Full-cost Pricing in the Classical Competitive Process: a Model of Convergence to Long-run Equilibrium

Enrico Bellino

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In the last 10–15 years a lot of attempts has been devoted to study the classical process of convergence of “market prices” toward “natural prices.” The two forces that one has thought could achieve this target were capital mobility, that determines the dynamics of output, and demand–supply forces, that determine the dynamics of prices. In this article a model of classical competition is proposed in which a full-cost pricing mechanism is adopted in the rule of evolution of market prices. An asymptotical stability result of long-run equilibrium is proved for a two-commodity model with and without a final demand.

Keywords: capitalistic competition, capital mobility, full-cost pricing, gravitation, market prices, natural prices, long-run equilibrium.

JEL classification: D4, D46, D50, E11.

1 Introduction

A well-known result in modern classical analysis of capitalistic competition is the instability of long-run equilibrium positions within the so-called “pure” cross-dual model, which is the simplest model that has been used to represent the classical competitive process. In this model a dynamic interaction between market prices and output is presented, which is based on two “principles”: the principle of capital mobility, for which capitalists allocate their capital funds across sectors according to their relative profitability, and the principle of supply and demand, for which firms set prices according to the imbalances between supply and demand of each commodity.

Both these behaviors of capitalists and firms have been borrowed from the classical analysis of competition. According to this view these forces should result in a tendency for the system to “gravitate” around “long-run equilibrium positions,” the latter being characterized by a uniform rate of profit across sectors and the equilibrium between supply
and demand of each commodity, so that there is no longer any incentive for capitalists to allocate their capitals differently, and for enterprises to vary prices of commodities deviating from their natural levels.

"If at any time (supply) exceeds effectual demand, some of the component parts of its price must be paid below their natural rate [...] if it is wages or profit, the interest of labourers in the one case, and of the employers in the other will prompt them to withdraw a part of their labour or stock from the employment [...]. If on the contrary, the quantity brought to the market should at any time fall short of the effectual demand, some of the component parts of its price must rise above their natural rate [...] if it is wages or profit, the interest of all other labourers and dealers will soon prompt them to employ more labour and stock in preparing and bringing it to the market" (Smith, 1974, p. 160).

"Suppose [...] that a change of fashion should increase the demand for silk, and lessen that for woollens; their natural price, the quantity of labour necessary to their production, would continue unaltered, but the market price of silk would rise, and that of woollens would fall; and consequently the profit of the silk manufacturer would be above, whilst those of the woollen manufacturer would be below, the general and adjusted rate of profits. [...] This increased demand for silks would however soon be supplied, by the transference of capital and labour from the woollen to the silk manufacture; when the market prices of silk and woollens would again approach their natural prices, and then the usual profits would be obtained by the respective manufacturers of those commodities" (Ricardo, 1951, pp. 90f).

1.1 The "Pure" Cross-dual Model

At the beginning of the eighties, after about one century since when these ideas were conceived, some questions concerning the validity of this classical conjecture have been raised. The problems resulted from the fact that the attempts to formalize, by means of dynamic equations systems, the process sketched above showed a lot of problems in proving the convergence – in mathematical terms – of market prices and output toward the long-run equilibrium. The simplest coherent model used to represent the competitive process sketched above was the "pure" cross-dual model, in which the dynamics of actual output and of market prices is represented, in discrete terms, by the two difference equations systems:
\[ x_{i,t+1} = x_{it}[1 + r_t + \gamma(r_{it} - r_t)], \quad (1a) \]

\[ p_{i,t+1} = p_{it}\left(1 + \beta \frac{d_{it} - x_{it}}{x_{it}}\right), \quad (1b) \]

where \( x_{it} \) is the quantity of commodity \( i, i = 1, \ldots, n \), produced at time \( t \), and \( p_{it} \) is its market price;

\[ r_t := \frac{x_t^T (I - A) p_t}{x_t^T A p_t} \quad \text{and} \quad r_{it} := \frac{p_{it} - a_{it}^T p_t}{a_{it}^T p_t}, \quad (2) \]

are, respectively, the average and the sectoral rates of profit, where \( x_t = [x_{it}] \), \( p_t = [p_{it}] \), and \( A = [a_{ij}] \) is the augmented technology matrix,\(^1\) with \( a_{it}^T \) as \( i \)-th column;

\[ d_t = [d_{it}] := x_{t+1}^T A \]

is the vector of demanded quantities at time \( t \) and \( \beta \) and \( \gamma \) are two positive reaction parameters which measure, respectively, the reactivity of enterprises in changing prices in response to imbalances between supply and demand and the reactivity of capitalists in moving capital funds across sectors in response to profitability differentials.\(^2\)

