A STRUCTURAL MODEL OF PRODUCTION*

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It is one sign of the greatness of any achievement, practical or theoretical, scientific or artistic, that its creative stimulus extends far beyond the area for which its author designed it originally. The ultimate intention of Keynes' *General Theory* was certainly practical-political: the etiology and therapy of capitalist underemployment. It is true, the work abounds with novel scientific tools. But they were forged or, as in the case of the multiplier concept, borrowed for the purpose of illuminating specific problems in applied economics rather than of enriching our tool chest in general. And yet concepts like the consumption function, the marginal efficiency of investment, or the aggregate supply function have become the cornerstone for a general macro theory of the market, not to mention the strategic role which Keynes' national-income analysis now plays in such diverse fields as economic budgeting and the study of economic change.

The last reference is perhaps the most surprising. It has often been pointed out that Keynes' analytical framework is basically static. Not only are the main data—tastes as well as obstacles—taken as given, but the variables are all related to the same point of time. To be sure, there are Notes on the Trade Cycle and "occasional digressions," which concern themselves with "the slow effects of secular progress" (*General Theory*, p. 109). But these passages are too vague to have stimulated more than general speculations on the dynamics of the economic process. In this field the real advances that the Keynesian work has promoted stem from the subsequent "dynamization" of some of its static concepts.

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Harrod's synthesis of the multiplier and the accelerator, later refined by Samuelson, Hicks' "multiplier in a changing economy," and again Harrod's dynamization of the saving-investment identity are cases in point. A book like Hicks' recent study on the trade cycle is hardly conceivable without the building blocks of Keynesian economics. Only on these foundations could he succeed in formalizing his model to such a degree that it can serve as illustration for quite a number of seemingly contradictory theories.

In long-term dynamics, progress along these lines has only started. Of the typical dynamic relations that rule in business cycles, namely lags and derivations, only the latter appear, on a priori grounds, to be relevant also in secular change. But, as Harrod and Domar have shown, certain circular mechanisms, like the interaction between multiplier and accelerator, are useful tools also in the study of self-reinforcing growth.

The more we learn about the various modes of change and their interaction, the more important it becomes that we form a clear notion of the substratum of such changes. More precisely, our ultimate success in handling dynamic phenomena analytically will depend upon a full grasp not only of the change processes themselves, but also of the strategic variables of the economic system which are exposed to change.

In this respect Keynes' contribution has been even more authoritative, though perhaps, as this paper is to show, less definitive. At any rate, the skeleton of practically all dynamic models designed in recent years is Keynes' national-income identity, or the saving-investment identity derived from it. Thus the basic variables

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1 These and subsequent references are meant as random examples of the indirect effect of Keynes upon modern dynamic theory, rather than as a survey of the influences that have shaped the latter. Otherwise pre-Keynesian contributions like those of J. M. Clark, Frisch, Tinbergen, Kalecki, and especially the Swedish School could not be passed over. See in this respect the interesting methodological introduction in William Fellner's "Employment Theory and Business Cycles," in A Survey of Contemporary Economics, edited by Howard S. Ellis (Philadelphia 1948) pp. 49-98.
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with which modern dynamic analysis operates are aggregate income or output and its components: consumption and saving, or consumption and investment. In the models of secular growth, induced investment is usually distinguished from autonomous investment, and a variable for the capital stock and its depreciation is added. The same specifications are found in certain business-cycle models, some of which take account also of employment, wage and profit rates, and the general price level. More often employment is, in Keynesian fashion, supposed to be uniquely correlated with national income, whereas the basic wage rate is treated as a datum, in accord with Keynes' use of the wage unit as the fixed value unit of the system.

The variables enumerated here are fairly representative of the models devised, for example, by Goodwin, Hansen, Harrod, Hicks, Lange, or Metzler in business-cycle analysis, and by Domar and Harrod in growth analysis. One may wonder whether the choice is wide enough for the study of all dynamic phenomena—wide enough for such problems as the employment and income effects of technical changes, or the peculiar demand and supply conditions affecting the early development stages of so-called backward regions. Nevertheless, our interest here does not center on possible gaps in the set of variables on which post-Keynesian dynamics builds. What provokes comment is the peculiar manner in which the elements composing the set appear in the various models.

As a rule, all these variables are treated solely as value aggregates or price sums, to the exclusion of the physical-technical properties which attach to them in an industrial system. The implications of this practice can best be shown by considering the one magni-

2 I am disregarding certain empirical models, like those of Tinbergen or Klein relating to fluctuations in specific economies. These models—which, for example, subdivide income according to sources, and savings and money stocks according to holders, and introduce the government sector as well as foreign economic relations—are much more highly differentiated than the theoretical models referred to above. From what follows it will become clear, however, that the particular proposal for refinement submitted here goes in a different direction.
tude which even conventional analysis subdivides into physically distinct components: national income, as made up of consumption and investment. There the difference in the technical functions which certain groups of commodities and services perform has been fully recognized, and the integral concepts of income and output have been disaggregated accordingly. But no further reduction, in physical terms, of the component parts themselves—that is, consumption and, above all, investment—is customary. This is quite surprising considering the importance which certain modern business-cycle theories, for example, attach to the physical form taken by investment: fixed capital or working capital, for either additional consumer goods or additional capital goods. Once we treat the law of diminishing returns less cavalierly than has become customary in building long-term dynamic models, we shall have to distinguish between the output of raw materials and that of manufacturing. Still more complicated physical differentiation is required if, in dealing with technical changes, we give up the simple notion of “neutral” inventions, which are on balance neither labor-saving nor capital-saving (Harrod). Then it becomes essential to diagnose the precise locus of the technical reorganization: in the sphere of consumer goods or of capital goods, in the stage of finished output or in some earlier stage of production.

Such studies of the physical-technical relationships among the main sectors of the productive structure would by no means have only academic interest. Twice during the last decade a speedy adjustment of American industry to the requirements of defense was impeded by a peculiar physical bottleneck, the lack of machine tools. What stands in the way of an accelerated development of backward regions is not so much general poverty as scarcity of specific commodities, namely capital goods, the physical form of which changes typically with the progress in development.

It is obvious that the distinction between induced and autonomous investment is of little help in this respect, since it refers to the dimension of values rather than to that of physical aggregates.
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Still, it is not difficult to understand why the physical aspects of the productive order, and thus the physical conditions for dynamic equilibrium, have received so little attention in both neoclassical and Keynesian economics. Obviously their significance is directly related to the extent to which the dynamic process is exposed to large internal shifts of specific resources more or less fully utilized. The whole issue can safely be disregarded if resources are by and large non-specific, or if the normal state of the system is one of underemployment. The former condition is assumed in most neoclassical reasoning, whereas the latter proposition is in the center of Keynesian analysis. Certainly the premise of non-specificity, which was taken over into neoclassical theory from the older classical notion of the "natural" mobility of the factors of production, has never been realistic, and has become less so with the progress of large-scale industrialization. On the other hand, the Keynesian proposition of underutilization of resources pretty well pictured the facts of the nineteenth and early twentieth centuries, except in boom periods. Such underutilization, though logically contradicting the full-employment framework of neoclassical analysis, made many of the latter's conclusions empirically valid, among them the thesis of the short-run adaptability of the system to large shifts. Most internal adjustments took the form of different rates of growth of the strategic sectors, brought about within a secular process of aggregate expansion. By focusing attention on aggregate changes, Keynesian economics enhanced the traditional belief in the ease with which sectional shifts can be carried out.

As no one realized more clearly than Keynes himself, the practical success of his teaching, brought about through an effective full-employment policy, would make classical theory "come into its own again." In the present context this means that in the absence of short-run variations of aggregate employment and output, internal adjustments have to take the Ricardian form of shifting the factors of production among the sectors—but this must now occur under structural conditions in which the Ricardian
The premise of perfect mobility has been superseded by the specificity of most inputs, not to mention the monopolistic and monopsonistic obstacles to speedy adjustment.

