excess of supply (assuming as the unit of value the value of any of the products produced by expenditure of the product $A$), in which case, however much of product $B$ is produced, $N$ will also be zero, and consequently so will the entire expression

$$N[(1 + r_B)^T - (1 + r_A)^T] = 0,$$  \(72\)

i.e. all motive for the movement of producers from $A$ to $B$ is destroyed. Therefore, when the value of product $B$ (and of others produced by its means), expressed in terms of product $A$ (or by some other commodity produced by its means), falls to zero, the incentive for conversion from $A$ to $B$ will cease, despite the fact that the rate of profit continues to remain higher in $B$ than in $A$ (since the rate is not dependent on price). Consequently, when different constant profit rates exist in different branches of production, a balance will be established either when products yielding a high profit rate pass into the realm of free goods or when the production of products with a low rate of profit is discontinued. What is actually produced in such a specific case is a question of fact and is dependent on the form of $f_A(D_A), f_B(D_B), \ldots, f_M(D_M), f_N(D_N), \ldots$ expressing the price of products $A, B, \ldots$ as a function of their sale: $D_A, D_B, \ldots$. The smaller is the group represented by the goods $A, \ldots, L$ (by comparison with the group $M, \ldots, P$) and the smaller is the demand for these goods (i.e. the smaller is the quantity of these goods which completely satisfies the demand for them), the greater is the probability that they will become ‘free’ goods before all capital leaves the branches $M, \ldots, P$ (and vice versa). Therefore, even were there actually to exist at present some exceptional production processes which were able without the participation of human labour to reproduce their real production costs in natura (and not in the form of equivalent value) and consequently to determine an independent level of profit unrelated to the production costs of the means of subsistence of the workers, the only result of such a situation, in view of the limited nature of the demands which these processes could satisfy, would be to render these products completely valueless and to transfer them to
2. Ricardo's theory of value

the realm of free (non-economic) goods. There is, therefore, no foundation for any of the references to various ‘natural’ processes (such as the breeding of animals and yields which do not necessitate human tending of the plants etc.) as independent sources of ‘profit on capital’.¹

Let us now present our ‘production costs’ formulas in a more general form (more general than the formula \( X_A = A_A X_a (1+r)^t_A + A_{A1} X_a (1+r)^t_{A1} + \cdots \)), namely let us set:

\[
A_A X_a = P_A; \quad A_{A1} X_a = P_{A1}; \quad \ldots
\]

(73)

in which case we obtain:

\[
X_A = P_A (1+r)^t_A + P_{A1} (1+r)^t_{A1} + \cdots
\]

(74)

where \( P_A, P_{A1}, \ldots \) will directly denote the number of units of value expended in production or, in other words, will denote the real production costs expressed in a common unit of value with the finished product.² Equation (74) is the most general expression of the connection between the price of a product and production costs, and therefore it enables us to extend our analysis beyond present-day forms of production.

Let us imagine⁵ a situation in which manpower is withdrawn from market circulation (for whatever reason: without or by means of legislation) so that it is impossible to buy or sell human labour on the market. In that case, obviously, it will no longer be possible to reduce the real production costs of products to the expenditure of products (the means of subsistence of the workers); human labour will be the last level to which they can all be reduced. Let the value of a unit of labour expressed in the same common measurement unit in which the values of the finished

¹ See pp. 77-78 for the characteristics distinguishing ‘profit on capital’, as a special form of income, from other forms of ‘income from ownership’.

² This formula may be derived directly from the definition of the concept of ‘profit’ as the difference between the value expended in production and the value obtained as a result of production:

\[
A_A = P_A + Z + P_{A1} + Z_{A1} + \cdots
\]

where the sums of the profits are replaced by expressions corresponding to them in terms of the rate of profit \( r \).

⁵ Ed. note. In this passage Dmitriev argues that if labour ceased to be a commodity (i.e. if it were ‘withdrawn from market circulation’) all other commodities would exchange at prices equal to their direct and indirect labour contents (labour values). Obviously Dmitriev needs an additional assumption, that labourers as free (i.e. non-hired) producers should not be able to rent capital goods from capitalists (or from each other) and sell capital goods to capitalists (or to each other); for otherwise there would be no reason why relative prices should be different, whether workers are hired by capitalists or machines are hired by workers. If we add this assumption then we must be either in a world without capital goods or in a world without capitalists (including the State among capitalists). In either case there would be no profit, hence relative prices could conceivably be equal to relative values. However, under this necessary additional assumption Dmitriev’s proposition seems somewhat trivial.
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products $X_A, X_B, \ldots$ are expressed by $K$. In that case, if the quantity of labour used (directly and indirectly) in the production of a unit of the products $A, B, \ldots$ is expressed by $M_A, M_{A1}, \ldots, M_B, M_{B1}, \ldots$, we shall have a series of equations:

$$X_A = M_AK(1 + r)^{t_A} + M_{A1}K(1 + r)^{t_{A1}} + \cdots$$
$$X_B = M_BK(1 + r)^{t_B} + M_{B1}K(1 + r)^{t_{B1}} + \cdots$$

(75)

The unknowns in the equations for $X_A, X_B, \ldots$ will be $K$ and $r$. Having regard to the fact that we shall have one equation in the equation system (75) (for the commodity used as a measure of value, i.e. a commodity whose value is taken to be unity):

$$1 = M_pK(1 + r)^{t_p} + M_{p1}K(1 + r)^{t_{p1}} + \cdots$$

(76)

which does not add a new unknown, we shall therefore have a total of $n$ equations with $(n+1)$ unknowns. In our previous analysis we excluded the superfluous $(n+1)$ unknown by means of the equation:

$$X_a = A_aX_a(1 + r)^{t_a} + A_{a1}X_a(1 + r)^{t_{a1}} + \cdots$$

(53)

which directly yielded us $r$ as a function of the known quantities. It is natural to consider whether it is also possible to find a similar equation in the system (75). Clearly this is not possible; for this to be possible one would have to obtain, as a result of production, the same 'production good' to which all real production costs could be reduced. But this is impossible, because production costs will always consist of labour (since labour cannot be bought by the price of its means of subsistence), and the result of production will always be a product, and not labour. Therefore the production cost equation for product $a$ will be included in labour [since labour cannot be bought by the price of the means of its subsistence], and the result of production will always be a product, and not labour. Consequently, the production costs equation for the product $a$ will be:

$$X_a = M_aK(1 + r)^{t_a} + M_{a1}K(1 + r)^{t_{a1}} + \cdots$$

(77)

where $X_a$, $K$ and $r$ are unknowns. If we take the value of the product $a$ to be the unit of value (i.e. if we take the commodity $a$ to be the commodity used as a measuring unit), the equation becomes:

$$1 = M_aK(1 + r)^{t_a} + M_{a1}K(1 + r)^{t_{a1}} + \cdots$$

(78)

In order to determine $r$ from this, $K$ would have to be a known quantity. However, for $K$ to be known, $r$ would have to be known. Therefore, the question apparently remains unresolved, at least within the limits of the
2. Ricardo's theory of value

data of production conditions (expressed by the production costs equation).
This is, however, only apparent. In reality, the quantity \( K \), expressing
the equivalent ratio of product \( \alpha \) and labour, cannot be determined on the
market, by virtue of the assumption made at the beginning of the present
analysis (since labour has been removed from market exchange). It
follows therefore that the only process by means of which the two different
goods (the product \( \alpha \) and labour) may replace each other in equivalent
quantities will be the process of production of product \( \alpha \).

