Pasinetti, Keynes and the Multiplier

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ABSTRACT This paper explores the relationship between the Keynesian multiplier and Pasinetti's model of pure production. Key assumptions of Pasinetti's model are its multisectoral structure, the definition of all income as a reward to labouring activities and, as a consequence, the operation of a pure labour theory of value. A translation between these models is effected by introducing investment as an exogenous determinant. By drawing from Keynes to apply his concept of the wage unit, it is possible to aggregate from Pasinetti's multisectoral model to a genuinely macroeconomic multiplier. This provides a way of using the scalar Keynesian multiplier without making the restrictive one-commodity assumption. In addition, this formal demonstration enhances our understanding of the relationship between the wage unit and the labour theory of value. Finally, critics have argued that Pasinetti downgrades the importance of institutional analysis; in contrast, the derivation of a scalar Keynesian multiplier contributes to an understanding of how relevant Pasinetti's approach is to the analysis of a monetary production economy.

1. Introduction

A key difficulty faced by Keynes in writing The General Theory was 'the choice of units of quantity appropriate to the problems of the economic system as a whole' (Keynes, 1936, p. 37). He argues that since heterogeneous physical outputs cannot be aggregated, labour should be used as a unit of account. Dillard (1984) and Wray (1998) interpret Keynes as adopting a labour theory of value that enables a macroeconomic level of analysis in which total employment can provide an appropriate index of aggregate output. Instead of using the usual price index in an attempt to measure physical outputs, Keynes divides aggregate money output by the wage unit, the wage rate paid to a unit of unskilled labour.

A possible reason for the suppression of this somewhat classical interpretation of Keynes is the predominance in Keynesian economics of a one-commodity macroeconomic model, which 'is generally legitimized in terms of its simplicity' (Kurz, 1985, p. 121). By making this simplifying assumption, macro theorists can sidestep the problem of aggregation across heterogeneous commodities and ignore the attention paid to this issue by Keynes.
In a bold and incisive attempt to re-introduce multisectoral foundations into Keynesian economics, Pasinetti (1981) proposed a simple alternative to the one-commodity model by deriving a genuinely macroeconomic relationship that holds regardless of the level of sectoral disaggregation. Central to this relationship is the definition of a macroeconomic condition for full employment under which there must be full expenditure of national income. Pasinetti has generated some controversy (Davidson, 2001; Hodgson, 1994) by claiming that this condition constitutes the pre-institutional essence of Keynes’s principle of effective demand – a theoretical starting point that cuts through the institutional complexities of a monetary production economy.

Pasinetti (1997) notes the connection between his model and the Keynesian cross diagram with its associated multiplier relationship. When this simple Keynesian diagram is stripped of behavioural properties, Pasinetti argues, the core pre-institutional principle of effective demand can be identified. The problem, however, is that Pasinetti does not make a direct translation between the Keynesian cross and his own multisector model. If the same key relationships hold for both models then such a translation should be possible.

The present article undertakes to establish this translation, thereby providing a more accessible illustration of Pasinetti’s contribution, and showing how Keynes’s multiplier can be derived from multisectoral foundations. Using this translation we demonstrate that the multiplier can embody complex intersectoral relationships which are genuinely macroeconomic but at the same time retain the simplicity of the Keynesian approach. Moreover, these foundations suggest an analytical starting point for examining the role of institutions in a monetary production economy, a contribution that casts light on recent debates between Pasinetti and his critics. Finally, our analysis also provides a mathematical formalisation of the relationship between Keynes’s wage units and the labour theory of value.

The article consists of three parts and a conclusion. In Section 2 we introduce Pasinetti’s multisector pure production model and identify the macroeconomic condition for full employment. In Section 3, a multisectoral Keynesian multiplier is derived by modifying the pure production model. Section 4 considers the role of the wage unit in this multisector model and from this discussion we develop a more accessible derivation of Pasinetti’s macroeconomic condition.

