3. Economic Systems

*The whole is more than the sum of the parts.*

(Aristotle, *Metaphysica* 1045a, 10f)

3.1 Introduction

There is no such thing as description without language. But different languages recognize the existence of different things. That was the point of the previous chapter. The use of the real field as a generalized analytical space (a paradigm, a language) did not enable expression of the concept of connection in much the same way that some natural languages have no verb tense; they function without them, but are expressively and conceptually limited because of that fact. To express the idea of a connection we need a language that has this concept as part of its structure. That language is graph theory.

A number of prominent theoretical biologists and systems ecologists have recognized that graph theory is a universal framework for the study of complex systems (for example, Grøen 1996, Kauffman 1993). An economist, Alan Kirman (1983, 1987b) has too. Graph theory is a powerful and obvious foundation for an evolutionary microeconomics. It enables the representation of a complex object as composed of two primitives: elements and connections. It enables us to track changes in this object as the discrete dynamics in either elements or connections. It enables representation of elements embedded within systems, which are themselves elements in higher-level systems (I refer to this as a hyperstructure). As such, it provides a foundation for the study of the dynamics of networks, hierarchy and, most significantly, of emergence. Finally, graph theory provides the geometric foundation for models constructed in terms of algorithms. I shall also show, as something of an aside, how a graph-theoretic framework can make sense of Aristotle's proposition about summation (at the head of this chapter).

When you learn a new natural language the objective is usually explicit and functional; you struggle for months with French, say, so you can communicate with the French. Graph theory, understood as a language, does not have such a purpose (and nor will it take so long to grasp the basics). Graph theory is a way of thinking, and it is a way of thinking, as I argued in the previous chapter, that most heterodox economists already do. Its rationale, rather, is to sharpen that
thinking, to provide a way of saying exactly what you mean, and, as a secondary objective, to enhance communication both within the heterodox communities and between heterodox and orthodox economists. It is a way of saying—this is what I mean, here is my model. And I would emphasize that this is precisely the same reason that powered the accession of field theory in the first place.

A graph-theory model looks like this: $S = (V, E)$. The crux of this logic for the evolutionary microeconomics is that connections ($E$) are afforded the same ontological and analytical status as elements ($V$). This concept can only be defined over a non-integral space, thus suggesting an important relationship: connections are the building blocks of systems, and these systems then become elements in higher-level systems. Once clear of the erstwhile field framework, we do not plunge headlong into some ill-defined analytical oblivion but emerge into a topographic framework ordered according to the logic of local structure, nested hierarchy and emergence. The purpose of this chapter, then, is to set these principles into the foundations of an evolutionary microeconomics.

First, we discuss the concept of connections afforded ontological existence. Second, the basic graph theory model is introduced and employed to define the analytical concept of a system. Section 3.4 then extends this framework to define a hyperstructure, which is a system of systems. The concept of system-element duality is defined in Section 3.5, and a discussion of the arithmetic of evolutionary systems is provided in Section 3.6. Section 3.7 concludes.

3.2 On Connections in an Economic System

Following from Chapter 2, the second of the ‘first principles’ of an evolutionary microeconomics is that connections exist. In the evolutionary economics research programme this ontological assertion is in the hard core. There are two hard-core propositions, both of ontological bearing.

Evolutionary-HC1: There exists a set of elements.
Evolutionary-HC2: There exists a set of connections.

E-HC1 does not differ fundamentally from the first proposition in the neowalrasian hard core, in that both essentially acknowledge that there exists a set of things that we may infer as agents, goods or some such. E-HC2, however, is fundamentally different from the neowalrasian hard core and is in contrast with the concept of a field. The concept of a field, as I have shown, is set up by the axioms of continuity, transitivity, reflexivity and completeness, plus the hard core propositions (Hausman 1992: chs 1–3). According to Weintraub (1985: 109), the hard-core propositions of the neowalrasian framework are:
HC1: There exist economic agents.
HC2: Agents have preferences over outcomes.
HC3: Agents independently optimize subject to constraints.
HC4: Choices are made in interrelated markets.
HC5: Agents have full relevant knowledge.
HC6: Observable economic outcomes are coordinated, so they must be discussed with reference to equilibrium states.