Every man who has in his possession some quantity of units of labour
has no means of replacing them by the product \( \alpha \) other than by expending his labour
in the production of the product (he cannot sell his labour on the market).
The coefficient \( K \) cannot therefore remain undetermined, but will have a
quite precise (and unique) value determined by the conditions of pro-
duction of product \( \alpha \). If \( N \) units of labour are capable of producing \( S \) units
of the product \( \alpha \), it follows that \( K = S/N \).\(^1\) Consequently, we shall have
\( M_A K + M_{A1} K + \ldots = 1 \) in our equation (78), from which the only value
for \( r \) satisfying this equation will be \( r = 0 \). Therefore despite the apparent
inadequacy of equation system (75), we obtain a fully determined magnitude
for \( r \) which, by the law of the equality of the profit level in all industries,
is also established in the industries \( A, B, \ldots \).

Setting \( r = 0 \) in the system (75), we obtain:

\[
\begin{align*}
X_A &= M_A K + M_{A1} K + \ldots \\
X_B &= M_B K + M_{B1} K + \ldots \\
\vdots \\
X_{AB} &= \frac{X_A}{X_B} = \frac{M_A + M_{A1} + \ldots}{M_B + M_{B1} + \ldots}, \text{ and so on,}
\end{align*}
\]

From this, i.e. the exchange ratio of commodities will be determined exclusively by the quantity
of labour used in their production, irrespective of the time that will have elapsed
between the time labour was expended and the time when the finished product was
obtained.\(^2\) Therefore, the law of 'labour value' would always hold were
human labour to be withdrawn from circulation in the market (whereas
in the present state of affairs it holds only for products produced by
capital goods of the same organic composition, as is noted and emphasised
by Ricardo).

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\(^1\) \( N \cdot K = S \cdot X_A \), from which \( K/X_A = S/N \) and, since \( X_A = 1 \) (because we have taken
the product \( \alpha \) as the commodity used as a unit of measure), we have: \( K = S/N \).

\(^2\) In the absence of hired labour, the introduction of capital goods (capitale tecnico) will
not therefore serve to infringe the 'labour theory of value' (see, in this respect, the remarks
by A. Loria, Analisi della proprietà capitalista, 1889. Loria does not directly assume the
absence of hired labour, but this stems indirectly from the other conventional assumptions
which he makes, such as free land (terra libera)).
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The same conclusion may be arrived at by a different route. For there to be equilibrium in the sphere of production it is essential that entrepreneurs should be identically rewarded in all branches of industry. This condition is satisfied if equal amounts of value expended in equal periods of time yield equal amounts of value in all branches of production. Let us assume that we have two industries A and B; let the process of production of product A require \( t \) units of time for its completion and let the process for B require \( nt \) units of time. Let us assume that when \( N \) units of labour have been expended we obtain \( M_A \) units of the product A or \( M_B \) units of the product B.

Let \( K_N, X_A \) and \( X_B \) respectively denote the value of a unit of labour and of units of the products A and B expressed in some arbitrary but common unit. In that case the amount of value expended in the productions A and B will equal \( K_N N \); the amount of value obtained in the production of B on completion of the production process, i.e. after \( nt \) units of time (from the expenditure of labour) will equal \( X_B M_B \). For equilibrium to exist between sectors A and B it is essential that the value of the total quantity of the product obtained in sector A over the same period of time should also equal \( X_B M_B \) (since otherwise there would be infringement of the condition that equal quantities of value yield equal quantities of value in all branches in equal periods of time). In order now to determine the value of one unit of the product A, it is necessary to divide \( X_B M_B \) by \( y \), the number of units of the product A obtained in sector A for \( N \) units of labour expended in \( nt \) units of time. The question which arises is how large \( y \) will be in the absence of hired labour? It is not difficult to see that in this case (in contrast to what is observed under present-day conditions) \( X_A M_A \) units of value obtained in the production of A at the end of \( t \) units of time cannot be exchanged in the market for an equivalent quantity of labour (which, when \( r > 0 \), is always greater than \( N \); i.e. is greater than the quantity of labour expended in the production of \( M_A \) units of product A). Therefore, \( N \) units of labour will yield as many units of the product in the production of A in \( nt \) units of time as will be yielded in \( t \) units of time, i.e. \( M_A \) since the production process for A cannot be repeated without the expenditure in production of \( N \) further new units of labour.

Hence, for sectors A and B to be in equilibrium, the value of a unit of product A should equal:

\[
\frac{X_B M_B}{M_A} = X_A \quad \text{from which} \quad \frac{X_A}{X_B} = \frac{M_B}{M_A} \tag{81}
\]

1 By virtue of the law of the 'equality of profit rates' already established by Adam Smith.

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and since the quantities of labour needed for a unit of product A and of product B are respectively

\[ N_A = \frac{N}{M_A} \quad \text{and} \quad N_B = \frac{N}{M_B} \]  \hspace{1cm} (82)

we have:

\[ \frac{N_A}{N_B} = \frac{N \cdot M_B}{M_A \cdot N} = \frac{M_B}{M_A}, \quad \text{from which} \quad \frac{X_A}{X_B} = \frac{N_A}{N_B}, \]  \hspace{1cm} (83)

i.e. the relation between the values of units of products A and B, expressed in any common unit, should be equal to the relation between the quantities of labour used in their production, irrespective of the length of the production processes of A and B.

Let us now proceed from this general analysis of the conditions affecting the appearance and rate of profit to the present state of affairs. Hardly anyone will dispute\(^1\) that the only process determining the level of profit at the present time is the process of production of the means of subsistence of the workers (capitale alimenti\(^2\)). Let us consider this special case of the existence of profit on capital in greater detail. Take the production costs equation of the means of subsistence \(a\) of the workers:

\[ X_a = n_a aX_a(1 + r)^{t_a} + n_{a_1} aX_a(1 + r)^{t_{a_1}} + \cdots \]  \hspace{1cm} (84)

which, when shortened, yields

\[ 1 = n_a a(1 + r)^{t_a} + n_{a_1} a(1 + r)^{t_{a_1}} + \cdots \]  \hspace{1cm} (85)

from which

\[ r = F_{a}(t_a, t_{a_1}, \ldots ; n_a, n_{a_1}, \ldots ; a). \]  \hspace{1cm} (86)

It is evident from equation (85) that the derivatives of \(F_a\) with respect to the variables \(t_{a_1}, t_{a_2}, \ldots, n_{a_1}, n_{a_2}, \ldots, a\) will all be negative. This means that the quantity \(r\) will be smaller: (1) the greater is the labour expended on production of a unit of a subsistence product of the workers, (2) the greater is the time elapsing between the moment that the labour is expended and the time when the finished product is obtained, (3) the greater is the amount of the consumer product of the workers consumed per unit of work.

In Ricardo’s opinion, the most important factor affecting an increase

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\(^1\) Although discussion of this question is not within the competence of political economy.

\(^2\) The terminology of Loria; see his Analisi della proprietà capitalistia, 1889.
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in the quantity of labour expended on the production of a unit of a product consumed by the workers is the need to go over to the cultivation of less fertile land as the population increases. This point can be largely nullified by improvement in the techniques of land cultivation and, in particular, by the acceleration of production processes.

The quantity denoting the amount of the product consumed per unit of work, when the iron law of wages prevails, will be dependent on the level of needs of the worker and will increase together with them. If we imagine a situation in which the iron law of wages does not hold, the quantity \( a \) will in general be determined by the actual struggle of the mutually opposed interests of the capitalists striving to establish the greatest possible value for \( r \) and therefore striving to reduce the quantity \( a \) to the minimum possible, and of the workers striving conversely to raise \( a \) to the greatest possible value. The level of \( a \) at which equilibrium is established is a question of fact and is dependent on the strength of the opposing parties. In this state of affairs investigation of the conditions affecting the level of \( a \) falls outside the scope of political economy and within that of other disciplines; in this case also, as when the iron law of wages prevails and \( a \) is determined by the physiological needs of the worker’s body, political economy should take the quantity \( a \) to be given in its analysis. To proceed in any other way would be to offend against the requirements of correct methodology, by virtue of which every science should have its own special subject and corresponding strictly defined limits.