If this diagnosis of the adjustment problem under "regulated" capitalism is correct, the need for a model which takes account of the sectional differentiation of the process of production can hardly be denied. As we have seen, the task is not one of merely adding more variables to the Keynesian set. Nor is it one of disaggregation in general. In the latter case Leontief's input-output table would present the ideal solution.¹ There, in conformity with the Walrasian model of general equilibrium, the physical interrelationships among all the elementary sectors of production, transportation, distribution, and consumption have been registered, with their quantitative interdependence described for particular years in the American economy. For the solution of practical problems of comprehensive planning and "programming," a detailed schema of the Leontief type seems indispensable. But for general analytical purposes its greatest advantage, the high degree of disaggregation, turns into an obstacle.

One reason for this is that it is extremely difficult to trace the dynamic path for such a large number of variables, and to impute their behavior to specific causes, especially if they are exposed to several stimuli simultaneously. For another thing, a study of the most typical dynamic processes—changes in aggregate demand, in the supply of labor, natural resources, savings, and money, and in the technique of production—makes desirable a higher degree of aggregation, in order to throw into clearer relief the pertinent

features of the physical structure of production. Certainly, specificity of inputs and outputs creates adjustment problems ultimately for each individual firm. But the ensuing frictions become relevant for the stability of the system only when their size and locus exert a marked effect on the strategic variables, such as aggregate income and its distribution, spending and hoarding, consumption, saving and investment. Thus the task consists in choosing a level of aggregation between those chosen by Keynes and Leontief—high enough to permit analytical manipulation of complex dynamic processes, low enough to reflect those physical properties of an industrial market which affect its general stability.

As a matter of fact, a model of the productive process satisfying these requirements has been available for well over half a century. But it has been stowed away in a place where academic economics, at least its Anglo-American brand, is unlikely to search for conceptual enlightenment. I refer to the schema of production, more precisely of reproduction, which is contained in the second volume of Marx's *Capital.* Inspired by Quesnay's "tableau économique," Marx's schema is the only comprehensive macro-economic model of the industrial process of production established before Keynes.

This claim may seem exaggerated in view of the so-called Austrian model of the structure of production, which was first expounded by Eugen von Böhm-Bawerk, only six years after the publication of the second volume of *Capital.* Through the work of Wickert, and the studies of von Mises and von Hayek on money and business cycles, the Austrian concept has had great influence on Anglo-American economic thinking. But as I shall presently

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* See his *Positive Theory of Capital,* first published in 1891, in Jena (Book II, Section IV).
have occasion to point out more precisely, the Austrian schema depicts only a certain segment of the industrial process, a segment, moreover, which Marx discusses at length in the *Theorien über den Mehrwert* (pp. 198-231). In fairness to Böhm-Bawerk’s contribution it should be mentioned, however, that the *Theorien* was published only in 1905, though Marx wrote it, according to Engels’ testimony, between 1861 and 1863. In this context a word of acknowledgment is due to John Bates Clark, who as early as 1899 gave a brief description of the flow of production which exhibits in essence all the basic features of Marx’s schema.¹ What prevented Clark from drawing the full benefit from his model is the fact that he used it as an expository device rather than as a tool of analysis.

The credit for having synthesized the original ideas of Marx and Böhm-Bawerk in an overall presentation of the problem belongs to F. A. Burchardt.² His study corrects some errors in Marx’s exposition, in particular his confusion of stocks and flows, and adds all that is valuable in Böhm-Bawerk’s version. Though mainly concerned with the static aspects of a schema of production, Burchardt leaves no doubt about its relevance as an instrument for dynamic analysis. In what follows I shall draw heavily on Burchardt’s essay, in the original drafting of which I was privileged to participate. I shall concentrate, however, on the positive solution of the problem under investigation, refraining from all polemical digressions. In particular, I shall not go into my reservations regarding the Austrian concept of the structure of production, an extensive critique of which can be found in Burchardt’s essay.

¹ John Bates Clark, *The Distribution of Wealth* (New York 1899) Chapters XVIII-XX; see also his *Essentials of Economic Theory* (New York 1924) Chapters IV, V, XV.
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III

If at any point of time we were to undertake a census of an industrial community's wealth in physical terms, we would find altogether four different stocks: a stock of labor; a stock of natural resources; a stock of equipment, such as plants, machinery, residential buildings; and a stock of other commodities, which shall presently be discussed specifically.

We would find these stocks distributed among a number of productive units, whose activity consists in converting, through certain technical processes, portions of some or all of the stocks into new commodities. This yields us the most general definition of "production," in the technical sense of the word—the only sense in which we are interested at the moment.

It will prove useful to subdivide these stocks in four progressive steps, as follows:

First, we can distinguish between "original" stocks—labor and natural resources, which are the ultimate data of the productive process—and "manufactured" stocks, represented by the last two types of commodities included in the list.

Second, concentrating on the manufactured stocks in particular, we can subdivide them according to the degree of "completion." Unfinished and finished commodities are easily distinguished, with the latter defined as those that are "ready for use." It is conventional to subdivide the group of unfinished commodities further into raw materials and semi-manufactured goods, but there is no easy criterion for this distinction. Since production as a technical process consists of an unbroken chain of operations, any caesura made in order to establish definite "stages" of completion is arbitrary. This will have to be kept in mind when presently, for reasons of exposition, we devise a "stage" model. As for transportation and commerce, it is convenient to include these activities in our definition of production, and to regard a commodity as completed only when it has reached its final place and time of use.

Third, narrowing down our field of vision still further, we can
classify the group of finished commodities according to their specific use as either consumer goods or equipment goods. The former are used up in the households, whereas the latter are used up during the process of production. (The term "used up" defines precisely what happens to equipment and consumer goods in distinction to what happens to the stock of unfinished goods, which are "used" but not "used up" during the process of production.) It must be stressed that the physical appearance of a commodity does not always clearly indicate in which group it belongs. The obvious example is coal. It can serve, on the one hand, as a consumer good in the household, on the other hand as an auxiliary material and even as an unfinished commodity (chemical industry) in the productive process. In any concrete case it should be possible, however, to establish the function of a good unambiguously.

Fourth, our final distinction refers to what figures in the literature as the difference between "durable" and "non-durable" commodities. The terminology is not a particularly useful guide to what the distinction is meant to convey. It is true that the number of services a finished commodity is capable of rendering is of great importance for the study of dynamic processes; this is true, for example, of most equipment goods. But some of the essential problems that arise in this connection should be treated from the point of view of "divisibility" rather than "durability." Divisibility as an economic concept depends, however, on the prevailing terms of purchase or hire, no less than on the technical properties of the good in question.

A distinction which takes account of these latter complications is that between goods bought with "income" and goods bought with "capital," in the sense of savings funds and business funds. This classification is especially helpful for deciding on certain borderline cases. I refer to residential buildings and a number of other "durable" and "semi-durable" goods like automobiles which, though intended for use in final consumption, are as a rule...
financed by means other than current income. They should be treated like equipment goods, and only their services entered on the side of consumption. And also certain auxiliary materials—like coal applied as industrial fuel, or lubricant oils, which are used up in one single act of production—though certainly non-durable in the conventional sense, are, because of their economic functions, preferably classified with productive equipment, which is what our definition does.10

Thus far we have dealt with the types of goods and services that make up the physical wealth of a community. It will also be necessary for us to distinguish the productive units in which these goods and services are combined, or from whose productive activity they emanate. In this respect our fundamental distinction is between consumer-goods and equipment-goods industries. Above we have confined the terms "consumer good" and "equipment good" to the finished products ready to be "used up." In now classifying the two basic industrial groups, it is convenient to extend the meaning of these terms so that they include all stages of production that lead up to the finished goods. In other words, our category of consumer-goods industries includes not only those productive units that actually "complete" the finished good, but all other units preceding the final stage in the process of completion and thus producing unfinished goods which are to mature into finished consumer goods. The same considerations are valid for our definition of equipment-goods industries.

Since the physical distinction between the goods belonging to the two groups is blurred, it cannot be expected that the respective industries are clearly divided up. On the contrary, certain industries, like coal mining or chemical production, belong to both categories, and it is again only the concrete situation which, revealing the purpose, can tell us where to draw the exact dividing line.