It can easily be shown that the solutions of the two following linear systems:

\[ p = Ap(1 + r), \quad (3) \]

\[ x_t = x_{t+1}^T A \quad \text{and} \quad x_{t+1} = (1 + r)x_t, \quad (4) \]

which are, respectively, the production-price vector and the balanced-growth path, constitute an invariant set for system (1). But this equi-

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1. Coefficients \( a_{ij} \) of matrix \( A \) include workers' subsistences.

2. The formulation of the capital-mobility principle by means of Eq. (1a) has been proposed by Duménil and Lévy; it is based on the idea that the "class" of capitalists can be conceived as a centre for the allocation of capital (big enterprises, banks, financial institutions, etc.) that decides on the amount of investments in the various sectors. This is not the only way by which capital mobility can be formulated; for example Boggio uses a different formulation that, in its linearized form, appears as \( x_{t+1} = x_t(1 + r_{it}) \) or \( x_{t+1} = x_t[1 + r_{it} + \gamma(r_{it} - r_t)] \); these formulations reflect the idea that capital mobility is mainly carried out by the re-investment of profits earned by the firm. In this work I will use the Duménil and Lévy's formulation, as in my opinion it is more suitable to describe the contemporary ownership arrangements of enterprises and their linkages with financial institutions.
librium has turned out to be asymptotically unstable for the dynamic process represented by system (1) for whichever level of the reaction coefficients $\beta$ and $\gamma$ (for details, see Boggio, 1984, 1985, and 1992; Duménil and Lévy, 1993, appendix to chap. 6). For the continuous-time version of the model the asymptotical stability of the long-run equilibrium can be proved if $\det A < 0$, but this assumption limits considerably the relevance of the classical notion of long-run equilibrium, even if it is possible to give an economic interpretation of this assumption (see, e.g., Hosoda, 1985).

A lot of attempts have been made to deal with these problems, and some results of convergence have been proved by introducing some reasonable changes of the structure of the "pure" cross-dual model: for example by introducing a final demand with strong price-substitution effect (Hosoda, 1985; Boggio, 1985), or by introducing a direct adjustment of output in response to imbalances between supply and demand (see Duménil and Lévy, 1987b, p. 152; 1993, chap. 6), or by using the notion of a "realized" rate of profit, based on actually cashed profits (see Duménil and Lévy, 1993, chap. 6).\(^3\) Alternatively, gravitation has been conceived as a permanent oscillation around the long-run position, and one has studied the existence and the asymptotic stability of periodical orbits that surround the equilibrium (see, e.g., Franke, 1987; Kubin, 1991).

1.2 Full-cost Pricing

In this work we want to consider another way in which the problem of the instability of long-run equilibrium can be overcome in cross-dual models, by offering a more satisfactory description of the evolution of market prices. More precisely we will introduce a full-cost pricing mechanism into the dynamics that describes the evolution of market prices.\(^4\) By looking at the two forces that constitute the motion laws of the model one can observe that, while on one side the principle of capital mobility seems to be a first acceptable basis to describe capitalists' investment decisions, the law of supply and demand seems rather unrealistic and quite inadequate to describe only by itself firms' behavior in facing imbalances between supply and demand and their price.

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\(^3\) Two good surveys of this literature are Boggio (1992) and Duménil and Lévy (1993, appendix to chap. 6).