At all events we invariably have two limits for \( a \): a lower limit, which will be the quantity \( a \) established when the iron law of wages (determined

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Figure 1.1

Clearly when the wages rate rises from \( OA \) to \( OB \) not only will the supply of labour not increase, but on the contrary it will fall. A detailed and thorough analysis of the ‘labour supply curve’ (and also of the prototype curves from which it is derived) is to be found in W. Launhardt: *Mathematische Begründung der Volkswirtschaftslehre*, Leipzig, 1885, pp. 94–5 and also p. 90 [the figure is from Launhardt, p. 95].
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by the physiological needs of the worker’s body) prevails, and an upper limit, which will be the total quantity of the product produced per unit of work.

A question which arises is whether it is possible to find a value for \( r \) between these limits which, even if only in some hypothetical ideal system, would be simultaneously the most advantageous both for the capitalist employers and for the workers and consequently would be determined by an economic factor.

We know that this is a problem which the celebrated economist J. H. von Thünen, one of the first economists to decide to apply higher mathematical analysis to economic problems, set himself. Thünen refers to the wages which satisfy such conditions as ‘natural wages’ (in contrast to the wages established by the struggle of the mutually opposed interests of capitalists and workers). Thünen concludes as a result of his study that it is possible to achieve complete harmony of the interests of capitalists and workers under certain ideal conditions; Thünen defined the level of wages most advantageous both to the capitalists and to the workers by the formula \( \sqrt{ap} \), where \( a \) are the means of subsistence necessary to the worker in a unit of time and \( p \) is the entire production of the worker in the same unit of time.

Unfortunately, despite its apparent rigour, Thünen’s research suffers from many omissions, each of which is sufficient in itself (owing to the very nature of the study) to make his conclusion unconvincing. We shall not here make a detailed analysis of Thünen’s work (see the criticism of Thünen’s theory in Launhardt)\(^1\) but shall confine ourselves to an indication of his principal error (thanks to which alone he was able to obtain a definite answer to the problem which he had set himself), which is of general fundamental (methodological) importance. We refer to Thünen’s incorrect use of maximum formulas. There is no doubt that the techniques made available by higher mathematics for determination of the value of a variable which maximises a

\(^1\) See Launhardt, 1885, p. 150. ‘This doctrine is erroneous. By contenting themselves with the necessary wages, the workers employed in the factory incur a privation which cannot be measured by the number of days during which they are subject to this privation but by the magnitude of this privation which is equal to \( y \) for each day. The magnitude of the privation is therefore equal to \( na(a + y) \); if this is divided by the profit of the firm one obtains \[ 1/(a+y) - 1/y; \] this expression is greatest for \( y=0 \). And this cannot be otherwise if one considers the question of the wage rate from the viewpoint of the entrepreneur, since the entrepreneurial profit will be greater the smaller the wage.’

Launhardt’s correction would be right if people did strive under the influence of economic calculation to the highest level of income on their capital; in fact this occurs only when capital is a constant or a variable independent of the rate of interest. We are not justified in assuming either case on the basis of the main assumptions made by Thünen. And that is why the aspiration of people toward the greatest well being may be regarded as satisfied (and consequently a balance of interests as established) only when the total sum of income on capital is greatest. The maximum formula given by Launhardt does not satisfy this requirement.
function ought to find extremely important application in political economy, which studies the actions of individuals under the influence of their striving for the greatest advantage. Nevertheless, great care ought to be taken to ensure that the differential formulas which serve to determine the value of a variable corresponding to the maximum of a function should not be applied purely mechanically to economic problems. Under the influence of the pursuit of the greatest benefit every economic subject in fact strives (as far as this is dependent on his will) to impart to all variables on which his net income depends, values such that the total sum of this net income should be greatest, but from this it does not at all follow that the same may be accepted in relation to any partial income (revenue partiel) of the subject. A value of the variable which yields the maximum value for some one partial income of the subject may completely fail to correspond to the maximum value of his total income and, consequently, may contradict the basic striving of every economic subject toward the greatest benefit. The sole exception is the case in which a given partial income does not stand in any functional relationship to the other parts of the total income.

The differential equation \( d\left(\frac{(p - a - y)y}{q(y + a)}\right) = 0 \) from which Thünen determines the most advantageous level of wages \((a + y)\) for a hired worker obviously contradicts this basic methodological rule: in fact, given the conventional assumptions which Thünen makes in order to present the worker as completely free of the effect of the 'iron law of wages', the formula \( \left(\frac{(p - a - y)y}{q(y + a)}\right) \) expresses only a part of the worker’s total income and moreover a part which cannot be accepted as an independent variable relative to the remaining income (since the income from previously accumulated capital is also a function of the variable \(y\)).

It was only thanks to the methodological error that Thünen was able to obtain a definite answer to the task which he has set himself. Had he based his analysis on an expression of the worker’s total income, and not on an arbitrarily selected part of it, he would have found that the question which he had formulated did not have and could not have any definite solution since, given the assumptions made by him, the sum of total income is a quantity independent of \(y\) and consequently also of \((a + y)\) (despite the fact that taken separately each of the parts of the total income is a function of the variable \(y\)), so that in such a hypothetical state of affairs the amount of wages would be a matter of complete indifference both for the workers and for their employers.

\footnote{This formula would express the total sum of the worker’s income only if the worker had no previously accumulated savings at all, but under such conditions it would obviously also not be possible to refer to any ‘natural’ wages.}
2. Ricardo's theory of value

Now let us turn to our equation (85), which defines the quantity \( r \) under present conditions:

\[
1 = n_a(1 + r)^t + n_{a1}(1 + r)^{t_{a1}} + \cdots \tag{85}
\]

Setting in it \( n_a(1 + r)^t + n_{a1}(1 + r)^{t_{a1}} + \cdots = 1 \), we have \( r = 0 \). With this state of affairs any extraction of profit from capital becomes impossible and, in consequence of this, any capitalist production (i.e. with hired workers) should cease (in fact it would cease even before \( r \) became zero). This at least is how it would be if the conventional assumptions made by us at the beginning of our analysis of the phenomenon of profit on capital were in fact to hold. But since in fact these conventional assumptions are realised only in exceptional cases, the incentive to continue production and exchange would still remain in force for some of the entrepreneurs even when \( r = 0 \).