2. Pasinetti’s Pure Production Economy

Pasinetti (1981) assumes a pure production economy in which labour is the only factor of production used in m sectors of production. For each industry i, define \( n_i \) as the vertically integrated labour coefficient \( (N_i/Q_i) \) where \( N_i \) is the amount of direct labour required for production of \( Q_i \) units of good \( i \) together with the indirect labour required to produce intermediate capital goods used in the production of good \( i \). All of the output is consumed by labour, although the workers employed in a particular sector consume only a part of that sector’s output. Thus, for each industry \( i \) we can define \( c_i \) as the per capita consumption coefficient \( (C_i/L) \) where the total amount of good \( i \) consumed, \( C_i \), is equal to \( Q_i \), and the scalar \( L \) is the amount of labour available for work in the economy as a whole \( (L = N_1 + \cdots + N_m) \). Now we can set out Pasinetti’s system using the slightly
more accessible format he adopted in a later contribution (Pasinetti, 1986). The quantity and price systems take the form:

\[
\begin{bmatrix}
1 & 0 & \cdots & 0 & -c_1 \\
0 & 1 & 0 & -c_2 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & \cdots & 1 & -c_m \\
-n_1 & -n_2 & \cdots & -n_m & 1
\end{bmatrix}
\begin{bmatrix}
Q_1 \\
Q_2 \\
\vdots \\
Q_m \\
L
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
0 \\
\vdots \\
0 \\
0
\end{bmatrix}
\tag{1}
\]

\[
\begin{bmatrix}
p_1 & p_2 & \cdots & p_m & w \\
0 & 1 & 0 & -c_2 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & \cdots & 1 & -c_m \\
-n_1 & -n_2 & \cdots & -n_m & 1
\end{bmatrix}
\begin{bmatrix}
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
0 \\
\vdots \\
0
\end{bmatrix}
\tag{2}
\]

It follows from equations (1) and (2) that

\[Q_i = c_iL\]

and

\[p_i = n_iw\]

with equation (3) expressing the relationship of physical quantities to labour, and equation (4) the relationship of prices to the price of labour \(w\). With \(c_i\) a parameter and \(L\) exogeneous, Pasinetti interprets equation (3) as representing a demand-determined theory of production, in which the absolute amount of each output produced \(Q_i\) depends upon consumer demand. In addition, because there is no capitalist class in the pure production economy and hence no category for profits or the rate of profit, equation (4) demonstrates the working of a pure labour theory of value in which the price of each commodity produced, relative to the price of labour, is proportional to the labour required for its production.

For a non-trivial solution to exist for equations (1) and (2), the determinant of the coefficient matrix must be zero which implies that:

\[\sum_{i=1}^{m} c_i n_i = 1\]

Pasinetti interprets equation (5) as the condition for full employment if \(L\) is set equal to \(L_f\), the total amount of labour available for work (see Pasinetti, 1981, p. 32).1 This can be shown by examining the binomial term \(c_i n_i\). Writing this

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1This condition ensures that supply automatically creates its own demand (Say’s Law) at all levels of employment. In the pure production model there is no barrier to the achievement of a full employment solution.
Explicitly:

\[ c_i n_i = \frac{C_i N_i}{L Q_i} = \frac{N_i}{L} \]  

(6)

where \( C_i \) is the total consumption of good \( i \), which is equal to \( Q_i \). Since a non-trivial solution for a homogeneous system of equations requires that the determinant of the augmented matrix be zero, the binomial terms must add up to 1, which also means that the proportions of labour employed in each sector must also add up to 1. With Pasinetti equating \( L \) to \( L_f \), this implies that all labour is fully employed. However, if \( L < L_f \) the same results will emerge as long as \( L \) is assumed to be exogenously given and therefore unaffected by the degree of unemployment (either directly or indirectly).