\(^4\) A brief synthesis of the full-cost principle and the supply and demand principle in setting prices is presented by Duménil and Lévy (1993, appendix to chap. 8).
decisions. In the short run, which is the horizon in which we suppose that firms face this disequilibrium, it seems more reasonable to think that they react mainly by varying their inventories rather than prices; on the other hand it does not appear realistic to think that firms set their prices without any consideration of production costs. In the literature it is known that full-cost pricing is a mechanism that permits to obtain the convergence toward a production-price system in models in which only price dynamics is considered (see Boggio, 1980, 1984). Moreover some studies of the interaction of full-cost pricing with the cross-dual model have already appeared. In Boggio (1986) a cross-dual gravitation model is presented in which the price dynamics is determined either by a full-cost component or by the excess-demand principle. Boggio proves the asymptotical stability of long-run equilibrium if the effects of relative excesses of demand on prices and on mark-up rates are sufficiently small and moreover if the effect on mark-up rates is sufficiently small with regard to the effect on prices. The analytics of the model is quite sophisticated as it considers a system with \( n \) commodities, in which the mark-up charged on production cost differs from industry to industry and the dynamics of these mark-up rates is determined endogenously to the model. This makes it sometimes difficult to pick up the stabilizing properties of the full-cost pricing mechanism within the classical competitive process. Moreover, the model considers only the case without consumption out of profits (a pure-accumulation system). Flaschel and Semmler (1988) and Semmler (1990) also inserted into a cross-dual model a full-cost pricing mechanism, together with a direct adjustment quantities → quantities. Their results are quite unsatisfactory: they proved that the long-run equilibrium is stable if the cross-dual dynamics interacts only weakly with the components representing the full-cost pricing and the direct adjustment of quantities. Another limitation of the model concerns the specification of all adjustment mechanisms: actual output levels and market prices are adjusted on the basis of their shifts from their long-run values; this entails that agents have to know the equilibrium magnitudes.

2 A Cross-dual Model with Full-cost Pricing

2.1 Competitive Dynamics

In this section we will present a model of capitalistic competition with final demand in which the dynamics of output continue to be described by Eqs. (1a), with \( r_{it} \) and \( r_t \) given by (2), while market prices are determined by production cost, on which a proportional mark-up is
charged, as well as by imbalances between supply and demand. For simplicity we will consider the case of a two-commodity model \((n = 2)\):

\[
p_{i,t+1} = (a_{i1}p_{1t} + a_{i2}p_{2t})(1 + \pi_{it})(1 + \beta \frac{d_{it} - x_{it}}{x_{it}}), \quad i = 1, 2 \tag{5}
\]

where \(\pi_{it}\) is the rate of mark-up of commodity \(i\) at time \(t\). We have now to explain how mark-up rates are determined. The literature on this field is very large. Perhaps the simplest assumption is to suppose that for both firms the rate of mark-up is equal to the average rate of profit of the system at time \(t\), which is a magnitude that can be calculated by firms. This would reflect the idea that each firm wants to obtain how much one obtains, on average, in the whole system. Therefore the market-prices equation (5) becomes

\[
p_{i,t+1} = (a_{i1}p_{1t} + a_{i2}p_{2t})(1 + r_t)(1 + \beta \frac{d_{it} - x_{it}}{x_{it}}), \quad i = 1, 2 \tag{6}
\]

In this way the mark-up rate that each firm charges on its cost is corrected by the supply and demand component. This entails that the actual mark-up charged on the cost of production is affected by the excess of demand for that commodity.

The demand for each commodity at time \(t\) is determined by the employment of that commodity as input in the production of period \(t + 1\) plus the final demand that comes from profits earners, \(c_i\):

\[
d_i^T = [d_{it}] := x_{i+1}^T A + c_i, \quad i = 1, 2
\]

where \(c_i = c(x_i, p_t)\) is a continuously differentiable map from \(\mathbb{R}^2 \times \mathbb{R}^2\) to \(\mathbb{R}^2\), defined for \(x_i > 0, p_t > 0\) with the following properties:

1. Nonnegativeness: \(c(x, p) > 0\) for all \(x > 0, p > 0\).
2. Homogeneity: \(c(\lambda x, \mu p) = \lambda c(x, p)\) for any \(\lambda > 0, \mu > 0, \lambda, \mu \in \mathbb{R}\).
3. Monotonicity: For any \(p > 0\) we have \(c(x^1, p) \leq c(x^2, p)\) for all \(x^1\) and \(x^2\) such that \(x^1 \leq x^2\).
4. Constant propensity to consume: \(c(x, p)p = (1 - s)x^T (I - A)p\), where \(s\) is the propensity to save out of profits, \(0 \leq s \leq 1\).