The point is that profit on capital is not the only form of income yielded by capital. Following Ricardo, we take profit on capital to mean only one quite definite form of income regulated by its own precisely defined laws. The characteristic distinguishing this form of income from the group of remaining (rent-like) incomes governed by their own laws is that the 'profit on capital' is obtained by virtue of the mere possession of capital, whereas all other forms of income connected with capital are obtained by virtue of the various advantages of some capitalists over others. These advantages may relate both to the sphere of production and to the sphere of sale (exchange) and even to the sphere of consumption; they may be either temporary or permanent (the former correspond to what are known as incomes depending on the general conditions of markets [kon'yunktural'nye dochody], and the latter to rent-like incomes in the strict sense). However they may be expressed, the incomes produced by them are subject to their own definite laws which have nothing in common with the laws governing the origin and level of the 'profit on capital'. It would be not merely unscientific but impossible to study these two groups of incomes together,\(^1\) since the difference existing between them is not merely superficial but fundamental. The actual classification of incomes (into profit on capital and rent-like incomes) cannot present the slightest difficulty: all that needs to be done to decide to which group a given income belongs in each specific case is to consider whether this income would be possible if all the capitalist entrepreneurs were placed under completely identical conditions both in relation to production and in relation to sale and consumption. Such a conventional assumption excludes all possibility of the occurrence of rent-like profits and the only possible

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\(^1\) For this reason the extension of the concept of 'rent' (made by Rodbertus) to 'all income which any person obtains without personal labour, solely on the basis of some possessions' is highly irrational. (C. Rodbertus, Zar Eichstaitis, 1842, p. 64; Zur Bisleitungen der socialen Frage, Berlin, 1875, Vol. i, p. 32.)
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Income from capital will be 'profits on capital' in the strictly scientific sense (i.e. understood as a quite definite form of income governed by its own unique laws). This is the approach which we have previously used in our analysis of the 'profit on capital'. Given such an assumption, all possibility of the extraction of income from capital is eliminated when r = 0 (since all the conditions for the development of rent-like income have thus been excluded beforehand and 'profit on capital' is the only possible income from capital).

We still have to consider whether or not our conclusion will be modified if, instead of calculating income in exchange units, we calculate it (as is in fact done by every economic subject) in its use value.* For this purpose we must calculate the sum of the use value (utility) represented to a given individual by the product expended by him in production and to subtract this sum from the sum of the use value represented to the same individual by the finished product of the production. If, in the interests of simplicity, the actual period of production is taken as the unit of time, the profit rate r may be arrived at by dividing the difference obtained by us by the sum of the use value expended.

Let the use value of a unit of the product α be K. In this case, under the production conditions we have assumed we shall have for the determination of r:

\[ K = n_0aK(1+r)^{k_0} + n_1aK(1+r)^{k_1} + \cdots \]

or \[ 1 = n_0a(1+r)^{k_0} + n_1a(1+r)^{k_1} + \cdots \]  

(87)

from which \( r = 0 \) when \( n_0a + n_1a + \cdots = 1 \). But this will be so only while we assume that the use of a unit of the given product is a constant for the given individual, at least within the effective limits of economic calculation. In fact, however, this is not so: the use value of a unit of a given product for a given individual is a function of time. It was on this basis that Böhm-Bawerk attempted to construct an independent theory of profit on capital (i.e. independent of production conditions) in his work Kapital.

* Ed. note. It is difficult to think what Dmitriev seeks in this passage (up to the end of Section 2 of the First Essay) by calculating income (and inputs) in terms of 'use value', unless he has in mind a situation of equal marginal and average (cardinal) utility of commodities, and admits the interpersonal additivity of utility levels. The weakest point of his analysis is his criticism of Böhm-Bawerk, on the ground that it is sufficient 'to postulate that all people 'overvalue' present goods by comparison with future goods to an equal degree for this overvaluation to cease to be a source of income'. In modern terminology 'overvaluation' of this kind does not require a rate of substitution between present and future consumption equal to \( (1+r) \), where \( r \) is the interest rate, whatever the relation between present consumption \( c_0 \) and consumption at a future date \( c_1 \); 'overvaluation' would be interpreted as \( (1+r) > 1 \) for \( c_0 = c_1 \). Even if economic agents had the same rate of 'overvaluation', or more generally if they had identical time preference functions, as long as their relative endowments of dated consumption differ there would be a positive mutual gain from exchange.
2. Ricardo's theory of value

und Kapitalzins. He states: 'Present goods are generally of greater value than future goods of the same sort and number. This proposition is the kernel and centre of the theory of interest, which I have to propose' (Positive Theorie, 1889, p. 248).\(^1\)

It is not difficult to demonstrate that although Böhm-Bawerk wished to indicate the source of 'profit on capital' he in fact indicated only a new source of rent-like (differential) income. In fact, we only have to postulate that all people 'overvalue' present goods by comparison with future goods to an equal degree for this overvaluation to cease to be a source of income.

In fact, let us assume that an individual A gives an individual B a sum of 100 roubles to be returned with interest a year later. Let the ratio of the usefulness of one rouble at the present time to the usefulness of one rouble after a year be 2:1, denoting these usefulnesses by \(K_p\) and \(K_f\) and we have \(K_p:K_f = 2:1\), from which \(K_f = \frac{1}{2}K_p\); if B agrees to give A 200 roubles at the end of a year, his income from this operation expressed in use value is:

\[
100K_p - 200K_f = 100K_p - 200 \cdot \frac{1}{2}K_p = 100K_p - 100K_p = 0. \tag{88}
\]

Were he to return more than 200 roubles at the end of a year, his income would be expressed by a negative quantity: therefore 200 will be the highest sum which B may in general return to the creditor A. Let us consider what will be A's income at this highest sum of 200 roubles which B may return to him. Since we assume that neither of the contracting parties A and B has an advantage over the other in the sphere of consumption, the coefficients \(K_p\) and \(K_f\) will be the same for both. Consequently, the benefit obtained from the operation by contracting party A will also be expressed by \(100K_p - 200K_f = 100K_p - 100K_p = 0\).

A transaction between A and B under conditions which exclude the

\(^1\) Böhm-Bawerk notes three main 'overvaluations' of goods in hand. It is only the second of these three bases (the difference in the use value of goods in hand and future goods) that is an essentially new factor capable of providing a basis for the construction of an independent theory of the origin of profit on capital. Böhm-Bawerk gives the following formulation of the second basis (Zweiseit Grund): 'We underestimate systematically our future needs and the means for their satisfaction' (p. 266). '... The existence of this fact is beyond doubt. It is more difficult to say why the fact exists' (p. 267). Böhm-Bawerk subsequently indicates three bases for this fact. 'It seems to me that a first reason stems from the fragmentary character of the idea we have about the future state of our needs' (p. 268). '... While this reason seems to amount to erroneous estimation, a second reason seems to stem from an erroneous decision. I believe that it occurs frequently that somebody who is faced with a choice between a present and a future pleasure or pain decides in favour of the lesser present pleasure although he knows exactly and is aware at the moment of his choice that the future disadvantage is greater and hence that his choice is disadvantageous for his wellbeing as a whole' (p. 268). '... Finally as a third reason, the consideration of the short duration and insecurity of our life seems also, to me, to be important' (p. 269).
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possibility of the development of rent does not therefore give any advantage to either of the contracting parties; it is further not difficult to prove that under such conditions an advantage will be completely impossible provided, of course, that the contracting parties are guided in their actions by correct economic calculus.

In fact, correct economic calculus is incompatible with any economically purposeless acts (i.e. acts as a result of which the benefits do not exceed the sacrifices), even when a given act does not entail any risk (since every act invariably entails some expenditure of energy, which could be otherwise used on something else with greater benefit or satisfaction). But every transfer of value to other hands always entails some risk. There is no point in incurring this risk if the transaction is unprofitable, and therefore the very transfer of value to other hands under such conditions is opposed to the economic calculation of a contracting party acting in good faith. Consequently, under conditions which exclude the possibility of the occurrence of rent, no overvaluation of goods in hand can provide an independent source of profit on capital. Therefore, if production conditions are such as we have assumed at the beginning of this section, profit on capital cannot arise whatever the units in which the balance of the economic operation is calculated (units of exchange or use).

All the foregoing is fully applied in the theory in which Launhardt attempted several years before the appearance of the second volume of *Kapital und Kapitaleins* (in which Böhm-Bawerk's own views are set out) to construct a theory of profit on capital on the same basis as Böhm-Bawerk.¹

Although the 'overvaluation' of goods in hand by comparison with future goods, which is noted by Böhm-Bawerk, does not contribute anything new to the theory of the development of profit and the level of profit, it is a significant factor under the given conditions of production in the question of the accumulation of capital. (For a detailed analysis of this question see Launhardt, 1885, pp. 67–9).