Condition (5) also implies, for Pasinetti, that full employment is contingent upon the expenditure of consumers. Using equation (4), the binomial term can be re-expressed as

\[ c_i n_i = \frac{C_i p_i}{wL} \]  

(7)

Each binomial term represents the proportion of national income \( wL \) that is spent as \( C_i p_i \) in sector \( i \). For full employment to be achieved these proportions must add up to 1, and hence there must be full expenditure of national income. Pasinetti interprets equation (5) to be the ‘Keynesian effective demand condition for full employment’ in which there is no saving in the economy as a whole (Pasinetti, 1986, p. 422).

3. Derivation of the Keynesian Multiplier

Pasinetti generated controversy by claiming that his macroeconomic condition for full employment is pre-institutional, a piece of pure economic theory that provides ‘sturdy and rocklike theoretical foundations’ to the study of more complex economic phenomena (Pasinetti, 1994, p. 40). This pre-institutional condition, he argues, provides a fundamental definition of the principle of effective demand, free from behavioural relationships, which Keynes was ‘able to perceive, but not explicitly state’ (Pasinetti, 2001, p. 384). Hodgson (1994), however, contends that the economic categories which define the pure production model – prices, wage rates and input–output coefficients – are inextricably bound up with a particular institutional context. It is difficult to envisage prices and wage rates, for example, without doing so in the context of a money-using market economy, which represents a particular type of institutional arrangement. From a similar perspective, Davidson argues that Keynes defined the principle of effective demand in relation to economic behaviour patterns specific to a monetary production economy. Pasinetti’s attempt to reduce the principle of effective demand to a ‘deeper level’ is rejected by Davidson (2001, p. 403).

A particular problem with this debate between Pasinetti and his critics is the inaccessibility of his macroeconomic condition. Compared with the analytical simplicity of Keynesian macroeconomics, the Pasinetti condition, being based
on the mathematics of a homogeneous system of equations, is more complex mathematically and not very familiar to many heterodox economists. To make his argument more accessible, Pasinetti has in recent years tried to draw the connection between his model and the conventional Keynesian cross diagram and multiplier relationship (Pasinetti, 1997). Despite his stated objective to clarify Keynes’s principle of effective demand, the relationship between his interpretation of the Keynesian cross diagram and the pure production model remains hazy. The two models appear to belong to qualitatively different modelling frameworks. In the analysis that follows, a more direct translation is suggested between Pasinetti’s pure production model and the Keynesian multiplier.

A key similarity can be identified between Pasinetti’s model and the approach taken by Keynes in parts of *The General Theory*. Like Pasinetti, Keynes assumes that labour is the only primary factor of production: ‘I sympathise, therefore, with the pre-classical doctrine that everything is produced by labour… It is preferable to regard labour, including, of course, the personal services of the entrepreneur and his assistants, as the sole factor of production’ (Keynes, 1936, pp. 213–214, Keynes’s emphasis; also see Reati, 2000). Using this similarity it is possible to maintain, in a Keynesian system, Pasinetti’s association of all income with wages paid to labour and the corresponding absence of categories for either profits, the rate of profit, or the capitalist class. Both the Keynes and Pasinetti systems model non-primary factors, that is, capital inputs which take the form of produced means of production; but they are non-recipients of (value added) income.²

Unlike Pasinetti, however, Keynes broadly accepted the prevailing theoretical view in the 1930s that capital goods (means of production) can be reduced in a finite number of steps to dated quantities of labour. Hence, conceptually, the most straightforward, albeit limited, way to construct a multisectoral model of Keynes’s system and its associated employment multiplier is with a two-period Austrian production model.³ But we propose to take a more general approach in translating the Pasinetti model into the Keynesian system by explicitly including the circular flow of intermediate commodities.