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5 In \(\mathbb{R}^n\) we will use the following conventional notation: let \(x = [x_i] \in \mathbb{R}^n\) and \(y = [y_i] \in \mathbb{R}^n\); then: \(x \geq y\) iff \(x_i \geq y_i\) for \(i = 1, \ldots, n\); \(x > y\) iff \(x_i \geq y_i\) for \(i = 1, \ldots, n\) and \(x \neq y\); \(x \gg y\) iff \(x_i > y_i\) for \(i = 1, \ldots, n\).
2.2 Long-run Equilibrium

It is easy to prove that a couple of vectors, \( \mathbf{p} \) and \( \mathbf{x} \), satisfying systems

\[
\mathbf{p} = \mathbf{A}\mathbf{p}(1 + r), \quad (7)
\]
\[
\mathbf{x}_t^T = \mathbf{x}_{t+1}^T + \mathbf{c}^T (\mathbf{x}, \mathbf{p}) \quad \text{and} \quad \mathbf{x}_{t+1} = (1 + sr)\mathbf{x}_t \quad (8)
\]
is an invariant set for the difference system constituted by Eqs. (1a) and (6). Systems (7) and (8) describe the long-run position of the economy. The following proposition deals with the existence and the uniqueness of the solution of systems (7) and (8).

**Proposition 1:** If \( \mathbf{A} \) is an indecomposable matrix with the dominant eigenvalue smaller than 1 in modulus, then systems (7)–(8) have an economically meaningful solution which is unique up to a scale factor.

**Proof:** The existence of a unique scalar \( r^* = 1/\lambda_M(\mathbf{A}) - 1 \) and of a unique positive vector, determined up to a scale factor, \( \mu \mathbf{p}^* \) with \( \mu > 0 \), \( \mathbf{p}^* \gg \mathbf{0} \), and \( \|\mathbf{p}^*\| = 1 \) that satisfies (7) is an immediate consequence of the Perron–Frobenius theorem on nonnegative matrices. Define \( \mathbf{h}^T(\mathbf{x}) := (1 + sr^*)\mathbf{x}^T \mathbf{A} + \mathbf{c}^T (\mathbf{x}, \mathbf{p}^*) \), where the map \( \mathbf{h}(\mathbf{x}) \) is continuous, differentiable, nonnegative, homogeneous of degree 1, monotonous [thanks to the properties of map \( \mathbf{c}(\mathbf{x}, \mathbf{p}^*) \)], and indecomposable (as \( \mathbf{A} \) is indecomposable). It satisfies therefore the assumptions of the Perron–Frobenius theorem for nonlinear maps;\(^6\) and hence there exists a positive number, \( \xi \), and a positive vector, \( \mathbf{z} \), unique up to a scale factor, satisfying \( \mathbf{h}^T(\mathbf{z}) = \xi \mathbf{z}^T \), i.e.,

\[
(1 + sr^*)\mathbf{z}^T \mathbf{A} + \mathbf{c}^T (\mathbf{z}, \mathbf{p}^*) = \xi \mathbf{z}^T. \quad (9)
\]

By multiplying (9) by \( \mathbf{p}^* \) one obtains \( \xi = 1 \). Hence (9) reduces to \( (1 + sr^*)\mathbf{z}^T \mathbf{A} + \mathbf{c}^T (\mathbf{z}, \mathbf{p}^*) = \mathbf{z}^T \). Therefore, there exists a unique positive vector, determined up to a scale factor, \( \mathbf{z} = \beta \mathbf{x}^* \) with \( \beta > 0 \), \( \mathbf{x}^* \gg \mathbf{0} \), and \( \|\mathbf{x}^*\| = 1 \), satisfying (8).