Among the equipment-goods industries I propose to distinguish

10 See the discussion of this category in Burchardt, op. cit., pp. 122, 127-28, where certain minor problems regarding it are satisfactorily disposed of. I propose to disregard them in the subsequent exposition.
between those that produce equipment to be applied in the production of consumer goods, and others that produce equipment for the equipment-goods industries themselves. Once more it is not postulated that this distinction is always reflected in the actual differentiation of the business units that make up the equipment industry as a whole. But for certain analytical purposes it is important to subdivide at least the output of finished equipment goods according to these two categories.

We now move on from the consideration of stocks to that of the relevant flows. A continuous flow of production requires that the industrial groups, the various stages of completion into which they are subdivided, and the portions of each of the four stocks of goods and services assigned to the individual productive units in every stage of every group, are arranged in a definite order. This order changes, as a rule, with changes in the technique of production. But given a certain technique of production, the relative size and the sequence of the groups and stages are determined.

This has never been contested for the size and sequence of the stages, and there the conceptions of Marx and Böhm-Bawerk coincide. Such an order of stages can easily be described, as shown in the accompanying model, in which N stands for the stock of labor, F for the stock of fixed-capital goods available at each stage (though in Böhm-Bawerk’s and Hayek’s models, F does not appear among the factors of production), and R for the stock of natural resources. Capital letters are used to indicate stocks, irrespective of how much of each stock enters the actual flow of production during any given period of time. Flows are symbolized by lowercase letters. The arrows signify no more than the causal relationship between stocks and inputs on the one hand and outputs on
the other hand, and at this stage of our analysis do not express any definite quantitative order. Similarly, the plus signs symbolize the technical fact of "combination," rather than any numerical addition; since $N, F,$ and $R$ represent stocks, whereas $w$ represents a flow, summation in the proper sense is obviously impossible.

Our stage model contains only one set of flow magnitudes: several items of working-capital goods, denoted by $w$. In the first stage this symbol appears only once, indicating the output of that stage. In each of the subsequent stages, however, it appears twice, both as an item of input and as an item of output. And at any stage the input $w$ is always of the same kind (though not necessarily of the same quantity) as the output $w$ of the preceding stage. This arrangement manifests the technical meaning of the process of production as the progressive transformation of the "passive" factor—natural resources—with the help of the "active" factors—labor and fixed-capital goods. In this process the factor $R$ plays its main role in the first stage, but since every plant has to have a location it cannot be eliminated from the later stages. Working-capital goods are in this sense nothing but, partially transformed natural resources, which in the final stage take on the form of the finished good—in our model, a finished consumer good.

To assure continuity of production, two conditions have to be fulfilled. The active factors have to operate in a predetermined sequence on the passive factors, among which we may now count, besides natural resources, also the inputs of working-capital goods. In addition, the stocks of active factors and of natural resources have to be continuously replenished in proportion to what they release into actual production. With regard to the renewal of the stocks of labor and natural resources, economic analysis has nothing to say. Except in a slave economy, in which labor would have to be treated as a fixed-capital good, a continuing supply of labor and natural resources belongs among the "data" of the economic process which will be taken for granted in our further exposition.

The case is different with regard to fixed-capital goods, and it
is at this point that the models of Marx and Böhm-Bawerk part company. Fixed equipment is as much the result of natural resources, gradually transformed with the help of labor and fixed-capital goods, as are dresses or hats. Therefore by changing the quality of the inputs in our stage model, we can arrive at a sequence of, say, ore-pigiron-steel-machinery, which illustrates the process of production of a fixed-capital good.

But we see at once that this procedure does not solve one basic difficulty. In adding a second stage model illustrating the output of fixed-capital goods, we tacitly assume the existence of further fixed-capital goods which can function as input factors in the production of such goods, and we seem to be involved in an infinite regress. To avoid this dilemma we might look for a kind of fixed-capital good which is produced with the help of labor and natural resources only. Obviously this would take us back to the beginning of history, when primitive man first picked up a stone to use it as a tool. This not only would teach us little about economic history, but would utterly fail to explain how, on our level of industrial civilization, the existing stock of fixed-capital goods is continuously renewed in the technical sense, not to say expanded, if we assume that the available stock of equipment is fully utilized.

Burchardt (pp. 137-40) has drawn attention to a more elementary situation which poses the same problem. We meet it in agricultural production, even under the most primitive conditions. How can the output of, say, wheat be maintained? Disregarding tools altogether, we require for this purpose labor, land, and seed wheat. A constant yield is assured only if part of the final product, wheat, is allotted every year not to consumption but, in the form of seed wheat, to the production of the next crop. And if the next crop is to be increased over the present one, the ratio of the productive use to the consumptive use of wheat has to be raised.

What makes this example illustrative for our problem, and points the way to its solution, is not the acts of gross saving and net saving as such, but the technical precondition on which they
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rest. The required shift in use is possible only because wheat and seed wheat are physically identical goods. Thus the primary condition for the economic reproduction of wheat is its physical capacity for self-reproduction.

The lesson is obvious. Only if we succeed in discovering in the realm of fixed-capital goods certain instruments which share with wheat the capacity for physical self-reproduction can our problem be solved. In other words, we have to look for a type of equipment which is technically suited to produce other equipment as well as its own kind. What we find, as a matter of fact, is not one single instrument, but the comprehensive group of instruments which are classified as machine tools. They are for industrial production what seed wheat or the reproductive system in animals represents for agricultural production. They form an indispensable part of input whenever an equipment good, including machine tools themselves, is to be produced. And in the industrial sphere, as is the case in the agricultural sphere, saving and investing rest on a technical foundation: the fact that the existing stock of machine tools can be used for the production of new machine tools.

IV

We have now collected all the basic elements with the help of which the main physical-technical conditions for a continuous flow of industrial production can be described. I propose first to illustrate the resulting model by a diagram, and then to present in algebraic form certain important relations among the variables. In the accompanying diagram of the model the essential features are as follows, if we disregard the arabic numbers for the time being.

The order of production has been vertically divided into the two elementary groups, equipment goods industries and consumer goods industries, henceforth denoted respectively by the Roman numerals I and II. To simplify presentation, Group II is drawn on a smaller scale than I, and is represented by one good only.
Group I, on the other hand, has been subdivided into 1a, which produces the fixed-capital goods applied in Group I, and 1b, which supplies Group II. The division is technically relevant only for the last stage, where the productive process in Group I divides into different types of finished goods. All groups are horizontally divided into four stages, representing the successive maturing of natural resources into finished goods. Again for the sake of simplicity, the number of stages is made the same in each group.

On both sides of the schema are recorded the active factors—labor, fixed-capital goods, and natural resources—the continuous application of which maintains the process of production. In each main group these three factors appear at each stage. Again they are denoted by capital letters (N, F, R), to show that we are dealing with stocks. For the fixed-capital goods the particular physical form in which these goods appear is indicated.

All other magnitudes measure flows—input and output flows—over a certain period of time. The input flows are the specific amounts of the stocks of active factors which enter the productive process at each stage during the given period; these are indicated by the small letters n, f, and r, representing, respectively, the hours of labor, the wear and tear on the fixed-capital goods, and the amount of natural resources which constitute the inputs of N, F, and R over the period. The output flows are represented by the shaded rectangles, which show, in each stage, the outputs produced by the inputs of active factors in combination with the working-capital goods of that stage.

It has already been mentioned that, from the technical point

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21 The Marxian schema depicts only the two main groups, a procedure to which also Schurardt adheres. As will be shown below, however, the path of many dynamic processes is determined by the relationships which exist between Ia and Ib and between Ib and II. I first pointed out the need for a tripartite schema in my paper "Wie ist Konjunkturtheorie überhaupt möglich?" in Wirtschaftliches Archiv, vol. 24 (1926) pp. 165-97. It need hardly be stressed that the tripartite schema discussed in the text must not be confused with another tripartite schema, which L. von Bortkiewicz used in his famous critique of the manner in which Marx transformed "values" into prices. A lucid survey of this discussion is contained in Sweezy, op. cit., Chapter VII.
of view, the number of stages and the points of separation between two stages are arbitrary, and this is true also of any attempt to construct a total from a combination of all stages. Certainly such a total must not be mistaken for a measure of the "flow of working capital" over a given period, a concept that will be examined more closely in the last section of this paper. To be sure, an exact meaning can be attached to the sum total of all stages if we interpret the order of stages as an indicator of business differentiation. Then the total measures the aggregate exchange transactions that occur during the process of transforming natural resources into finished goods. But this magnitude, though of great importance for monetary analysis, has no relevance for the physical-technical aspect of production.