3. THE THEORY OF MONOPOLY PRICES

Ricardo pays hardly any attention to the laws determining the price of scarce products. Nor is any clear distinction to be found in his writings between scarce products in the true sense and monopoly products (the

¹ Compare Launhardt, 1885, '... an enjoyment is the less appreciated the more distant the future when it can be had...' (p. 5). 'The secure prospect of an enjoyment in the future is thought to be of lesser value than the same enjoyment in the present...' (p. 6). This hypothesis is subsequently related to interest: 'Interest is the compensation for waiting for an enjoyment or for the temporary renunciation of an enjoyment...' (p. 7).
3. The theory of monopoly prices

quantity of which is arbitrarily limited by the individual in whose hands
the production of this product is exclusively concentrated).

We shall consider the first case in passing when we analyse the laws
determining the price of products whose production is under the influ-
ence of unlimited free competition. The second case, which is of interest
in its own right, we shall consider straight away. It will be seen sub-
sequently that it is impossible to proceed to a scientific criticism of the
theory of competition unless this problem is correctly solved.

How should a monopolist, motivated by the desire to achieve the
greatest advantage, decide how much to produce?

Adam Smith stated that a monopolist will restrict supply so as to
obtain for his goods the highest price the consumer will agree to pay. It is
not difficult to see that to proceed in this manner would not be in
accordance with the aspiration for the greatest advantage; a large price
does not in fact ensure an even greater advantage and would do so only
if the price and the amount sold were to be independent variables.
However, this is not so in practice, and price is invariably a function of
the quantity sold.

If we denote the amount sold in a unit of time by $D$ and the price by $p$,
we shall have $p = f(D)$; now let the production costs of each unit be $u$;
except for the category of products which we shall consider in the next
section, this quantity is independent of the amount produced and may be
taken by us to be a constant. In this case the total profit of the monopolist
in a unit of time will be given by

$$D \cdot f(D) - D \cdot u$$  (89)

If the quantity $D$ is determined by a single individual (the monopolist)
motivated by correct economic calculation, he will so determine it that
the value of the expression $[D \cdot f(D) - Du]$ will be as large as possible. This
quantity $D$ is determined from the equation:

$$d[D \cdot f(D) - D \cdot u] = 0.$$  (90)

Before proceeding with further analysis, let us pause to consider in
greater detail the relationship between price and the amount sold
(expressed by the function $f$). An excellent analysis of this question
is to be found in Cournot.¹

Even so, we are able to employ more illustrative mathematical tech-
niques. Instead of taking the various values of $D$ as the abscissa of the

¹ A. Cournot, Recherches sur les principes mathématiques de la théorie des richesses, Paris, 1838,
Ch. 4; English translation by N. T. Bacon, Researches into the mathematical principles of the
theory of wealth, New York, 1897.
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curve, as is done by Cournot, we can take the product pd as the ordinates; in this case the equation of the curve will be:

\[ Y = D \cdot f(D) = F(D). \]  
(91)

If we set \( D = 0 \), we have \( Y = 0 \); by successively increasing \( D \), we finally arrive at a value of \( D \) at which \( Y \) once again vanishes; this will occur at the value of \( D \) for which \( f(D) = 0 \); since the demand for each product is limited and since no one will pay money for a product which he does not need at all (and there cannot be several prices for one product in the market), the price of the product will vanish at some finite value of \( D \),\(^1\) and therefore the form of the curve defined by the equation \( Y = F(D) \) will be that shown in Fig. 1.2. If sales \( D \) are taken as the abscissae and costs \( Du \) as the ordinates, we shall obtain a straight line \( OA \) running at an angle to the axis of the abscissae. The vertical distance between the lines \( ON \) and \( OA \) will denote the net profit corresponding to each scale of production. This distance will be greatest at the point on the curve \( ON \) where the tangent to it become parallel to the straight line \( OA \). If this point is \( C \), the quantity yielding the greatest net income will equal the abscissa \( OB \); the price corresponding to this quantity will equal the tangent of the angle \( \phi \) (which will equal \( BC/OB \)).

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Therefore, if the monopolist has a correct understanding of his advantage, this is the price which he will set. Were the production of the given product to be influenced by free competition, the scale of production would be increased until the market price of the product exactly equalled the necessary production costs (this, at least, is what Ricardo's theory asserts; we shall see subsequently whether or not it is correct), which would occur when the amount produced equalled \( OE \); in this case the

\(^1\) For the present this will be taken to be an empirical fact. For the explanation of this fact the reader is referred to the third essay, in which there is a detailed analysis of \( f(D) \).
price of a unit would equal the tangent of the angle $\phi_1$ and would always be less than the tangent of the angle $\phi$, as is shown by Figure 1.2.

4. RICARDO'S THEORY OF RENT

We shall not make a detailed analysis of the theory of rent in general at this point, but shall deal with Ricardo's theory of rent only to the extent to which it is an element of his general theory of value (for this reason, for example, we shall not deal at all with the question of the importance, to Ricardo's theory, of land rent, of the historical sequence of the occupation of lands of different quality, or with whether rent necessarily swallows an ever-increasing portion of the product produced as population increases: it is not even necessary to understand Ricardo's theory to imagine that it is connected with some sequence or other of the occupation of lands.\(^1\) Although Ricardo does not himself clarify the laws governing rent in a special case of the origin of this kind of income (namely in the case of land rent and rent from mines, see Ricardo, *Principles*, Chs. 2 and 3), this does not prevent his theory from being of general significance for all possible cases of the origin of rent-like incomes.\(^2\) Yu. Zhukovsky states\(^3\) that

'According to Ricardo's theory, the fundamental condition for the origin of rent is in no sense the correct transfer of cultivation from some portions of land to others, but the general law of the equality of profits. According to this law the same capital and labour cannot yield different profits in the same production, since competition will hasten to equalise these profits if they can occur. If this law holds not even for different productions, but for the same production, an unavoidable consequence of this should be the subtraction of all differences in profits in favour of the monopolist of the cheap method and in the particular case of the land owner, and it is this which constitutes rent.'

Ricardo himself was well aware of this when he stated that 'If air, water, the elasticity of steam, and the pressure of the atmosphere, were of


\(^2\) The statement by A. Miklashevsky, that 'the law of rent on capital expended in manufacturing industry differs fundamentally from the law of rent in general' merely proves that this author has not sufficiently understood the concept of a scientific 'law' (he confuses the law itself with the conditions under which its operation is manifested). See A. Miklashevsky, *Den'gi - Opyt izuchenii tannynikh polezhenii ekonomicheskoy teorii classicheskoy shkoly* [Money - An examination of the basic propositions of the classical school of economic theory], pp. 246–7.

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various qualities; if they could be appropriated, and each quality existed only in moderate abundance, they, as well as the land, would afford a rent as the successive qualities were brought into use. 1 which would be governed by the same laws as land rent. 1

We shall not repeat Ricardo's theory of rent, which is well known, but shall proceed directly to a close analysis of it (an excellent analysis is to be found in Zhukovsky's book to which reference has been made), and in our analysis we shall make use of the constructions of Auspitz and Lieben (Auszitz and Lieben, Untersuchungen über die Theorie des Preises, 1889). Although Auspitz and Lieben were not specially concerned with the theory of rent, their constructions nevertheless provide excellent material for an analysis of this phenomenon because in a general analysis of the phenomenon of value they arbitrarily and completely incorrectly give to the curve of production costs the form which is appropriate for branches of production yielding rent; this arbitrary assumption, which deprives their own conclusions on the question of value of any generality, enables us to use their constructions for an analysis of the phenomenon of rent.