In his exposition of the pure production model, Pasinetti (1981, p. 30) is clear that ‘it is always possible, when needed, to re-introduce intermediate stages and intermediate commodities by linear algebraic transformations’. This can be achieved by writing equation (1) in block matrix form such that:

$$
\begin{bmatrix}
I & -c \\
-n & 1
\end{bmatrix}
\begin{bmatrix}
Q \\
L
\end{bmatrix}
=
\begin{bmatrix}
O \\
0
\end{bmatrix}
$$

(8)

where $I$ is an $m \times m$ identity matrix, $c$ is an $m \times 1$ vector of consumption coefficients, $Q$ is an $m \times 1$ vector of final outputs, $O$ is an $m \times 1$ vector of zeros, $n$ is a

²This contrasts with the simpler but more restrictive model developed by Pasinetti (1993), in which labour is the only input.

³An ‘Austrian’ exposition of the Keynesian system and employment multiplier is given in the Appendix.
1 \times m \text{ vector of vertically integrated labour coefficients, and } L \text{ is the scalar representing the amount of labour available for work. Since vertically integrated labour coefficients can be expressed in terms of an explicit input–output structure, in equation (8) we can re-express the labour coefficient vector as}

\[ n = l(I - A)^{-1} \]  

(9)

where \( A \) is an \( m \times m \) matrix of interindustry technical coefficients and \( l \) is a \( 1 \times m \) vector of direct labour coefficients, each element \( l_i \) representing the direct labour required to produce each unit of gross output in sector \( i \). With \( X \) defined as the \( m \times 1 \) vector of gross outputs, the vector of final outputs can be re-expressed as

\[ Q = (I - A)X \]  

(10)

Combining equations (8), (9) and (10) yields:

\[
\begin{bmatrix}
(I - A) & -c \\
-l & 1
\end{bmatrix}
\begin{bmatrix}
X \\
L
\end{bmatrix} = \begin{bmatrix}
O \\
0
\end{bmatrix}
\]  

(11)

In this expression the role of intermediate (circulating) capital inputs is shown explicitly in the \((I - A)\) matrix.\(^4\)

A modifying assumption is required to render the pure production model open with respect to investment. Assume that investment in the current period becomes new capital inputs in the next period and that the rate of depreciation is 100\% (that is, all capital is circulating capital).\(^5\) With \( M \) representing an \( m \times 1 \) vector of physical quantities of investment goods produced in each sector, Pasinetti’s quantity and price systems can be modified to take the form:

\[
\begin{bmatrix}
(I - A) & -c \\
-l & 1
\end{bmatrix}
\begin{bmatrix}
X \\
L
\end{bmatrix} = \begin{bmatrix}
M \\
0
\end{bmatrix}
\]

(12)

\[
[p \quad w]
\begin{bmatrix}
(I - A) & -c \\
-l & 1
\end{bmatrix} = \begin{bmatrix}
O \\
pM/L
\end{bmatrix}
\]

(13)

where \( p \) is an \( m \times 1 \) row vector of money prices.

In equation (13), income is allocated either to consumption or to saving, which finances investment \textit{ex post}:

\[ w = pc + pM/L \]

(14)

\(^4\)Torr (1992) has examined the relationship between the input–output model and Keynes’s definition of user cost.

\(^5\)Throughout the analysis that follows we maintain the assumption, embodied in equation (11), that the intermediate capital inputs \((AX)\) are produced and used up during the current period.
Let \( C = cL \) be the \( m \times 1 \) column vector containing the total quantities consumed of each sector’s output. Multiplying equation (14) throughout by \( L \), we obtain:

\[
Y = Lw = pC + pM
\]  

Equation (14) shows that the wage rate per unit of labour \((w)\) is made up of total money consumption per unit of labour \((pc)\) plus total money investment per unit of labour \((pM/L)\). Equation (15) shows that the entire wage bill \((Lw)\) or national income \((Y)\) equals consumption plus investment. With investment funded out of savings from wages, we preserve the assumption that all income is paid to labour. It should be emphasised, of course, that the magnitude of investment is determined exogenously by entrepreneurs, that is, it is not constrained by savings. But once the amount of planned investment is set, equation (14) determines the wage rate required to finance this expenditure.