The situation is different with the concept of a stock of working-capital goods. Such a stock must indeed be available continuously if the flow of production is to be continuous. It is not immediately visible in our schema, but it can easily be derived from its elements. To do so we had best start from the moment when the production flow is generated for the first time. To set it in motion, a certain amount of active factors has to be combined with a certain amount of natural resources in the first stage. The resulting output moves to the next stage, to be combined there with another set of active factors, while at the same time a second set of such factors appears in the first stage—and so on until after a certain lapse of time the first quantity of finished output becomes available. From then on, every additional unit of input is matched by a unit of output; but an amount of input equivalent to that accumulated in the various stages during the period of gestation remains permanently in the system so long as production continues in its original dimensions.

It is this amount of output of intermediate goods built up during the gestation period which measures the permanent stock of working-capital goods in a functioning system. This permanent stock depends on what can be called the period of maturation, that is, the average time it takes to transform a unit of natural
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resources into a finished good, and on the aggregate input of active factors which enter the system during this period. It is not easy to determine the period of maturation statistically; conceptually the term is unambiguous.13

V

The purpose of the schema is to represent the process of production as a number of interdependent flows which are themselves related to the basic stocks of the system. If we concentrate, to begin with, on Group II, we find there a vertical as well as a horizontal flow. Vertically, natural resources flow in progressive transformation down to the level of finished consumer goods. But the continuity of this vertical flow is assured only by a horizontal flow, from I to II, of certain amounts of fixed-capital goods—large enough at least to replace the fixed-capital goods used up in every stage during the vertical flow occurring in Group II.

The vertical flow is the same in Group I. At first glance there is, however, an important difference with regard to the horizontal flow. In Group I the currently used-up equipment is replaced from the output of that group itself—more precisely, from the output of Ia. Thus the latter horizontal flow—which is as necessary for the continuity of production in I as the above-mentioned horizontal flow is in II—consists of a portion of the vertical flow which "turns back"; that is, it is a circular flow.

This apparent difference between I and II can be resolved by a simple postulate, which has an analogy in the classical definition of "gross produce." Just as the output of I currently replaces the

13 Denoting by \( m \) the period of maturation, by \( p \) any period over which we wish to measure output, and by \( A \) the aggregate input of active factors during \( p \), we have for the stock of working-capital goods \( A_m/p \). The factor \( z \) enters the denominator because the stock of working-capital goods equals the successive factor input during the period of maturation, which is half the simultaneous factor input that occurs over that period in a continuous flow. The problem has been discussed by D. H. Robertson in his famous analogy with a "sausage machine" (Money, Cambridge, Eng. 1938, Chapter V, paragraph 7), and was earlier considered by Böhm-Bawerk (op. cit., Appendix XIV).
used-up fixed-capital goods, the output of II can be interpreted as the replacement of the input of the labor factor. Then we have, besides the vertical flows, a circular as well as a horizontal flow in each major group. In II the horizontal flow replaces the worn-out fixed-capital goods, whereas the circular flow restores the labor stock operating there. Conversely, in I the circular flow restores the stock of fixed-capital goods, whereas a horizontal flow from II replaces the labor used up in I.

Thus understood, any process of production, stationary or dynamic, consists in combining over a certain period a portion of the available stock of original factors (labor and natural resources) with a portion of the available stocks of fixed-capital and working-capital goods. Then all that production can achieve is to increase or decrease or keep constant, with the help of the free gifts of nature, the existing stocks of labor and capital goods. From this point of view the "purpose" of production can be seen in the maintenance or change of the productive capital of the system—a technological purpose. And this view can be taken with the same logical right with which we customarily establish a sociological purpose—relating to the maintenance or change of the "human capital"—when we make consumption the final criterion. As a matter of fact, from the point of view of economic accounting, both interpretations are defective, because they obscure the circular nature of the productive process.

It is the failure of most models based on the Austrian concept of

33 At this point I should make explicit a simplification which has been introduced into the schema for didactic purposes only. I have confined the number of "income-receiving" factors to two: labor and natural resources. In what follows I shall go even farther and treat natural resources as free gifts. All this, of course, is tenable only for a stationary process, and even then prevailing opinion insists on regarding interest as a positive magnitude. And once we pass on to dynamic conditions, the possible sources of net receipts multiply. The difficulty can be resolved, however, if we redefine the symbol N as standing for all "income-receiving" factors. The same arrangement makes it possible to dispense with the surplus-value item, which plays such a central role in Marx's formulation of the schema. In a stationary flow, surplus value represents if its existence can be demonstrated at all, only a specific form of income devoted to the purchase of consumer goods. The manner in which the saving and investing of parts of income affect the schema will be indicated below.
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the structure of production that they disregard the circular flows, concentrating rather on the linear ones. There is indeed within each group one linear flow, from the first down to the last stage. But to keep industrial production functioning, no less than five circular processes are required, in addition.

Two of these arise from the fact that within each major group one circular flow channels part of the finished output back to the stock of factors employed in that group: in Group II, consumer goods back to the labor employed; and in Group I, fixed-capital goods back to the equipment employed.

Two others arise from the fact that between the two groups circular flows channel part of the output of each group into the other: consumer goods flow to the stock of labor employed in I; and fixed-capital goods flow to the stock of equipment employed in II.

Finally, there is a circular flow within I, for which there is no parallel in II. More precisely, it concerns Ia, where machine tools and the equipment used in I are continuously replaced. From the purely technical point of view, as we have seen above, this last circular flow is the most important. It depicts the act of self-reproduction without which a continuous productive process, not to mention its expansion, would be physically impossible. In the labor factor it has its equivalent in the bio-sociological organization of man—which, however, is part of the schema of production only in a slave economy.

If the schema is to serve as a tool for dynamic analysis, we have now to transform it from an expository device into a set of quantitative relations. Both the linear and the circular processes lend themselves to such transformation. We shall concentrate here on the circular flows, that is, on the interrelations between the finished outputs of the groups and subgroups—and this for two reasons. On the one hand, modern theory, in spite of its stress on the distinction between consumption and investment, has paid
little attention to certain intergroup relations which prove requisite for equilibrium, stationary or dynamic. On the other hand, the answers to certain basic problems in business-cycle theory, as well as in the analysis of growth, hinge on an understanding of these very relations.

Our first step will be to consolidate the productive results of our successive stages, thus obtaining aggregates of group outputs. For this purpose we shall focus on the relationship which exists, in each group, between the aggregate output and the aggregate input of active factors in all stages during the period of observation.

In the following algebraic formulations the output of Group Ia has been designated as a, and that of Ib as b, while for the output of Group II the letter z is used (this choice of letters is intended to emphasize the technical propinquity of the outputs in the subgroups of I). The n, f, and r notations stand for the aggregate factor inputs (labor, fixed-capital goods, and natural resources, respectively) in all stages of the specified group, the relevant group being specified by enclosing its symbol in parentheses; thus n(a) signifies the labor input in Group Ia, and f(z) the input of fixed-capital goods in Group II. Successive periods of observation are designated as \( t' \) and \( t'' \).

In the three expressions numbered (1) we start from the most general causal relationship. The symbols on the right side of the arrows stand for the “progress in completion” which takes place during the period \( t' \) in the production of equipment goods (a) intended to make other equipment goods, equipment goods (b) intended to make consumer goods, and, finally, consumer goods themselves (z). These expressions, however, tell us only that the outputs are the effects of the inputs.

Much more can be said if we postulate continuous production; then a, b, and c assume the exclusive meaning of finished output
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in the respective groups, and we have the (2) set of equations (here, and in what follows, it is assumed that natural resources are free goods). These equations mean that the output of finished equipment (a and b) during one period must, at least, suffice physically to replace the fixed-capital goods worn out during the preceding period; and that, in accordance with our previous interpretation, the same relationship exists between the output of finished consumer goods (2) and the preceding input of labor. The (2) equations are strictly valid only over a period which covers the full life of the available stock of fixed-capital goods. During any period shorter than that it is possible, of course, to continue production without actual replacements. We also have to disregard the existence of liquid capital, that is, inventories of fixed-capital goods, working-capital goods, and consumer goods.