In what follows we will focus upon the dynamics of relative output and relative market prices:

\[
X := \frac{x_1}{x_2} \quad \text{and} \quad P := \frac{p_1}{p_2}. \quad (10)
\]

The existence and the uniqueness up to a scale factor of the solution of systems (7) and (8) entail the existence and the uniqueness of the long-run equilibrium expressed in relative terms; it can be found by solving the following systems:

\[ P = (a_{11} P + a_{12})(1 + r) , \]  
\[ 1 = (a_{21} P + a_{22})(1 + r) ; \]  
\[ X = (X a_{11} + a_{21})(1 + sr) + C_1(X, P) , \]  
\[ 1 = (X a_{12} + a_{22})(1 + sr) + C_2(X, P) , \]

where \( C_i(X, P) := c_i(x, p)/x_i, i = 1, 2 \). Let us denote by \((X^*, P^*, r^*)\) the economically relevant solution of systems (11)–(12). From (11) one easily gets the expressions of \( P^* \) and \( r^* \), \( 1 + r^* = 2/(a + b) \) and \( P^* = (\bar{a} + b)/\bar{a} \), with \( a := a_{11} + a_{22}, \bar{a} := a_{11} - a_{22}, b := \sqrt{\bar{a}^2 + \bar{a} \bar{a}}, \alpha := 2a_{12}, \bar{a} := 2a_{21} \).

2.3 The Recursion

The competitive dynamics of relative variables is obtained by re-expressing Eqs. (1a) and (6) in terms of \( X \) and \( P \) as follows:

\[ X_{t+1} = X_t \frac{1 + sr_t + \gamma (r_{1t} - r_t)}{1 + sr_t + \gamma (r_{2t} - r_t)} ; \]  
\[ P_{t+1} = \frac{a_{11} P_t + a_{12} 1 + \beta ((D_{1t} - X_t)/X_t)}{a_{21} P_t + a_{22} 1 + \beta (D_{2t} - 1)} \]

with

\[ 1 + r_{1t} = P_t/(a_{11} P_t + a_{12}) , \]  
\[ 1 + r_{2t} = 1/(a_{21} P_t + a_{22}) , \]  
\[ 1 + r_t = X_t P_t/[X_t(a_{11} P_t + a_{12}) + (a_{21} P_t + a_{22})] , \]  
\[ D_{1t} = X_t[1 + sr_t + \gamma (r_{1t} - r_t)]a_{1i} + [1 + sr_t + \gamma (r_{2t} - r_t)]a_{2i}, \quad i = 1, 2 . \]

The following proposition is devoted to the existence and the uniqueness of the economically meaningful equilibrium of the recursion (13).

**Proposition 2:** If \( A \) is an indecomposable matrix with the dominant
eigenvalue smaller than 1 in modulus, then difference system (13) has a unique economically relevant equilibrium which coincides with the long-run equilibrium of the economy, \((X^*, P^*)\).

**Proof:** By inserting \(X_{t+1} = X_t = X\) into Eq. (13a) one gets \(r_1 = r_2\), that thanks to (14a) and (14b) entails equation \(a_{21} P^2 - (a_{11} - a_{22}) P - a_{12} = 0\); by solving this equation one obtains: \(P = P^*\) and \(r_1 = r_2 = r^*\); by inserting \(P_{t+1} = P_t = P\) into Eq. (13b) one obtains \((D_1 - X)/X = D_2 - 1\), which is satisfied by \(X = X^*\); \(X^*\) is also the unique (positive) solution as for \(P = P^*\) condition \([D_1(X, P^*) - X] \times P^* + [D_2(X, P^*) - 1] = 0\) holds. \(\Box\)

### 2.4 Asymptotic Stability of Long-run Equilibrium

Before turning to the study of the stability of long-run equilibrium we have to prove the following.

**Lemma 1:** For consumption functions \(C_i\) the identity \(C_i(X, P) = XC_{iX} + (\partial c_i/\partial x_2), i = 1, 2\), holds, where \(C_{iX} := \partial C_i/\partial X\).