It should also be noted that, with the absence of the categories for profits and the rate of profit, the proportionality between prices and labour values (equation (4)) is maintained in this modified Pasinetti framework. Equations representing the first \( m \) rows and columns of equation (13) take the form \( p(I - A) = wl \) such that:

\[
p = l(I - A)^{-1}w
\]  

This derivation explicitly shows the proportionality between prices and the vertically integrated labour coefficients, \( n = l(I - A)^{-1} \).

To derive the Keynesian multiplier from this modified Pasinetti framework, the simultaneous equations collected in equation (12) can be written out such that

\[
X = AX + cL + M
\]  

\[
L = lX
\]  

Substituting equation (18) into (17) yields

\[
X = AX + clX + M
\]  

Since from equation (10), \( X = (I - A)^{-1}Q \) it follows that

\[
Q = cl(I - A)^{-1}Q + M
\]  

or\(^6\)

\[
Q = cnQ + M
\]

\(^6\)In this modified Pasinetti model, final output \((Q)\) is made up of consumption \((cnQ)\) and investment \((M)\), in contrast to the original Pasinetti model in which final output and consumption are identical. Equation (10), however, which defines the structural relationship between gross and final output, has the same structure in both models.
Multiplying throughout by $n$:

$$nQ = ncnQ + nM$$

and therefore:

$$nQ = \frac{1}{1 - nc} nM$$

This is a multisectoral multiplier relationship between scalars representing the labour required to produce investment goods ($nM$) and total labour employed ($nQ$). The multiplier ($1/1 - nc$) is a simple scalar magnitude.\(^7\) To specify a macro multiplier relationship between total employment and employment in the production of investment goods it is not necessary to specify a one-commodity model. Using Pasinetti’s pure production model as a starting point, the macro multiplier relationship can be derived from multisectoral foundations.

4. The Wage Unit and Multiplier

Subject to certain qualifications, Keynes assumes that the employment multiplier also represents an income multiplier (Keynes, 1936, p. 116). By dividing money income by the wage unit, the volume of employment, or labour units, provides an index of real income. This method of aggregation can be formally demonstrated by examining the employment multiplier derived in equation (23). Since $L$ represents the total units of unskilled labour employed in the economy, then the wage unit represents the money wage paid to each unit of unskilled labour.\(^8\) This wage unit is the same as the wage rate $w$ in Pasinetti’s model. Furthermore, by assumption, in Keynes and Pasinetti, all money income ($Y = pQ$) is paid to labour, which includes the payment to entrepreneurial labour.

\(^7\)This multisectoral structure can be illustrated in a two-sector model in which the key parameters take the form:

$$n = \begin{bmatrix} n_1 & n_2 \end{bmatrix};$$

$$Q = \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix}; \quad M = \begin{bmatrix} M_1 \\ M_2 \end{bmatrix}; \quad \text{and} \quad c = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}.$$  

Hence the elements of the aggregate multiplier relationship are defined as scalars:

$$nQ = n_1 Q_1 + n_2 Q_2;$$

$$nM = n_1 M_1 + n_2 M_2; \quad \text{and}$$

$$nc = n_1 c_1 + n_2 c_2$$

\(^8\)Keynes assumes that labour of higher than unskilled quality is weighted according to its additional remuneration: ‘i.e. an hour of special labour remunerated at double ordinary rates will count as two units’ (Keynes, 1936, p. 41).
The two quantities in the multiplier expression (23) can be written as

\[ nQ = \sum_{i=1}^{m} n_i Q_i \]  

(24)

\[ nM = \sum_{i=1}^{m} n_i M_i \]  

(25)

Since, from equation (4), \( n_i = p_i / w \), it follows that

\[ nQ = \sum_{i=1}^{m} \frac{p_i Q_i}{w} = Y_w, \quad \text{and} \]

(26)

\[ nM = \sum_{i=1}^{m} \frac{p_i M_i}{w} = M_w \]  