These qualifications vanish if we postulate stationary equilibrium. Then the finished output of Group Ia—henceforth called primary machinery—must, in any period, be physically equal to the equipment input in Group I as a whole during the preceding, as during the subsequent, period. A similar equality has to prevail between the output of Group Ib—henceforth called secondary machinery—and the equipment input in Group II, and also between the output of Group II and the absorption of consumer goods on the part of all income-receiving factors. But—and only this clarifies the physical meaning of the above equations—it is not enough that finished equipment goods and consumer goods are produced in amounts sufficient, and in technical forms adequate, to replace the wear and tear on the stock of factors applied in the preceding period. Parts of the finished output of both groups must, as it were, change places with one another in the order of production. It is a situation analogous to that in foreign trade when each of two countries exports a certain surplus output not
required at home. Here, as there, the condition for the "exchange" is that the physical form of the "exports" satisfies the requirements of the "importing" group.

In the present context the first condition for stationary equilibrium is that the current output of secondary machinery must be exchanged for that part of current consumer-goods output which is not retained for the next period's consumption in Group II itself. In symbolic terms—and using \( q(t) \) to denote the ratio of "domestic" consumption to total consumption—we can say that \( b_r \), which is equal to \( f(c)_r \), must be exchanged for \( z_r - q(t)z_r \). And analogously, that part of the output of primary machinery which is not required for the continuation of production in Group Ia has to replace the wear and tear on equipment goods in Group Ib—though here the goods "given in exchange" do not consist of the output of Group Ib itself. As we know, this output, consisting of secondary machinery, has to be wholly transferred to Group II. The goods given "in payment" from Ib to Ia consist of consumer goods intended to satisfy the income receivers in Ia, and they are acquired by Ib from II through the exchange process just described. Group Ib retains what is necessary to satisfy income receivers there, and exchanges the rest with Group Ia for primary machinery. For this process the symbolic expression is that \([a - q(a)z_r] + f(b)_r\) exchanges for \(z_r - [q(b) + q(t)]z_r\).

Thus the "balance of trade" is established for Ia by the exchange of primary machinery for consumer goods, and for II by the exchange of consumer goods for secondary machinery. Group Ib first balances its trade with Group II by exporting secondary machinery for consumer goods, and afterwards uses part of its imports for "reexport" to Ia.

Thus far we have concentrated exclusively on the physical-technical aspects of the group relations. But it is obvious that we cannot speak of "exchange" between aggregates of goods unless there exists, apart from their physical compatibility, an equality in value terms. To repeat once more, a purely physical-technical schema of production can yield very important insights into the
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qualitative interdependence and direction of a number of sectional input-output flows—insights to which there is no access in the dimension of values alone. The fact remains, however, that the size of the flows in question, and their equality or inequality in economic terms, cannot be determined unless we make physically heterogeneous quantities comparable in terms of prices.

The arabic numbers attached to our original model are an attempt at such a translation of the physical magnitudes into a value schema. They describe, as a basis for dynamic analysis, a stationary flow. As period for the measurement of the flows, the calendar year has been chosen, to which an average depreciation rate of 10 percent has been applied. Moreover, it is assumed that in each group each of the two priced factors—labor and equipment—makes the same contribution in each stage. Finally, the system is a closed one. Needless to say, the schema is in principle capable of reproducing a much more highly differentiated order of production, especially in the consumer-goods group, and of including also a government sector and interrelations with other systems.

The order of the annual production flow can now be represented by the equations in sets (3) and (4), and the two “exchanges” noted

\[ n(a)_r(8) + f(a)_r(z) = a_r(10) \]
\[ n(b)_r(32) + f(b)_r(8) = b_r(40) \]
\[ n(z)_r(160) + f(z)_r(40) = z_r(200) \]

\[ a_r(10) = f(z)_r(z) + f(b)_r(8) \]
\[ b_r(40) = f(z)_r(40) \]
\[ z_r(200) = n(a)_r(8) + n(b)_r(32) + n(z)_r(160) \]

\[ f(z)_r(40) = b_r(40) \]
\[ = z_r(200) - n(z)_r(160) \]
\[ = n(z)_r(8) + n(b)_r(32) \]

\[ f(b)_r(8) = a_r(10) - f(a)_r(z) \]
\[ = z_r(200) - n(b)_r(32) - n(z)_r(160) \]
\[ = n(a)_r(8) \]
above take the form of the equations in sets (5) and (6). The (3) equations can be called "group cost-price" relationships, since they express the value relation between aggregate inputs and outputs for the technically relevant groups. The (4) equations, establishing the conditions under which production is continuous in the relevant groups, can be labeled "group reproduction" relationships. The same name may be given to the (5) and (6) equations, which describe the reproduction process among the groups with reference to both simultaneous and successive relations between priced physical aggregates.

These conditions for a stationary process of reproduction can be briefly stated as follows. The value of secondary machinery required per period in Group II must equal the output of such equipment produced previously in Group Ia, but must also equal the value of a definite amount of consumer goods—that is, the total previous output in Group II minus that group’s present requirements, which must equal the amount at present demanded by all income receivers in Group I as a whole. Furthermore, the value of primary machinery required per period in Group Ia must equal the surplus of such equipment produced previously in Group Ia over and above that group’s own present requirements, and must also equal the value of consumer goods at present demanded by the income receivers in Group Ia. This latter magnitude, in turn, must equal the total previous output in Group II minus the present requirements of consumer goods in both II and Ia.

This formulation points to a characteristic of the schema which comes into full evidence only in its algebraic formulation: the threefold meaning of each term as a measure for input (or output), for receipt, and for expenditure. In the “cost-price” equations the n and f variables, for example, have to be interpreted as inputs of a physically distinct nature in every group, but at the same time as the money receipts which accrue to the “holders” of the respective stocks in return for their input services. In other words, n and f represent, respectively, specific quantities of “labor
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bestowed" and specific pieces of equipment, as well as the wage sums and amortization paid in return. In the "reproduction" equations the same symbols represent the expenditure on consumer goods and equipment goods respectively. The a, b, and z variables, on the other hand, denote, in the cost-price equations, physically distinct outputs as well as receipts from sales, whereas in the reproduction equations they measure certain aggregates of expenditure.

This "ambiguity" has a parallel in the Keynesian model, which also has a "supply" as well as a "demand" meaning. In the first interpretation we deal with aggregate output, consumer-goods output, and investment-goods output. The second interpretation gives us aggregate income, divided into that part which is spent on consumer goods and another part called savings, which is spent on investment goods. Far from prejudicing analytical clarity, this change of meaning of the basic variables in successive transactions only emphasizes the circular nature of the exchange process.

VII

What now is the significance of the schema of production for dynamic analysis? To answer this question we have to supplement the formal exposition of the schema in two directions. On the one hand, we have to demonstrate that our model is capable of reproducing all relevant processes of change in a manner accessible to exact calculation. On the other hand, we have to show that the schema enables us to solve problems which prove refractory under conventional treatment. The present section of this paper is to serve the former purpose. In the concluding section certain specific topics will be discussed which the schema is particularly well adapted to clarify.

The price-cost equations and reproduction equations established above describe stationary equilibrium. An important step is to transform them into a picture of dynamic equilibrium, in which all data vary proportionally; this will be discussed in the next section. Here we focus our attention on dynamic disequilibr-
rium, that is, on disproportional changes in the data and the resulting processes of adjustment. For all conceivable processes of this kind the schema can serve as a tableau économique. The data changes themselves, the quantitative variations through which the system adapts itself, and finally the new state toward which the system gravitates—all these can be read in typical changes of the priced physical magnitudes composing the schema.

Looking first at the relations that prevail among the different flows, we can discern the effect of three types of data changes. There are, to begin with, shifts in the quality of demand, following either shifts in taste or such changes in technology as induce the substitution of one kind of machinery for another. These variations are reflected in shifts in factor employment and production flow within either the consumer-goods group or one of the two subgroups of the equipment-goods group. To calculate the effects of such changes we must, of course, disaggregate our group equations into price-cost and reproduction relationships valid for the individual industries in question. We can do so by elaborating on the stage model presented in Section III above. This yields us, for the industries concerned, sets of equations depicting concomitant shifts among those stages of production that are implicated in shifts in demand for the final goods.