**Proof:** From the homogeneity properties of functions \(c(x, p)\) one has \(c_i(x_1, x_2, p_1, p_2) = x_2 C_i(X, P)\) for \(i = 1, 2\); by differentiating with respect to \(x_i\) we get \(\partial c_i/\partial x_1 = x_2(\partial C_i/\partial X)(\partial X/\partial x_i) = x_2(\partial C_i/\partial X) \times (1/x_2)\), i.e.,

\[
\frac{\partial c_i}{\partial x_1} = \frac{\partial C_i}{\partial X}.
\]

On the other hand, as \(c_i(x, p)\) is homogeneous of degree 1 with respect to \(x\), Euler's theorem holds, i.e., \(c_i(x, p) \equiv x_1(\partial c_i/\partial x_1) + x_2(\partial c_i/\partial x_2)\); hence because of (15), the lemma follows. \(\Box\)

The Jacobian matrix of system (13) is:

\[
J = \begin{bmatrix}
1 & \gamma U \\
-\beta(V - C_X) & W + \beta Y Z + \beta C_P
\end{bmatrix},
\]

where

\[
U := \frac{1}{1 + sr^*} \frac{X^*}{P^*} \left(\frac{2}{a + b}\right)^2 b > 0,
\]

\[
V := (P^*/X^*)[1 - (1 + sr^*)(a_{11} - X^*a_{12})] > 0.
\]
\[ C_\xi := (P^*/X^*)(C_{1\xi} - X^*C_{2\xi}), \quad C_{i\xi} := (\partial C_i/\partial \xi)|_*, \quad \xi = X, P, \]
\[ W := (a - b)/(a + b), \quad |W| < 1, \]
\[ Y := (s - \gamma)P^* \frac{2}{a + b} \frac{1}{X^*P^* + 1} [X^* - (1 + r^*)(X^*a_{11} + a_{21})] \times \]
\[ \times [a_{11} - X^*a_{12} + (a_{21}/X^*) - a_{22}], \]
\[ Z := \left( \frac{2}{a + b} \right)^2 \frac{a_{12}}{P^*} (a_{11} - X^*a_{12}) - a_{21} P^* \left( \frac{a_{21}}{X^*} - a_{22} \right). \]

The long-run equilibrium is stable if the following three conditions hold:

1. \( Q(1) > 0 \): \( \beta \gamma U(V - C_X) > 0 \); for \( \beta > 0 \) and \( \gamma > 0 \) the sign of \( Q(1) \) depends on the sign of \( V - C_X = (P^*/X^*)[(1 - (1 + s r^*) \times a_{11} - C_{1X})] + P^*[(1 + s r^*)a_{12} + C_{2X}] \); but for \( X = X^* \) the following relationship holds:

\[ X^* = (1 + s r^*)(X^*a_{11} + a_{21}) + C_1(X^*, P^*) \quad (16) \]

and from Lemma 1 we can write (16) as follows:

\[ 1 - (1 + s r^*)a_{11} - C_{1X} = \frac{1}{X^*} \left[ (1 + s r^*)a_{21} + \left. \frac{\partial c_i}{\partial x_2} \right|_* \right] > 0. \]

This entails that \( V - C_X > 0 \), thus for \( \beta > 0 \) and \( \gamma > 0 \) condition 1 is satisfied;

2. \( Q(-1) > 0 \): \( 2 + 2W + 2\beta Y + 2\beta \gamma Z + 2\beta C_P + \beta \gamma U(V - C_X) > 0 \); for \( \beta \) and \( \gamma \) sufficiently small the sign of \( Q(-1) \) coincides with the sign of \( 2 + 2W \), which is positive as \( |W| < 1 \); thus for \( \beta \) and \( \gamma \) sufficiently small condition 2 is satisfied;

3. \( Q(0) < 1 \): \( Q(0) = W + \beta Y + \beta \gamma Z + \beta C_P + \beta \gamma U(V - C_X) = W + O(\beta, \beta \gamma) \); for \( \beta \) and \( \gamma \) sufficiently small \( O(\beta, \beta \gamma) \) can be neglected and the value of \( Q(0) \) depends on \( W \), which is less than 1 in modulus; therefore for \( \beta \) and \( \gamma \) sufficiently small condition 3 is also satisfied.