(27)

where \( Y_w \) represents real income (nominal income deflated by the wage unit) and \( M_w \) represents real investment (nominal investment deflated by the wage unit). By substituting equations (26) and (27) into (23), the multisectoral multiplier relationship can be re-expressed as:

\[ Y_w = \frac{1}{1 - nc} M_w \]  

(28)

This is an income multiplier relationship, with \( nc \) representing the propensity to consume.\(^9\) Using some of the shared assumptions of Pasinetti and Keynes, the employment multiplier in equation (23) is identical to the income multiplier in equation (28).\(^10\)

It follows that we have established what Pasinetti refers to as a genuinely macroeconomic relationship, in this case between real income and investment. The structure of this relationship holds regardless of the degree of disaggregation. Although the number of elements of the vectors \( n \) and \( c \) may vary with the number of sectors, the structure of the relationship remains unchanged. This interpretation of the Keynes multiplier differs markedly from the assumption in much of

\(^9\)From Equation (7),

\[ nc = \frac{\sum_{i=1}^{m} C_i p_i}{w L} \]

the ratio of total money consumer expenditures (aggregated across sectors) to total income.

\(^10\)It has previously been established by Kurz (1985) that the employment and income multipliers are identical when the share of all income is directed to wages, as assumed in the pure production economy.
macroeconomics that the Keynesian scalar multiplier embodies the assumption of a one-commodity world.

There are three main ways in which this result can contribute to the existing literature. First, there is an established matrix multiplier literature in which multisectoral models have structural similarities to the Keynesian multiplier (see, for example, Kurz, 1985; Miyazawa & Masegi, 1963; Goodwin, 1949). The novel result presented here is that the multisectoral model can be collapsed into a scalar. This result is of course achieved without modelling profits as an economic category, and should therefore be viewed as a conceptual contrast and more accessible background to the more complex family of multisectoral models.

Second, in addition to providing insights into the structure of the multiplier, Pasinetti’s analysis, which is underpinned by a production model in which the pure labour theory of value is in operation, also shows how the multiplier relationship in equation (28) could be consistent with an embodied labour conception of value. We therefore add to the largely textual analysis of Keynes’s writings, in Dillard (1984) and Wray (1998), by drawing upon well-known results in the Sraffian literature to provide a demonstration of how the relationship between the wage unit and the labour theory of value can be formalised.

Finally, this multiplier relationship also facilitates a clearer exposition of Pasinetti’s contribution to Keynesian macroeconomics. We have seen that in Pasinetti’s pure production model there is no exogenous investment demand, which in a Keynesian macroeconomic system means that \( M_w = 0 \). Applying this condition to equation (28), and taking the denominator to the left-hand side yields the expression:

\[
(1 - nc)Y_w = 0
\]  

(29)

This represents a macroeconomic quantity system comparable to Pasinetti’s pure production model, which was presented in equation (1). A non-trivial solution for the homogeneous system of equations requires the identity

\[
1 - nc = 0
\]  

(30)

to be satisfied. This identity is the same as Pasinetti’s non-trivial solution in equation (5), that is: \( \sum_{i=1}^{m} c_i n_i = 1 \). If, as before in our interpretation of equation (5), the total quantity of labour \( (L) \) is set equal to the total amount of labour available for work \( (L_p) \) then Pasinetti’s effective demand condition for full employment is established using the simple Keynesian multiplier relationship derived from multisectoral foundations.

In claiming that his pure production model is pre-institutional, Pasinetti has given the possibly misleading impression that he seeks to diminish the importance of institutional analysis (see Section 3). However, our comparison of equations (28) and (29) allows an alternative emphasis on what are the key institutional characteristics of a monetary production economy. The only mathematical difference between the Keynesian model and the pure production model is the assumption in the former of exogenous investment with positive aggregate savings. This
distinction suggests an institutional focus on the way in which investment is financed, pointing to net borrowing from financial institutions,\(^{11}\) and a focus on the way in which aggregate savings are held and managed as assets in the financial sector. Instead of downgrading the importance of institutional analysis, the comparison of the Keynesian and Pasinetti models provides a possible analytical starting point for institutional analysis.