1 Let us construct a system of coordinates [Figure 1.3] where the abscissae denote quantities of the article A and where the ordinates denote quantities of money. If we draw the various possible quantities of the yearly product horizontally, and the corresponding cost of production vertically, we obtain a series of points which, connected, present a curve OA, which we shall call the curve of total cost of production' (Auszitz and Lieben, p. 6).

1 Ricardo, Principles, Ch. 2, p. 75 of the Sraffa edition. Buchanan remarks quite correctly, commenting on Adam Smith, that the excess income obtained by the possessor of any secret in manufacturing industry is, in its essence, rent governed by the same laws as land rent and any other rent. See Buchanan, Observations . . ., pp. 39-41.
4. Ricardo's theory of rent

to the axis of ordinates (this latter assertion is equivalent to the assertion that there is a limit to every production, beyond which it cannot be expanded whatever the expenditure), (4) the curve is convex throughout, i.e. each successive tangent is less inclined to the axis of abscissae than all the preceding tangents. Consequently, an increase in the quantity of the product causes a greater increase of cost, the higher is the annual production of the product to which this increase is added.

In Figure 1.4 let OA be the 'cost curve' of a product A; let OA' be 'the quantity of the article A under consideration actually produced and turned over during one year. We can then imagine it being partitioned into a number of small parts of equal length. Each of these small parts will yield the same revenue since the whole quantity will be sold at one and the same price. The costs of production however are different, for each further part necessitates a greater cost of production than the one which precedes it. If the producers are to be induced to produce the quantity OQ, the revenue from the last part, which we denote by TS, has to be as big as the additional costs necessitated by the production of this last part, for otherwise it could not be produced.'

'On the other hand, if the revenue from this last part were greater than the cost which it causes, the producers would be led to increase their production still further by the free competition prevailing between them. Since we have now assumed a stable situation and OQ represents the appropriate annual quantity, the costs TR necessary for the production of the last part ST which has actually been produced have to be equal to the revenue generated by it. The length TR therefore represents the income generated by the last unit of production ST; but since there exists only one price for the whole quantity OQ produced during the year, and since there is also only one price for the last unit ST of that quantity, the total quantity produced during the year must bear the same proportion to the total revenue generated by it as ST to TR. If we now draw a parallel to the curve SR through the origin, and if the chosen unit into which the quantity annually produced is partitioned is small enough, the curve SR will indicate the direction of the tangent to the cost curve at point R. This parallel cuts the ordinate continued beyond the point R at R', and the length of QR' indicates the total revenue corresponding to the above proportions which the producer must attain to induce him to produce the quantity OQ. If we repeat the same construction for each other quantity to be produced during the year, from the smallest to the biggest, we obtain a series of points which represents a new curve OA' . . .'

'... the ordinates of this curve indicate the quantity of money for which the annually produced quantities shown on the abscissae are

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offered. We call this curve, therefore, the curve of total supply’ (Auspitz and Lieben, pp. 12–14).

Consequently, the ordinates of this curve show the sum which consumers should pay for the amount of the product on the market (the annual amount or in general the amount in a unit of time) to correspond to the given abscissa. The sum which consumers agree to pay will, in its turn, depend on the quantity of the product bought by them; if we take the quantities purchased as the abscissae and the sum which the consumers agree to pay for these quantities as the ordinates, we obtain the total demand curve already used in the preceding analysis. (Auspitz and Lieben construct this curve in a different manner, deriving it from the general utility curve, but the form which they take for this curve is quite incorrect and has no data in support of it (cf. Auspitz and Lieben, pp. 14–16).)

According to Ricardo’s theory, when free competition prevails the scale of production should be established at a level OE (see Figure 1.5) at which, as the figure shows, the market price paid for a unit of the product (equal the tangent of the angle $\theta$) will equal the production costs of the last unit of the product to be produced (i.e. the unit produced with the greatest costs; it is evident from the construction that these costs will also equal the tangent of the angle $\theta$).

The direction of the price line OP (Preislinie) is defined by the intersection of the curves OA and ON, and therefore the price of the product (which equals the tangent of the angle formed by the line OP and the axis of abscissae) will also be dependent on the form of both curves and consequently production conditions alone cannot suffice for its definition, as is asserted by Ricardo.
4. Ricardo’s theory of rent

Consequently, Ricardo gives us an impeccable analysis of the law governing the value of products when individual portions of these products are produced with different production costs (in particular with different expenditure of labour when production costs may be reduced to labour alone), but this analysis definitely does not prove, as Ricardo assumes, that the value of such products is not ultimately dependent on the conditions of supply and demand and will tend to settle at a level exclusively dependent on production conditions. On the contrary, we have seen that any alteration in the conditions of supply and demand (leading ultimately to an alteration in the sphere of consumption, as we shall see when we analyse the demand curve in the third essay) leads inescapably to an alteration in the price of the product (= value in some product, whose own value is assumed to be constant). This is the case even if all production conditions (which are not dependent on the economic calculation, but only on the state of technology and the availability of the various natural factors of production) have remained unchanged.

All that we have said above concerning the law of value established by Ricardo for products when individual portions of them are produced with different production costs is also fully applicable to the theory which states that the value of products is determined by the amount of socially necessary labour expended on their production. In fact, by relating value to the amount of socially necessary labour, this theory makes value dependent (as we have demonstrated in the analysis of Ricardo’s theory of rent) on the condition of supply and demand\(^1\) (ultimately on the conditions of consumption).

In order to free the definition of the amount of ‘socially necessary’ labour from the conditions of supply and demand, some advocates of this ‘developed’ form of the labour theory of value attempt to equate the amount of socially necessary labour with the average amount used in the production of a given commodity. To assert this is, however, to deny everything which Ricardo did to clarify the laws governing the value of those products, individual portions of which are produced with different production costs. Ricardo’s analysis leaves no doubt that the value of commodity is determined by the quantity of labour expended on its production not under average but under the most disadvantageous conditions of its production.

The amount of socially necessary labour could be equated to the average amount only in the exceptional special case when the sum of positive rent in a given sector was exactly equal to the sum of negative rent (negative rent may rise only in exceptional cases when, owing to various obstacles to the free movement of entrepreneurs from poorly rewarded

\(^1\) Since it is only the conditions of supply and demand which determine how much labour is ‘socially necessary’ in each specific case.
brances of industry into better rewarded branches, the scale of production is expanded beyond the limit at which the least favourably placed entrepreneurs obtain all the production costs in the price of the product: Ricardo does not analyse this case, since he invariably assumes complete (juridical and actual) mobility of entrepreneurs from one branch of industry to another. Our construction and the corresponding analysis present all the data needed for clarification of the phenomenon of negative rent.

Let us assume that all the production conditions and, consequently, the form of the \( OA \) curves and of the derived \( OA' \) curve remain unchanged, and let us give various positions, denoted by a dashed line, to the \( ON \) curve, which is not dependent on production conditions. The point of intersection of the two curves will then assume different positions \( C_1, C_2, C_3, \ldots \), and by linking these points with \( O \) we obtain a series of lines \( OC_1, OC_2, OC_3, \ldots \). The tangents of the angles between them and the axis of abscissae will denote the price established in the market under the influence of free competition for given conditions of demand. Clearly by arbitrarily varying the form of the curve \( ON \) we may also arbitrarily vary the price of the product, and therefore the assertion that this price is determined by the conditions of production is based merely on a misunderstanding; the price of the product will be affected by all conditions which affect the form of the overall demand curve. Therefore, any change in the sphere of supply which affects the value which consumers place upon a given good, and which consequently affects the sum they will agree to expend in order to acquire it, will affect the price ultimately established in the market. This price will be determined only if the equation of the curve \( ON \) is given: \( Y = F(D) = Df(D) \), for which the function \( f \) expressing the relationship between price and the quantity sold needs to be known. Consequently, in this case, as in the monopoly determination of price, the price cannot be determined independently of the form of the function expressing price as a function of the quantity sold, \( p = f(D) \).