It is apparent, I trust, that the reader will now perceive that propositions HC2–HC6 are simply the dimensioning of the underlying field axiom into its intuitively relevant economic aspects. They are all statements to the effect that there exists a set of connections and that this set is complete in all dimensions. Ergo, and this is the sleight of hand, it is as if, for all analytical purposes, there does not actually exist a set of connections. At the risk of labouring this point, it is not the case that the neoclassical framework denies the existence of connections, clearly it does not, but in axiomatically treating them as complete it effectively denies that they exist in that they have no explanatory content. Instead, it is broadly within the heterodox schools of thought and in particular within the more explicitly evolutionary approaches that connections have re-emerged as central to analytical representation and explanation. Admittedly, a reading of this level of generality is not explicit in any particular heterodox treatment, but, rather, is the point of synthesis that I am advancing. As such, it will be worth briefly reconsidering the treatment of connections in heterodox theorizing.

Following the seminal work of Nelson and Winter (1982), which re-established the agenda of an evolutionary economics, theoretical approaches to evolutionary model building have proceeded in an explicitly microeconomic manner. A microeconomic model is a type of story that is centred about what agents do and why they do it. And in the context of an evolutionary framework, and thus a process dynamic, a microeconomic model must further find some measure or identification of what is actually changing in the dynamic process of change. If we are to theorize about evolution then we must be clear about what it is that is actually evolving, which is to say that we must be able to distinguish the dynamic null state of 'no evolution' because a certain phenomenon has not changed (compare, Bedau et al. 1998). This is perhaps an obvious or trivial statement but it is absolutely fundamental and, it seems, has been precisely the lack of explicit formulation of this basic prerequisite that has thus far impedied the development of a general theoretical framework of evolutionary microeconomics. The answer is simple: it is connections that change.

It was not until the 1980s that an evolutionary economics was reformulated to a sufficient degree to present itself as a definite theoretical possibility. Prior to then, Kenneth Boulding (1966) had performed a critical service in gathering the momentum of the evolutionary idea by making this point of critique the
necessary basis of such a framework. For Boulding, the answer to 'what is actually changing?' is knowledge, a fundamental point also stressed by Loasby (1976, 1991) and Hayek (1945, 1974) and both in a similar metatheoretical context. Romer (1994, 1996) has also argued this point in terms of theoretical treatment of the growth of an economic system. Yet in pushing forth a highly eclectic fusion that lent heavily upon analogy with evolutionary biology, Boulding (1978) then made what I believe was a wrong-turning, emphasizing a genotype/phenotype distinction analogous to that pertaining in biology (also Alchian 1953, Gowdy 1985, Boulding 1989, Faber et al. 1996). The wrong-turning, however, was not obviously wrong, for the metaphorical concept of a genotype and a phenotype is clearly illustrative of the distinction between knowledge of how-to (genotype) and the resultant product (phenotype), and, moreover, the logic of evolutionary biology places the moment of evolution expressly in the genotype, which is also clearly applicable to the locus of economic evolution (Vromen 1995: 92).

In this respect it was an obvious yet difficult path to follow, a thick tangle of metaphor, metaphysics and metamathematics with only the foggiest sight of reference points. Yet it is Loasby (1976, 1986, 1991, 2000) who has most clearly seen through the haze, fixing 'the growth of knowledge' as the fundamental of economic change and evolution (see Harper 1994). My point of criticism does not pertain to this essential insight, but with the difficulties of analytical formulation it then requires. Identifying that knowledge is 'what is changing' centres the paradigm, but exposes a wide ontological and analytical flank. It begs the question 'What, then, is knowledge?'.