Moreover, this approach can throw some light on the debate between Pasinetti (2001) and Davidson (2001). Different analytical foundations are proposed in each approach. Whereas the classical system considered by Davidson is decidedly neoclassical, with individual utility-maximising agents, the pure production model is claimed by Pasinetti to have a more traditional classical structure, with a focus on production that has its roots in Smith, Ricardo and Marx.\(^{12}\) In addition, whereas the Pasinetti approach leads to a consideration of the multiplier as a key analytical tool, Davidson (2001, p. 406) argues that there has been an ‘overemphasis’ on the importance of the multiplier in Keynesian economics. However, both follow a similar methodological approach in that the monetary production economy is compared with a narrow limiting extreme: an alternative economy which is not a monetary production economy. Indeed, there are some areas of interface that can be examined.

Davidson provides a rigorous analysis of the axioms that hold in a (neo-) classical economy, and the institutional consequences of relaxing these axioms in a monetary production economy. The limiting extreme is an economy in which there are utility-maximising individuals who are assumed to operate on their budget constraints. ‘For classical utility-maximizing agents, therefore, the marginal propensity to spend out of current income must be equal to unity’ (Davidson, 2001, p. 397). Once we move away from this limiting extreme, once ‘these restrictive axioms are jettisoned ... the marginal propensity to spend out of current income on the products of income is less than unity’ (Davidson, 2001, p. 399). It should be noted that this is the same distinction captured by the relaxation of the zero aggregate savings condition in our comparison of the Pasinetti pure production and Keynesian models. From different analytical starting points, the relaxation of the zero aggregate savings condition is common to both the Pasinetti and Davidson specifications of a monetary production economy.

This contribution provides some basis for engagement between these alternative Keynesian perspectives. Rather than focusing upon the differences between the two approaches, as evidenced by the dispute between Pasinetti and Davidson about the importance of the multiplier, our approach shows how robust the aggregate saving condition is to these different analytical frameworks.

\(^{11}\)As pointed out by Chick (1983, p. 121), ‘it is typical of the business sector as a whole that it is a net borrower.’

\(^{12}\)Both strands of classical analysis can be found in Keynes’s comparison, in his 1933 draft of *The General Theory*, of a ‘neutral’ economy with the monetary production ‘entrepreneur’ economy (Keynes, 1979). The structure of the neutral economy is specified using both neoclassical utility maximisation and the classical commodity-money circuit developed by Marx.
5. Conclusions

Using Pasinetti’s pure production model we have derived a simple multiplier relationship that has some resemblance with that used by Keynes in *The General Theory*. There are three main results from this derivation. First, we show that using Keynes’s wage unit, a genuinely macroeconomic relationship between real income and investment can be established. At one and the same time this aggregate multiplier relationship can embody intersectoral transactions and a pure labour theory of value. This examination of the multiplier provides a formal representation of recent attempts to examine the relationship between the labour theory of value and the wage unit in Keynes’s writings. Second, the analysis helps to clarify some of the issues raised in recent debates between Pasinetti and his critics. Instead of focusing upon the perceived lack of institutional relevance of Pasinetti’s pure production model, more emphasis is placed upon the institutional structure of a monetary production economy. As a contribution to this institutional analysis, the Keynesian multiplier demonstrates Pasinetti’s result that a monetary production economy is characterised by positive aggregate savings. The robustness of this result is suggested by its constituent role in the model of a monetary production economy defined by Davidson (2001). Although Davidson and Pasinetti adopt different analytical frameworks, with and without the multiplier, this finding provides some basis for engagement between these alternative Keynesian perspectives.