We have thus proved the following:

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7 A second-degree polynomial, \( Q(\lambda) \), with the coefficient of the term in \( \lambda^2 \) equal to 1, has roots having modulus smaller than 1 if and only if \( Q(1) > 0 \), \( Q(-1) > 0 \), and \( Q(0) < 1 \); see, e.g., Gandolfo (1996, pp. 58f).
Proposition 3: The equilibrium \((X^*, P^*)\) of system (13) is locally asymptotically stable for \(\beta\) and \(\gamma\) sufficiently small.

3 Conclusive Remarks

The statement of Proposition 3 constitutes the main result of the present paper: the long-run equilibrium of the economy is a (local) attractor for the competitive process described by system (13); if the actual system is not in its long-run equilibrium (and not too far from it) it will converge towards a price system that assures the prevalence of a uniform rate of profit and a quantity system that guarantees the equilibrium between supply and demand. The only condition required is that the reaction parameters \(\beta\) and \(\gamma\) are “sufficiently small” in order to avoid overshooting.\(^8\)

It could be noted that in this model any ad hoc assumption on final demand, such as in particular strong substitutability, is necessary to achieve the main result. Obviously, if this assumption should hold, i.e., if \(C_{1P} < 0\) and \(C_{2P} > 0\), \(C_P\) would be negative and thus the term \(\beta C_P\) in the expression of \(Q(0)\) would play a role in favor of the fulfilment of condition 3. This is a well-known result: strong substitutability has a stabilizing effect on equilibria. It appeared in stability analyses of neoclassical general-equilibrium models as well as in cross-dual gravitation models with consumption (see, e.g., Boggio, 1985, 1992, theorem 8; Duménil and Lévy, 1993; Hosoda, 1985). But in the present model this assumption is no more necessary.

It is also interesting to note that the long-run equilibrium remains asymptotically stable also in the two extreme cases of a simple reproduction system \((s = 0)\) or of a pure accumulation system \((s = 1)\).\(^9\) In the latter case the analytics of the model becomes simpler, thanks to the perfect duality between the systems that describe the long-run equilibrium for quantities and for prices: the long-run levels for \(r^*, X^*,\) and \(P^*\) are \(1 + r^* = 2/(a + b)\), \(X^* = (\bar{a} + b)/\alpha\), and \(P^* = (\bar{a} + b)/\bar{a}\) and the Jacobian matrix assumes the form:

\[
J = \begin{bmatrix}
1 & \gamma U \\
-\beta V & W + \beta \gamma Z
\end{bmatrix},
\]

\(^8\) This condition is not extraneous to gravitation models: it has already emerged in several models in which a convergence result was obtained.

\(^9\) This is not always the case in gravitation models, see, e.g., Duménil and Lévy (1987a, pp. 141f).
where

\[ U := \frac{2}{a + b} \frac{a}{\alpha} b, \quad V := \frac{\alpha}{\bar{a}} \frac{2b}{a + b}, \]

\[ W := \frac{a - b}{a + b}, \quad |W| < 1, \quad Z := \frac{2b}{a + b} W. \]

In this case the conditions under which the characteristic polynomial of \( J \) has roots smaller than 1 in modulus reduce to \( \gamma < 1/\beta \) and \( \beta > 0 \), and the stability locus in the parameter space is delimited by the two positive semiaxes and the positive branch of the hyperbola represented in Fig. 1.

The treatment of the general case with \( n \) commodities complicates the analytics, as it requires the study of a higher-dimension recursion. A lot of computer simulations of the above model with a higher number of commodities has been performed that have widely confirmed the convergence result. In Fig. 2 we present one of the simulated time evolutions of sectoral profit rates for a model with 4 basic commodities which are also demanded as final goods; in the simulation we have supposed a fixed-proportion consumption function,

\[ c = (1 - s) \frac{x^T (I - A) p}{\delta^T p} \delta. \]

where \( \delta = [\delta_i] \) is a fixed vector which represents the proportions in which the commodities are consumed.
Fig. 2: Dynamics of $r_t$

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Address of author: Enrico Bellino, Institute of Economic Theory and Quantitative Methods, Università Cattolica del Sacro Cuore, Largo Gemelli 1, I-20123 Milano, Italy.