I suggest a more abstract approach. Admitting that what is changing is indeed knowledge, we may yet invest a conception of knowledge with a far wider meaning than either Boulding or Loasby, amongst others, attribute. Knowledge, in the abstract, is a specific instance of association. Technological knowledge, of the blueprint kind realized by moving discretely through production possibility space, is a set of combinations between factors (elements), and can be represented in geometric form as the specific connections between the specified elements in a space. This corresponds with the genetic meaning (and metaphor) that inspired Boulding, which is a DNA sequence as a set of elements (bases) in particular association (strings of base-pairs). Information is, in part, structure. Knowledge is a structure that can interpret structure.

Yet this knowledge does not map the integral of a production function, but rather consists of independent points each realizing a geometry of connections that we may classify as an instance of knowledge, or, as I shall propose in Chapter 6, a potential competence. A firm, in this sense, is both a set of resources (factors, in the orthodox model) and the connections between them. And this is the crucial distinction, for in the evolutionary microeconomics these connections are also to be regarded as resources of the firm. Structure has value,
and can be thought of as an asset. The value of connections derives from the fact that they too are building blocks of such assets.

It is in this sense that we shall regard a firm as a system which is a more encompassing object than a simple factor endowment. The connections also form an endowment of the system, such that different connections would form a different system. In the neoclassical and neowalrasian models the firm is in a technology field so that the particular point it occupies on the production possibility map includes all other points (from a single point it 'knows' all other technologies and can implement them at will and without cost). Retreating from field theory, a firm is not effectively at all points in the space and therefore the particular connections it makes must be recognized to be inclusive in the resources of a firm as a system. Competence, then, is a (knowledge) resource. Foss (1996) and Lomasby (1996) have both suggested that a competence-based theory of the firm will most likely be furnished with theoretical underpinnings from an evolutionary perspective (see also Hodgson 1999).

Technology is a particular set of connections, as is competence. Skills, habits and routines, in the context of a firm, can also be directly interpreted in this manner (Nelson and Winter 1982). But how far can we actually extend this conception? Is it the case, for instance, that all is knowledge, and that in the final analysis firms are to be understood as knowledge structures for creating further knowledge? If we follow this line of abstraction it becomes apparent that commodities, capital and labour must also then be knowledge. But this is not what we mean. Consider the concept of knowledge as constructed from a reduction to physical essentials. This would refer broadly to a stable relationship between a set of neuron connections which then form a cognitive interpretation system which then orders signals from the environment into a meaningful structure that can then be epistemologically codified and tested (Simon 1957). Knowledge is not a simple quantitative concept in this respect (and unlike mass or charge, for instance), and very much a subjective and constructed human phenomenon. But this is not to say that the concept of knowledge is hollow, totally without material basis. For when we generalize the concept of knowledge, as above, we take purchase upon what is surely its most objective aspect, namely the concept of specific connections. When we argue that technology, competence, skills, habits and suchlike are instances of knowledge, we mean by this metaphor that they are structurally similar to the epistemic phenomenon of knowledge in that they are instances of specific connections that seem to work in a particular environment.

The key transfer is the concept of a connection. Neurologists can point to synapses, identifying specific connections. Cognitive psychologists can map associations, as mental connections. Behavioural psychologists can identify constructs, as sets of connected associations. And Lomasby speaks of knowledge precisely in this sense; not as an epistemic phenomenon, not an encyclopaedia of
unrefuted facts, but as a structure of connections. The growth of knowledge, then, is the accumulation and flux of connections, wherever these may occur. The growth of knowledge is a metaphor, but its analytical bearing into objectivity fixes on connections.