The final result is that by adopting Pasinetti’s pure production model as a simplifying device, it is possible to derive the simple multiplier relationship from multisectoral foundations. That is, the paper shows that a scalar multiplier can legitimately be applied in a multisector economy. These results, which potentially provide an accessible foundation to the established matrix multiplier literature, are derived under Pasinetti’s assumption that all income is paid to labour as wages. Whether these results hold in a class-based system in which capitalists receive profits, is an open question and hence the agenda for further work.

References

Appendix: A Two-Period Austrian Production Model

Assume that production in each industry is an Austrian two-period process. In the first period a given amount of unassisted labour ($L_{im}$) produces a given quantity of the machine or investment good ($M_i$): $L_{im} \rightarrow M_i$. Then in the second period the machine is used (with 100% depreciation) with a given amount of labour ($L_i$) to produce an amount of consumption good $Q_i$. Consequently, in the current period of production, the economy produces $m$ different machines (or investment goods) and $m$ different consumption goods. Thus, the modified Pasinetti quantity and price systems take the following form:

$$
\begin{bmatrix}
1 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & \cdots & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & \cdots & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & \cdots & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \cdots & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\
-\z_1 & -\z_{1m} & -\z_2 & -\z_{2m} & \cdots & -\z_m & -\z_{mm} & 1
\end{bmatrix}
\begin{bmatrix}
Q_1 \\
M_1 \\
Q_2 \\
M_2 \\
\vdots \\
Q_m \\
M_m
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
M_1 \\
0 \\
M_2 \\
\vdots \\
0 \\
M_m
\end{bmatrix}
$$

(A1)
\[ \begin{bmatrix} p_1 & p_{1m} & \cdots & p_m & p_{nnm} & w \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 & \cdots & 0 & 0 & -c_1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -c_2 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 & 0 & -c_m \\ 0 & 0 & 0 & 0 & \cdots & 0 & 1 & 0 \\ -z_1 & -z_{1m} & -z_2 & -z_{2m} & \cdots & -z_m & -z_{mm} & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & \cdots & p_m M / L \end{bmatrix} \] (A2)

where

\[ z_i = L_i / Q_i; \quad z_{im} = L_{im} / M_i; \]

\[ p_M = \begin{bmatrix} p_{1m} & p_{2m} & \cdots & p_{nm} \end{bmatrix}; \] and

\[ M = \begin{bmatrix} M_1 \\ M_2 \\ \vdots \\ M_m \end{bmatrix} \]

Wages are allocated between consumption and investment such that:

\[ w = pc + p_M M / L \] (A3)

where \( p = [p_1 \quad p_2 \quad \cdots \quad p_m] \).

Equation (A3) shows that the wage rate per unit of labour (\( w \)) is made up of total money consumption per unit of labour (\( pc \)) plus total money investment per unit of labour (\( p_M M / L \)).

To derive the Keynesian multiplier from this two-period Austrian model, the expression for total labour in equation (A1),

\[ L = z_1 Q_1 + z_{1m} M_1 + \cdots + z_m Q_m + z_{mm} M_m \]

can be re-expressed, since \( Q_i = c_i L \), such that:

\[ L = z_1 c_1 L + z_{1m} M_1 + \cdots + z_m c_m L + z_{mm} M_m \] (A4)

Collecting these terms together in matrix notation:

\[ L = zb L + zM M \] (A5)
where

\[ z = \begin{bmatrix} z_1 & z_2 & \cdots & z_m \end{bmatrix}; \]
\[ z_M = \begin{bmatrix} z_{1m} & z_{2m} & \cdots & z_{mm} \end{bmatrix}; \text{ and} \]

\[ b = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_m \end{bmatrix} \]

The employment multiplier can therefore be written as:

\[ L = \frac{1}{1 - zbM} \tag{A6} \]

With scalars representing the labour required to produce investment goods \((z_M M)\), total labour employed \((L)\), and the propensity to consume \((zb)\), equation (A6) is a macro multiplier relationship derived from Austrian two-period multisectoral foundations. Making use of Keynes’s wage unit, equation (A6) can easily be transformed into an income multiplier relationship, the same result established in Section 4.