We have seen that, on the assumption that both curves are continuous, production will settle down in such a way that the price for a unit of the product in the market will equal the production costs of the last unit produced. The situation will be different if (as is the case in reality) the line \( OA \) depicting the increase of production costs as the total amount produced increases is discontinuous (at least at some points), i.e. if the tangents to two points infinitely close together on this line form an angle of finite magnitude; in this case, by virtue of the conditions of its construction, the curve \( OA' \) will necessarily be discontinuous, and the points of discontinuity of the curve \( OA' \) will correspond to the points of discontinuity of the original curve \( OA \) (Figure 1.6).
4. Ricardo's theory of rent

For example, let the amount produced be \(OE\) when the price equals the tangent of the angle \(EOH\), and let us now assume that in order to cause the appearance on the market of a larger quantity differing from the quantity \(OE\) by an infinitely small amount, the price should be immediately increased by some finite amount. In that case the ordinate corresponding to the abscissa larger than \(OE\) by an infinitely small amount will be larger than the ordinate \(EH\) by some finite amount, since the curve \(OA'\) will assume the form \(OHJAJ'.\) Obviously, under such conditions, production cannot expand beyond \(OE\), since in this case the costs of the last units to be produced (above the amount \(OE\)) would be greater than the price paid for a unit of the product in the market. It is evident from Figure 1.5 that when production is \(OE\) the price paid in

![Figure 1.5](image)

the market will be greater than the costs of the last unit produced; it will be greater by as many times as \(EI\) is greater than \(EH\) (since the market price of a unit of the product will equal the tangent of the angle \(EOI\), and the production costs of the last unit produced will equal the tangent of the angle \(EOH\)). Consequently, the law which states that the price of a product, individual units of which are produced with different production costs, will equal the production cost of the part of the product produced under the most disadvantageous conditions, holds only when \(\Phi(Q)\), expressing the production costs of the last unit of the product to be produced as a function of the total quantity produced, is a continuous function (i.e. when every infinitely small increment to the amount is matched by an infinitely small increase in the production costs of the last unit to be produced).

In reality we frequently encounter instances of the discontinuity of \(\Phi(Q)\) both in agriculture and in the manufacturing industry. This occurs
especially in manufacturing industry: here it is only in rare instances that slow gradation of an increasingly less efficient mode of production may be observed. It is only when rent arises solely from a difference in the distance between production points and the market (the case investigated in detail by Thünen) that the curve $OA'$ can in fact be taken to be continuous and that, consequently, the Ricardian law of the price of products which yield rent is fully applicable.

Let us return to our original construction (Figure 1.5). The segment $CL$ will express the total sum of rent yielded by $OE$ units of the product; the distribution of this sum between the individual producers has no bearing whatsoever on the amount of the rent.

The only important factor is whether the production of the given product takes place under the influence of free competition or in the monopoly possession of a single individual. In the latter case the scale of production will be determined (in accordance with the principles in the preceding paragraph) by the abscissa $OB$, for which we have tangents to the curves $ON$ and $OA$ at the points $G$ and $F$ which are parallel and consequently the distance between these curves (expressing gross revenue and actual total costs incurred in the production of $OB$ units) is greatest. Clearly $OB$ will always be smaller than $OE$; also $FG$ is invariably greater than $CL$, i.e. the sum paid by the consumer over and above the actual production cost will be greater in the case of monopoly than under the influence of free competition.

As is shown by Figure 1.5, the magnitude of $CL$ will be smaller, the less is the curvature of the curve $OA$. If the curve $OA$ is ultimately converted into a straight line, the curve $OA'$ will also be converted into a straight line, by virtue of the conditions of its construction from the original curve $OA$, and it will merge with $OA$. This will happen when total production costs become proportional to the quantity produced, i.e. when all units of the product are produced with the same costs. In this case rent will vanish (since the vertical distance between the curves $OA$ and $OA'$ will be zero). The next chapter will be devoted to an investigation of this case. We have just shown that production costs cannot be recognised as the sole regulator of value for a product when price includes rent, and that every change in the sphere of demand (independently of production conditions) alters the price of such a product even if no changes have occurred in production conditions. This objection to the theory of production costs is not applicable when the production costs curve becomes a straight line. In fact, let the straight line $OA$ be the production costs line and let the curve $ON$ remain the demand curve. In this case, as is asserted by Ricardo, given that free competition prevails, production will expand until the price paid in the market is no more than sufficient to cover the essential production costs. Consequently, the amount produced will be $OE$ (Figure 1.7).
5. Theory of value of infinitely reproducible goods

price of a unit of the product will be \( EC/OE \), i.e. will equal the tangent of the angle \( \phi \).

Now let us assign different positions denoted by a dashed line to the curve \( ON \). Clearly the price will remain permanently equal to the tangent of the angle \( \phi \); changes in demand conditions do not affect the price until there are changes in production conditions, i.e. until the position of the straight line \( OA \) is altered. Consequently, price is apparently actually determined in this case exclusively by production costs.\(^1\)

And this would be so were the assertion by Ricardo italicised by us above in fact to be correct.

5. RICARDO’S THEORY OF THE VALUE OF INFINITELY REPRODUCIBLE GOODS (BY THE APPLICATION OF LABOUR AND CAPITAL UNDER ZERO RENT CONDITIONS)

We have seen that the value of scarce goods, i.e. of goods whose quantity is limited by natural conditions (scarce in the true sense), or artificially as

\(^1\) Here and elsewhere in our account of Ricardo’s theory of value we invariably refer to the costs of production rather than of reproduction, since we invariably assume in the interests of simplicity of analysis that no changes occur between the time of manufacture of the product and the time of its sale under the technical conditions of a given branch of industry. This does not, however, mean that Ricardo did not evaluate and take into consideration in his theory the effect of technical progress on the value of stocks previously produced but still not sold. On the contrary, only one completely ignorant of Ricardo’s work would think that Carey had made a significant correction on this point to Ricardo’s theory of value (see the
The theory of value of David Ricardo

a result of their production under the monopoly possession of a single economic subject, cannot be determined independently of \( F(D) \). Other things being equal, any change in \( F(D) \), i.e. in the demand conditions, will also entail a change in the value of the good.

We have further shown that the exclusion of \( F(D) \) from the equations of the goods (as essential production costs related to the quantity produced by the function \( \Phi(Q) \)) is only an apparent, verbal exclusion in Ricardo's theory; in fact the value of these goods cannot be determined independently of the form of \( F(D) \) and, other things being equal, any change in \( F(D) \) necessarily entails a corresponding change in the value of these goods as in the previous case of scarce goods. We now have to consider the last case, namely the formula for the value of goods which are infinitely reproducible by labour under conditions excluding the possibility of the occurrence of rent. The market price of goods in this category, like the market price of all goods in general, is determined by the 'conditions of supply and demand' (see Ricardo on market prices, Principles, Ch. 4), and therefore, if we denote the price of a good by \( X_A \) we shall have, *saepe paribus*, \( X_A = F(D_A) \), where \( D_A \) is the 'actual supply' (in Smith's sense) of product \( A \). It is evident by definition of the goods under consideration that \( D_A \) may be varied at will between 0 and \( +\infty \). What value is in fact established for \( D_A \)? Ricardo states that \( D_A \) is *ultimately* established as if \( F(D_A) = u \), where \( u \) stands for the essential production costs of the product \( A \). Direct substitution yields \( X_A = F(D_A) = u \). The new expression \( X_A = u \) no longer includes \( F(D_A) \), and consequently \( X_A \) is no longer directly dependent on the conditions of 'supply and demand', but for \( X_A \) to be completely independent of market conditions it is essential that the magnitude of \( u \) should also be independent of these conditions.