In this respect the significance of an economic agent's assembly and use of knowledge is far wider than the theory of production and also encompass the skills, decision heuristics and habits of an agent, be they in the consumer, input factor or entrepreneurial context (see Boisot 1995). For example, Earl (1986) conducts an extensive critique and reconstruction of the theory of consumer choice, drawing crucial attention to the structure of decision heuristics and issues of information sufficiency and complexity. Extending the seminal work of Simon (1959) and Kelly (1963), Earl implicitly but outrightly rejects the field model of consumer theory so that the economic agent is functionally charged with the information tasks of gathering, filtering, organizing, processing, acting and reviewing as an interplay of imagination and habituation. The rejection of the field construct is achieved by incorporating specific connections pertaining to the agent's sources of information, the connections forming the agents information processing (decision heuristics), and the set of constructs (mental connections) with which the agent acts. The economic agent is completely reconstructed so that all internal and external connections are explicit. The framework is a synthesis of psychological foundations (Scitovsky 1976, Leibenstein 1976) and epistemic considerations (Shackle 1972), and also lends synthesis to newly emergent approaches to algorithmic modelling (Dosi et al. 1999) and consumer innovation and learning (Swann 1999). Like both Lawson's (1997) and Langlois and Robertson's (1995) fusions of Institutional, Austrian and Post-Keynesian foundations, Earl's eclectic framework is fundamentally singular and coherent, but not with respect to a field-theory framework. From a neoclassical perspective such instances appear rather ad hoc, but the point is that they are wrongly conceived from such a perspective. These three instances—Earl, Lawson, and Langlois and Robertson, among others—are concordant with respect to a framework of knowledge as connections, and structure as connections, and dynamics as change in the connections. Ultimately it is an ontology of connections that underpins the theoretical substance of these heterodox advances.

If an evolutionary microeconomics is to encompass the full domain of heterodox concern with the nature and dynamics of information, knowledge and structure then it will be based upon an ontology of connections. That is, a theoretical system in which connections between elements have the same ontological bearing as the elements themselves.3 From the perspective of first principles, then, we define two components to existence in the specification of an economic system: the elements and the connections (E-HC1 and E-HC2) and
it is the conjunction that forms the system, and the dynamics of the system are to be represented in terms of changes in the set of connections.

3.3 The Definition of an Economic System

What is curious nevertheless is that in a world where the importance of the organisational structure of an economy is being more and more explicitly recognized, where external effects and who causes what to whom are more frequently discussed, where the question of the search for opportunities in a world of uncertainty receives considerable attention, and the question as to who learns what from whom is increasingly posed, in such a world a mathematical tool, graph theory, particularly apt to handle such problems remains largely unused.

(Alan Kirman 1987b: 559)

If we want to think in abstractions about interaction then we should use the mathematics of specific connections says Kirman. But we do not. Why not? From the perspective of the previous chapter, it can now be seen that the reason graph theory remains unused is simply that its proper place is fully at the level of microeconomic foundations. Attempts to fit it anywhere else will inevitably result in confusion unless it is plainly understood that the concept is incommensurable with all field-based theory.

A graph, $G$, is a mathematical object composed of two classes of sets—vertices $V$, and edges $E$—such that $G = (V, E)$. I shall modify the terminology immediately to reflect the specific ontological purposes required of it for economic analysis, the reasons for which will become apparent soon. A graph I will call a system, and denote it $S$. The other symbols I leave untouched, but will refer to the vertex set as elements, and the edge set as connections. A system, then, is an object made of elements and connections:

$$S = (V, E)$$  \hspace{1cm} (3.1)

The evolutionary microeconomics is a study of systems, and in that way it is a study of a set of things (elements) and the relations (the connections) between them. Let there be $n$ elements in a system, which defines the order of the system. The set of elements is denoted $V(S)$, but I shall refer to it simply as $V$, such that:

$$V = (v_1, \ldots, v_{n-1}, v_n)$$  \hspace{1cm} (3.2)

Let each element $i$ be connected to $k_i$ others, defining the degree of the element. The set of connections in the system $E(S)$ is then a set of binary elements, such as $v_1v_2, v_nv_b$, indicating a connection between elements 1 and 2, or $a$ and $b$, and
so forth. When a connection exists between, say \( a \) and \( b \), \( a \) and \( b \) are said to be \textit{adjacent}. If two elements are not connected they are \textit{non-adjacent}. We denote these individual connections \( e_{ij} \):

\[
E = (e_{ij}, \ldots)
\]  

(3.3)

When all elements are of the same degree \( k \), the system is a \textit{k-regular} system. It follows that the relation between \( n \) and \( k \) determines the total number of connections in the system, termed the size of the system, and denoted \( M \).