The same stage model can serve for the analysis of changes due to depletion of, or decreasing returns from, natural resources. There the shift concerns the transfer of labor and equipment from the later to the earlier stages within a single industry. If we know the size of the original output of final goods in any industry, and the distribution of factors over the different stages of production, we can devise a simple formula with the help of which the fall in output and the new distribution of factors can be calculated for any given fall in returns.

Finally, all technical changes that alter the ratio of the technical coefficients for labor and equipment—movements along, as well as shifts of, a given production function—express themselves as relative changes in factor employment and production flow among
the main groups. If the technical coefficients vary in the production of equipment, a similar change occurs also among the sub-
groups of the equipment group as a whole. Since the schema
registers, besides the values, also the quantities of the different
inputs and outputs, the analysis can be elaborated further. We
can calculate, for any given factor distribution, the residual amount
of factors which a special change in productivity displaces in one
group or subgroup, and which has to be transferred to another
group before equilibrium of output and employment can be
restored.

The schema, however, is not confined to the reproduction of
flows. It also depicts the magnitudes of the basic stocks, and the
relations and possible variations among stocks and flows, in terms
of both values and quantities. This makes it possible to apply
the schema to a step-by-step analysis of the most important dynamic
phenomena: changes in labor supply and in saving, and those
technical variations that are accompanied by a change in the stock
of fixed or working capital. In all these cases a change of some
flow magnitude (rate of population increase or of saving, labor
input or capital input per unit of output) induces a change in
the capital stock or its utilization, via a change in the current output
of Group Ia relative to the output of Groups Ib and II. Unless
the initiating change is continuous, a backward shift of factors
from Group Ia to Groups Ib and II has to occur, once the capital
stock has been expanded to the new equilibrium size. We know
from the discussion of the acceleration principle that this back-
ward shift may lead to the abandonment of part of the newly built
capital stock. Analysis with the help of the schema enables us to
establish the precise conditions under which such "waste" occurs,
and to calculate its size. If additional assumptions are introduced
concerning demand and supply elasticities, we can also take into

14 In other words, one group "broadens," whereas another group "narrows" proportionally. The notions of "lengthening" and "shortening," basic in the Austrian theory of capital, can be reasonably applied only to changes in the period of maturation, that is, to changes in the stock of working-capital goods relative to other stocks of factors—a type of change to which we shall refer presently.
account the concomitant changes in the relative prices of commodities and factors.

This survey of change processes amenable to schema analysis is not meant to be exhaustive. But a word should perhaps be added concerning so-called endogenous changes, which arise from lags between some of the variables and from variations in such lags, independently of any change in the variables themselves. As was shown above, by the (4), (5), and (6) equation sets, the schema lends itself particularly well to sequence analysis. Moreover, it contains all the variables that have proved relevant in lagged processes, such as receipts, expenditures, inputs, and outputs, and in a form which is from the outset “dated.”

VIII

It is obvious that the actual application of a tool can prove its usefulness more convincingly than the most elaborate methodological considerations are able to do. Therefore I propose in conclusion to employ the schema for a more detailed discussion of a few controversial issues. Of course, all that can be attempted in the framework of an article is an indication of the particular services which the schema can render over and above the contribution of other models. A systematic exposition must be reserved for a more comprehensive presentation.

The selection of the problems will be made with regard for the two main features that distinguish the schema from other economic models, such as the Keynesian national-income identity or the Austrian concept of the structure of production. These features are disaggregation in accordance with certain physical characteristics of the productive process; and the elaboration of the circular flows which, by linking up the basic groups, assure the continuity of reproduction. As we know from our previous considerations, both peculiarities gain importance with the specificity which, under industrial conditions, attaches to equipment as well as to labor. Over the Marshallian long period, and certainly over the secular period, most specificities can be overcome, and therefore
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the schema appears at first sight most useful for the study of the short-term effects of economic change, in which we have to include business cycles. Even over the secular period, however, a self-propelling system can maintain itself only through the medium of successive short-period adjustments. Hence we shall begin with an example which is to prove the schema's value in solving a problem relevant for both short-run and long-run analysis: the conditions of dynamic equilibrium.

(1) In the late 1930's, when Harrod and Samuelson performed the "marriage between the multiplier and acceleration theories," a new chapter was opened in economic dynamics. It had been the merit of the older acceleration analysis to show that the dynamic path of the system is greatly influenced by the manner in which the rate of change of some magnitudes affects the level of others. But so long as the causal nexus was treated as onesided only—the rate of change of demand for consumer goods being the stimulus, and an exogenous one at that—no self-reinforcing process could be derived from acceleration. Only when, through the multiplier concept, the dependence of consumer demand upon investment was taken into account, could true interaction be established, both for the short and for the long run.

Whereas the older "accelerationists" (Aftalion, J. M. Clark) had been interested mainly in the destabilizing effect exerted on the level of the system's activity by variations in the rate of change of the stimulus, Harrod and, later on, Domar have derived from the interaction analysis an essential condition for dynamic equilibrium. If full utilization of resources is to be maintained through time, the capital stock of the system must grow at such a rate that the current investment necessary for such growth equals current savings provided at full-employment income. Or, what

16 For the similarities and differences between the two approaches see Harrod's paper on "Notes on Trade Cycle Theory," in Economic Journal, vol. 61 (June 1951) pp. 261-75.
amounts to the same, if the average propensity to save (savings ratio) and the average output-capital ratio for the corresponding investment (investment productivity) are given for any period, investment and output will have to rise over that period by a rate which equals the product of the savings ratio and investment productivity.

The above formulation does not contain any assumption as to the stability of the savings ratio or investment productivity. Either factor may change from period to period, and the possibility of such variations only emphasizes the importance of the rule. Quite a different problem arises if the coefficients in question not only change autonomously but also adjust themselves to any discrepancy between the rate of output and the rate of capital formation. If potential "oversaving" causes investment productivity to fall—or, in other words, induces via a fall of the rate of interest a deepening of investment—the above condition of dynamic equilibrium loses its significance. The same is true if any potential "overinvestment" raises the savings ratio, via a rise of the rate of interest. We are then back at the classical mechanism, which, through short-run adjustments of prices and interest rates, maintains the equilibrium of full utilization against any conceivable distortion.\footnote{Comparable objections to the condition of dynamic equilibrium were first raised by E. Stern in his polemics against Domar; see Stern's "The Problem of Capital Accumulation," in American Economic Review, vol. 39 (December 1949) pp. 1160-69. That problem has since been reviewed systematically by William Fellner in "The Capital-Output Ratio in Dynamic Economics," in Money, Trade and Economic Growth: In Honor of John Henry Williams (New York: 1953) pp. 105-34. Fellner also stresses the difference between "changes" and "adjustments" of the critical coefficients.}

The empirical validity of the Harrod-Domar condition can easily be defended against the classical proposition. The latter rests on the customary classical assumption of perfect mobility. When production is not conditional on the long-lasting commitment of large private funds, the risks of changes in output and investment productivity are small, and entrepreneurs' expectations will stimulate equilibrating behavior. But such structural con-

\footnote{Comparable objections to the condition of dynamic equilibrium were first raised by E. Stern in his polemics against Domar; see Stern's "The Problem of Capital Accumulation," in American Economic Review, vol. 39 (December 1949) pp. 1160-69. That problem has since been reviewed systematically by William Fellner in "The Capital-Output Ratio in Dynamic Economics," in Money, Trade and Economic Growth: In Honor of John Henry Williams (New York: 1953) pp. 105-34. Fellner also stresses the difference between "changes" and "adjustments" of the critical coefficients.}
ditions for short-run adjustment do not exist in an industrial
system. There the productivity coefficients, in particular, are
largely fixed over the short period, and the Harrod-Domar rule
becomes indeed a necessary condition for dynamic equilibrium.

Nevertheless—and here we come back to our main subject—
the postulate that output and investment in the aggregate grow
at a rate that equals the savings ratio times investment productivity
is not a sufficient condition. The absence of perfect mobility and
the difficulty of short-run adjustment make it imperative that
definite relations be maintained among the main groups of
production.