The whole of this analysis of Ricardo's theory of value has led to the conclusion that Ricardo in fact succeeded in excluding the element of price from production costs and, consequently, in formulating the quantity \( u \) independently of demand conditions (this is the main difference between Ricardo's theory of value and Smith's theory, which still had to

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comment by Zalesky, 1893, p. 244. 'It is greatly to Carey's credit and recognisably by all that he drew attention to the importance of the costs of reproduction.') Suffice it to refer to Chapter 20 of Ricardo's work, where he states directly that any technical improvement raising the productivity of labour in a given industry will necessarily also have an effect on the portion of goods still unconsumed, which were manufactured previous to the improvement; the value of those goods will be reduced, in as much as they must fall to the level, quantity for quantity, of the goods produced under all the advantages of the improvement and the society will, notwithstanding the increased quantity of commodities, notwithstanding its augmented riches, and its augmented means of enjoyment, have a less amount of value. By constantly increasing the facility of production, we constantly diminish the value of some of the commodities before produced... ' (Ricardo, Principles, Ch. 20, p. 274 of the Sraffa edition [emphasis added]; see also Ch. 6 'On profit').
5. Theory of value of infinitely reproducible goods

relate one of the elements of production costs, namely profit, to the conditions of ‘supply and demand’, namely the supply and demand of capitals). Therefore, provided that the hypothesis \( F(D_A) = u \) is correct, we shall have an expression of value, for the kind of goods we are considering, which is independent of market conditions and determined exclusively by ‘objective’ production conditions. On what basis does Ricardo consider it possible to assume \( F(D_A) = u \), i.e. assume that the ‘actual supply’ of a good \( A \) produced under free competition without natural limits will expand to the point where the market price barely covers essential costs? Why is it not possible to establish for \( D_A \) some quantity \( D_{A0} \) for which we shall have \( F(D_{A0}) > u \), i.e. at which the producers will still make some surplus over and above the essential production costs? Nothing new concerning this question is to be found in Ricardo; he accepts as true the theory of Smith which we have already set out above. Smith and Ricardo reason as follows: if the price of the product in a given industry \( A \) exceeds essential production costs, the producers of this industry will receive more profit on their capital than do the producers of other industries. This surplus is the prize which compels the producers of other industries to transfer to the production of \( A \); as a consequence competition will increase in industry \( A \) and the price of product \( A \) will fall. This fall in price will continue until the movement of producers ceases. The influx of new producers will continue until the motive for making the change, i.e. the advantage, disappears. But the advantage will not be destroyed until \( F(D_A) = u \), and consequently the fall in the price of the product \( A \) will not cease until \( F(D_A) = u \), i.e. until the price of the product equals the essential production costs.

There is an undoubted logical omission in this reasoning; the final conclusion holds if the tacit assumption that ‘competition lowers prices’ also holds.

In fact, why should the movement of new producers into industry \( A \) lower the price of product \( A \)? After all, whatever part of the net profit earned in the whole industry \( A \) was received by each producer, it appears that the price most advantageous for each should be that at which total net profit is greatest.

If we assume that the total sum of the profit in a given industry is 100,000 roubles at the price which yields the maximum net profit (i.e. at a monopoly price), and if we assume that there are 1,000 isolated producers, we obtain a profit of 100 roubles for each. At any price below the monopoly price, the total sum of the net profit will be less than 100,000 roubles and, consequently, each of the 1,000 competing producers will have a share of net profit smaller than 100 roubles. But why should 1,000 isolated producers settle on a price less than the monopoly price, despite the fact that this lower price is far less advantageous for each of them
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than the monopoly price? And, in general, what is the relationship between the number of competing individuals and price?

We have sought in vain for a clear answer to these questions in the writings of Smith and Ricardo and their followers. The arbitrary nature of the assumption that the movement of new entrepreneurs into a given industry $A$ produces a reduction in the price of the product $A$ is concealed in the writings of the classical school. This is done first by a word play; reference is made to the movement of 'capitals' instead of to the movement of 'capitalist entrepreneurs'; this substitution is equivalent to the arbitrary assumption that any increase in the numbers of producers occupied in a given industry is invariably accompanied by an increase in the total amount of capital invested in the given industry. Secondly, there is an implicit arbitrary assumption that any extension of production also increases the supply of the given product on the same scale. Thanks to these two arbitrary assumptions the truism that 'competition lowers prices' receives an apparent basis (see our account of the way in which Smith reasons), but this does not make the assumption any the less arbitrary. The contradictory nature of the two principles on which all the conclusions of the classical school are based, namely the principle that every individual tends to pursue the greatest advantage and the assumption that competition tends to minimise prices, was evidently overlooked both by the members of the classical school themselves and by their numerous commentators (with the exception of Thornton, who enjoys no reputation as a theoretician of political economy; see W. T. Thornton, On labour; its wrongful claims and rightful dues, its actual present and possible future). The tendency of competition to lower prices has been accepted as some sort of 'spontaneous' phenomenon completely independent of the economic calculation of competing individuals; at any rate we cannot find in the writings of economists of the classical school even the hint of an attempt to separate the effect of competition from the basic principle of the pursuit of the greatest advantage (or even an attempt to reconcile it with this situation, by accepting the effect of competition on prices as empirically given). The very possibility that such an important omission in the reasoning of the deductive school should be overlooked was undoubtedly a result of the imperfection of the 'dialectical' method which its members used. The honour of having constructed a completely scientific theory of competition belongs entirely to members of the

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1 The thought which runs as a guiding line through all Thornton's reasoning is that 'Dealers do not undersell each other merely for fun. Each is quite content that all the rest should sell dearly, provided be himself can sell as dearly'... (Book III, Ch. 1, p. 61) The sole object of every merchant is to earn the greatest sum for all his goods, and therefore he will lower the price only if he can calculate that with this price reduction his sales will increase so that the total sum of his earnings will rise.
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Mathematical school of political economy, and mainly to the most talented of its members, the great 'forgotten' economist Augustine Cournot. Unfortunately, however, Cournot's immortal work had no effect on contemporaries and has been forgotten by new generations of economists.¹

The next essay will be devoted to a critical account of Cournot's theory of competition. We shall attempt to prove by rigorous analysis that although it is generally accepted and has become a truism that unrestricted free competition tends to lower prices to the essential production costs, this is no more than an arbitrary assumption. This is at variance both with the facts of economic reality and with the basic hypothesis of economic theory that each tends to pursue his greatest advantage. We shall attempt, on the contrary, to demonstrate tangibly that, as a general rule, unrestricted free competition invariably tends to raise actual production costs above the essential level, i.e. above the lowest level possible for a given state of production technique.

Thus the value of products of the third category (i.e. goods infinitely reproducible by the application of labour and capital under conditions of zero rent), like the value of products of the first two categories considered above cannot, as a general rule, be determined independently of conditions of supply and demand (i.e. ultimately independently of the conditions of consumption).

The Third Essay, which will conclude our examination of the general elements of value, will be devoted to an analysis of the dependence of price on conditions of supply and demand.

¹ See L. Slonimsky's article 'Zabytye ekonomisty Kurno i Thünen' [The forgotten economists Cournot and Thünen], Vostochnaya Europa, October 1878.