Our reproduction equations (5) and (6), above, point to the
additional conditions that must be fulfilled if dynamic equilib-
rium is to persist in an industrial economy in which, for physical-
technical reasons, the quantitative relations between the groups of
production are fixed over the short period. The (5) and (6) equa-
tions refer to stationary equilibrium. In order to serve as con-
ditions for dynamic equilibrium they have to be reformulated to
take care of saving and net investment, as shown in the accompa-
nying sets numbered (5a) and (6a), in which \( s(a) \), \( s(b) \), and \( s(z) \) stand

\[
\begin{align*}
(5a) \quad f(z)_t + s(z)_t &= b_t \\
&= z_t - n(z)_t \\
&= n(a)_t + n(b)_t \\
\end{align*}
\]

\[
\begin{align*}
(6a) \quad f(b)_t + s(b)_t &= a_t - f(a)_t - s(a)_t \\
&= z_t - n(b)_t - n(z)_t \\
&= n(a)_t \\
\end{align*}
\]

for the savings made and invested in the respective groups, and
\( n(a) \), \( n(b) \), and \( n(z) \) now signify only those parts of the respective
incomes which are consumed.\(^{18}\) In this form the equations
implicitly contain also the Harrod-Domar postulate, and therefore
represent sufficient as well as necessary conditions for dynamic
equilibrium. An additional expression in aggregate terms is

\(^{18}\) See also Joan Robinson, "The Model of an Expanding Economy" (cited above,
note 5), p. 46, note 1, and pp. 52-53.
therefore redundant, though for particular purposes that more condensed formulation is useful.10

(2) Our next example has been chosen from business-cycle analysis. It concerns Hicks' concept of a "strong" boom and the mechanism through which it is supposed to turn into a downswing.20

A strong boom in Hicks' sense is created by such coefficients for both accelerator and multiplier as would, in conjunction with autonomous investment, induce an explosive movement were it not for the approach to full utilization of resources, which sets an external limit. Once it strikes this "ceiling," output cannot simply "creep along the ceiling" but "must bounce off from it" (p. 98). The reason for this lies in the fact that up to this point it has been an increase in real output that has determined the rate of induced investment. Henceforth output can rise no more, because of lack of resources, and the previous rate of induced investment can no longer be maintained. Thus aggregate investment is bound to decline, and this in turn must depress output and initiate the downward spiral.

This analytical result is actually based on two special premises, one explicitly stated by Hicks, the other implied. Aggregate output is the equivalent of autonomous investment plus induced investment plus consumption, and any fall in induced investment can, in principle, be compensated by a simultaneous rise in one or both of the other components. Hicks expressly excludes a compen-

10 The fact that, and the reason why, no aggregate formulation can establish sufficient conditions for dynamic equilibrium have been clearly recognized by Fellner, in his "Capital-Output Ratio" essay (cited above, note 17; see especially pp. 116-22). But in the absence of a properly disaggregated model he could point only in literary terms to the additional condition: absence of specific scarcities and immobilities, which may distort a process even if the conditions for aggregate equilibrium are temporarily fulfilled. It is precisely this gap in exact analysis which the schema of production is designed to close. The issues referred to gain importance, of course, whenever the rates of saving and investment change, causing either bottlenecks or partial waste of the existing capital stock in Group Ia. In this respect see also a forthcoming study on models of industrialization by Julius Wyler.

20 See Hicks' A Contribution to the Theory of the Trade Cycle (Oxford 1930) Chapters VIII and X.
sating increase in autonomous investment, and rightly so, if his definition of autonomous investment is accepted (p. 59). From the above conclusion it follows that a similar assumption—that is, a fixed value of the consumption function all through the cycle—underlies his treatment of consumption. This latter assumption, however, can be maintained only if the propensity to consume is as little related to the behavior of induced investment as is autonomous investment. But it can easily be shown that under normal conditions a systematic relationship exists between these two factors. The connecting link is the behavior of the wage rate, which makes the multiplier move inversely with the accelerator.

To clarify this point we must first ascertain the principal cause of the ceiling; that is, we have to identify the particular resource which, for systematic reasons, grows scarce under the conditions described by Hicks. When he first introduces the concept of the ceiling he describes it, referring to Keynes, as one of “full employment” (p. 89). Subsequent references, among them one expressly mentioning population (p. 96), confirm the impression that the critical locus is the labor market. Later on, however, when Hicks inspects the ceiling in detail, he first defines it as “full employment or full-capacity output” (p. 124), and thereafter refers exclusively to “scarce resources” in general (Chapter X, passim).

As we shall see presently, a precise definition of the type of resource which establishes the ceiling is of more than semantic importance. We know from our previous considerations that, apart from labor, this role can be performed only by equipment, more precisely by primary machinery, and it is indeed in this direction that the reference to “full-capacity output” seems to point. I propose to comment on both alternatives, but it should be stated in advance that neither of them is a safe basis for Hicks’ analytical conclusions. If scarcity of labor creates the ceiling, Hicks’ theory of the downturn needs drastic revision. That theory might fare better if scarcity of primary machinery were the obstacle to further expansion. But it will be shown that primary
machinery—leaving aside accidental bottlenecks, which may occur also during a weak boom—cannot be scarce on systematic grounds so long as full employment has not been attained.

Our first proposition is based on the claim that the approach to full employment tends to stabilize rather than to destabilize the system. This can be demonstrated as follows. When the supply of labor grows inelastic all around, money wages are bound to rise. Hicks admits this fact, but all he derives from it is wage inflation, that is, a general rise in prices proportionate to the rise in wages (p. 126), disregarding all secondary effects of the wage rise on the structure of demand and the composition of output. In other words, he neglects the consequences of the change in income distribution which the general shift from profits to wages initiates. It is true that such a change in income distribution will affect the structure of expenditure and output only if the wage-earners' marginal propensity to consume differs from that of the receivers of profit. But on commonsense grounds it is precisely a change of this kind, and thus a rise in the ratio of aggregate consumption to aggregate savings, which we have to expect.

Once this is admitted it follows that the system need not "re-bound" from the full-employment ceiling. What happens to output is a shift in its composition rather than a change in its aggregate. The combined cost and demand effects of the wage rise will promote a transfer of resources from the investment sphere to the consumption sphere. In the former demand must fall, as a result of the fall in aggregate savings—and this in the face of rising costs. In the latter, given a high marginal propensity to consume, the rise in demand which emanates from wage earners in all groups of production exceeds the rise in costs, and thus stimulates expansion of employment.21 If the transfer of resources can be achieved promptly—we shall return to this point presently.

21 It can easily be shown that, in a system approaching full employment, monetary expansion cannot affect this shift in relative demand. We need not pursue this line of thought, since Hicks himself emphasizes the fact that, in the situation contemplated, monetary influences have little effect on the action of the real forces.
—there does not seem to be any systematic reason why the trend toward full employment should be reversed. And if from there on aggregate savings keep in line with aggregate investment, as determined by autonomous forces and the natural growth of the system, the "moving ceiling" becomes the path of a dynamic equilibrium.\footnote{In this connection see also S. S. Alexander, "Issues of Business Cycle Theory," in American Economic Review, vol. 41 (December 1951) pp. 861-76, especially 874-75. Alexander recognizes that "contrary to Mr. Hicks' argument, a downturn does not necessarily follow an encounter with the ceiling," but he fails to relate this fact to the normal order of the events which make up a strong boom.}

We have obtained our result by paying regard to the systematic shifts among the groups of production, whereas Hicks, in Keynesian fashion, concentrates on the forces that affect aggregate output so long as no shifts occur. In other words, we deal here with a problem that can be fully explored only with the help of a partly disaggregated model like the schema. Hicks must have felt this himself, as in his "Further Inspection of the Ceiling" (Chapter X) he devotes an extensive discussion to the differential behavior of prices and output in the investment-goods and consumption-goods industries respectively. But by lumping together Groups 1a and 1b, in the traditional manner, he not only fails to realize the locus of possible shifts, but even is caught in an apparent contradiction. In one context (p. 134) a spurt of demand for consumer goods is supposed to stimulate the production of investment goods, whereas at another place it is higher profits, due to the wage lag—that is, a lag of demand for consumer goods—which is to promote investment (p. 127).

The difficulty disappears as soon as we subdivide investment into primary and secondary machinery. During the upswing the demand effect of the wage lag keeps the output of consumer goods and of secondary machinery within definite limits. Its cost effect, on the other hand, stimulates investment in primary machinery. Here a terminological problem arises. Hicks agrees in the passage referred to (p. 127) that "the higher profits, which are a necessary consequence of the wage-lag, will stimulate investment."
But can we regard such investment as the "induced" type? If response to a preceding rise in output is the sole criterion, we must not do so. On the other hand, as Alexander has shown (op. cit., p. 869), there are good reasons for including in the concept the effects brought about "indirectly via profit, price, and credit movements."

Whether we wish to call such investment induced or autonomous, what matters is its behavior during the boom. Then the wage lag disappears, and demand for secondary machinery rises in proportion to the additional spending of wage earners. At this stage investment, far from contracting in the aggregate, shifts from primary to secondary machinery, and maintains its previous level until the backlog of the demand for the latter is worked off.

This discussion has implicitly shown that, so long as unemployment persists, equipment, and primary machinery in particular, cannot be generally scarce, and therefore cannot create an obstructive ceiling. So long as labor is plentiful, the ratio between wages and profits, and therefore between consumption and savings, is such as to assure a current output of primary machinery which exceeds the current needs for replacement and expansion of secondary machinery. This "lopsided" growth of Group Ia stops only, and indeed reverses itself, when full employment is realized, that is, at the moment when the rate of expansion slows down and the ceiling coincides with the equilibrium level.

Incidentally the schema provides us with an additional argument for the stabilization of a Hicksian strong boom. In telling us that the first systematic shift at the height of the boom is from primary to secondary machinery, it makes us realize that for once adjustment is not impeded by any specificities, as this shift amounts to no more than a different application of the same factors: primary machinery, and the labor force operating it. From the

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23 Hicks has a somewhat cryptic passage (p. 133, note) referring to the effect productive investment must have on the "movement of the ceiling itself," which may perhaps be interpreted in the above sense. If so, however, it should be clear that, contrary to Hicks' surmise, this effect makes a "decisive difference to the results obtained."
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previous task of producing additional primary machinery they shift to the production of secondary machinery. Not until the final stage, when part of the labor force in Group I has to move to Group II, in order to operate the newly produced secondary machinery, will any technical rigidities come into play. Whether at that moment such rigidities will by themselves be strong enough to disturb the otherwise equilibrating tendencies is at least open to doubt. It rather appears that after due qualification Hicks’ hypothesis, like all other variants of the “overinvestment” theory of the crisis, indicates at best a spark for the critical explosion, whereas the truly dangerous powder keg has to be looked for elsewhere.24

(3) Our third and final example is an issue which is not directly related to dynamic problems but has some bearing on their empirical-statistical presentation. It concerns the interpretation of the concept of gross national product. According to present usage, gross national product consists, in addition to the net change of inventories, of the sum total of all finished goods and services made available in a specific period, including those goods which are to compensate for the depreciation and depletion of fixed-capital stocks during that period.

It has again and again been asserted that this definition involves double counting; the reason that is given is that the value of the replacement goods is already contained in the remuneration of those income-receiving factors which produce the replacement goods, and in the equivalent value of goods and services purchased

24 It can be found only in the realm of autonomous investment, in the simultaneous onset of the “Schumpeter effect” when technically superior firms begin to eliminate marginal producers on a large scale. The particular conditions under which this process coincides with the approach to full employment, and other conditions under which it occurs earlier during the upswing, creating a “weak” boom, require, of course, further elaboration. In this respect see my paper on “The Turn of the Boom,” in Manchester Statistical Society, Transactions, Group Meetings, 1937–38 (Manchester 1938) pp. 10–15. See also Hanns-Joachim Rüstow, Theorie der Vollbeschäftigung in der freien Marktwirtschaft (1951) Chapter XI. A similar hypothesis has recently been indicated by Rendigs Fels in “The Theory of Business Cycles,” in Quarterly Journal of Economics, vol. 66 (February 1952) pp. 25–42, especially 27, 41.
by these factors. One glance at our schema shows, however, that this is not so. If we add up the right sides of our (3) equations, we obtain an expression for gross national product in accordance with prevailing usage—that is, the sum of consumer-goods and equipment-goods output. The left sides of the equations tell us that this sum is the equivalent, and no more than the equivalent, of the input of all active factors cooperating in producing output: income-receiving prime factors, and "amortization-receiving" supplementary factors. To impute the contribution of the latter to the value of the input of the prime factors in Group I, and to identify the goods value of this contribution with the consumer goods available to these prime factors, would amount to two untenable propositions.

First, it would imply that the value of all primary machinery equals the value of the input of the prime factors employed in their production—the "Austrian" notion that somewhere in the system goods are produced with labor and natural resources alone. Second, it would disregard the contribution of secondary machinery to the production of consumer goods. It is true that in stationary equilibrium the value of aggregate amortization in Group II equals the value of the aggregate income of the prime factors in Group I which is spent on consumption. But these equal values by no means refer to identical goods. The goods equivalents—equipment goods to replace the wear and tear in Group II, and consumer goods to feed the prime factors in Group I—are not only different but have to emerge from the flow of production simultaneously and side by side. Therefore they must both enter into the calculation of total output.

All difficulties disappear once we look at production in the manner proposed above, that is, as the replacement, and possible expansion or contraction, of the stocks of active factors, rather than

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as the procurement of a particular kind of output, that is, consumer goods and net investment. The latter view, which leads to the current concept of net national product, is useful for many purposes. But it cannot tell us how much of the stock of active factors has been used up in production, or what its output equivalent is. This total can be ascertained only if we adhere to the conventional definition of gross national product.26

Quite a different question might be raised: whether the conventional concept of gross national product is really comprehensive enough to measure all inputs and outputs. As we have seen, it measures the input of all active factors of production, that is, labor and equipment. But as was demonstrated in the exposition of the schema, continuous production depends no less on the maintenance and, in a dynamic flow, on the appropriate change of the passive stock of working-capital goods—a fact clearly acknowledged also by Kuznets (as quoted in note 26, below).

The main reason why this technically indispensable "flow" of working-capital goods is not accounted for in the conventional calculations of aggregate input or output is a practical one: it seems to defy measurement. We have seen that the value sum of all the outputs in successive stages of production cannot serve this purpose. This sum depends on the degree of business differentiation, and varies with it. Except in a fully integrated system it indeed contains duplications in the proper sense of the word, if it is taken as a measure of productive activity. We can expect to arrive at an exact account of all inputs and outputs only if we succeed, at least conceptually, in separating from this value sum

26 Kuznets (ibid., pp. 113-16) is fully aware of the postulative nature of any definition of national income or output which is based on the "goal" of providing goods to households. He expressly states that national income thus conceived "is not a measure of activity, of how much effort, toil and trouble economic activity represents" (p. 115). He even points out, in accordance with the view I have delineated, that, "if no ultimate goal is set to economic activity," net national income is reduced to the value of net additions to the stocks of factors (population and its efficiency, fixed-capital and working-capital goods, and foreign claims). Certainly, then, he should not speak of "duplication" (p. 117) if this net total is supplemented by a gross total in which, besides the "additions" to the stocks, the efforts necessary to their maintenance are counted.
of turnovers the indispensable contribution of intermediate goods to the continuance of the productive flow.

Once more the schema yields the clue to the solution. It was shown in Section IV above that we can measure the stock of working-capital goods necessary for continuous production if we know the input of active factors over the period under observation and the technically determined average "period of maturation." Assuming that the period of maturation equals the period of production over which we measure input and output, the value of the stock of working-capital goods is one-half the value of the input of the active factors (see note 12, above). But this value equals precisely the flow of working-capital goods which moves during the period of observation toward completion and is, at the same time, reproduced in the different stages. If the period of maturation is half as long as the period of production, the stock of working-capital goods equals only one-fourth of the value of the input of active factors. But then, during the period of observation, a stock of this size moves twice toward completion, and has to be reproduced twice. We arrive at the plain result that, over a given period, the indispensable contribution of working-capital goods to output always equals half the contribution of the active factors. The grand total of a "super gross national product" amounts, therefore, to one and one-half times the value of gross national product in the conventional sense.