\[
M = \frac{nk}{2}
\]  

(3.4)

A standard way to represent the connective structure of a system is with an \( n \times n \) matrix of 1s and 0s termed an adjacency matrix, \( S(A) \). Suppose the set of elements and connections are as such (which may be represented as in Figure 3.1):

\[
V = (a, b, c, d, e)
\]

\[
E = (ab, ac, cd, ae)
\]

\[\text{Figure 3.1 A system}\]

\( S(A) \) is then a \( 5 \times 5 \) triangular matrix with rows and columns corresponding to the elements of \( V \), and 1s as described by the set \( E \), else 0s. By definition the main diagonal consists of 0s (an element is not connected to itself).
The set of elements that are adjacent to any given element \( i \) defines the *neighbourhood of \( i \)*, denoted \( \Gamma_i \). We can now define two limiting case systems—a *null* system and a *complete* system—which serve to bound the *state-space* of a system, which is the set of all possible systems (connective structures) with \( n \) elements. The number of possible states (particular connective configurations) that a system can occupy is in most cases large. There are \( s \) distinct \( n \)-systems (Kauffman 1993).

\[
S(A) = \begin{bmatrix}
0 & 1 & 1 & 0 & 1 \\
1 & 0 & 0 & 0 & 0 \\
1 & 0 & 1 & 1 & 0 \\
0 & 0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

\[
s = 2^n
\]  

(3.5)

We may order the set of states of a system between two extreme states, a null system and a complete system. A null system is defined as the situation where for a set of elements \( V \) there are no connections between any of the elements. If interactions are interpreted to mean interactions, then in the null state no interactions occur, which is to say that no element is able to affect any other element. The null state is causally inert. For a null system then, the set \( E \) is the empty set and thus the adjacency matrix \( S(A) \) consists entirely of zeros. This is represented by Figure 3.2.

![Figure 3.2 A null system (no connections)](image)

A complete system is where every element is adjacent to every other element and is a special case on the spectrum of state-space, otherwise known as a field. Every element is adjacent to every other element because of the complete set of connections. In terms of causal structure, a change in any element affects every
other element. This can be described, for instance, by a system of simultaneous equations. A complete system might look like Figure 3.3, and is topographically equivalent to the topological construct of a field.

![Figure 3.3 A complete system](image)

State-space includes these extrema along with all other possible systems for a given number of elements. For the case above where \( n = 5 \) it follows that \( s = 32 \). This is to say that there are 32 possible systems, including the null system and the complete system, that may be constructed with five elements. The range between the null and complete systems we shall term the range of incomplete systems, and this is associated with the concept of a non-integral geometry of economic space. An incomplete system is defined when \( k < n \), mapping the domain of a non-integral space between the null system and the complete system. In this sense a non-integral space is associated with the domain of an incomplete system, and is the general case. The null system and the complete system (a field) make the special case. Figure 3.4 illustrates some of the states near the null state.

![Figure 3.4 Some states in state-space](image)

It is to be noted that of the three systems sketched above, the middle one is adjacent in state-space to the other two because it differs only by one connection. It is evident, then, that each system will be adjacent to \( n-1 \) other systems, so defining the neighbourhood \( \Gamma \). This relation is crucial to the nature of dynamics in state-space. For although it is the case that for a set of five
elements in \( V \) there are 32 possible systems \( S \) that we may represent, from any particular state of a system there are only nine states that are adjacent and can be reached by a single change in the connective structure of the system. The significance of this relation becomes more apparent, I think, if we use a larger set \( V \), say 20. For this there will be 1,048,896 states, but from any particular state only 189 will be adjacent (that is, approximately 0.02 per cent of all states!). The logic of a field, however, hides this problem by collapsing state-space such that all possible states are simultaneously potential. That is, there is no such concept of adjacency and every point in the field can be reached by a single change (as the logic of a technology or utility field). Further, if there is no such concept of adjacency (or neighbourhood) then there can be no concept of search strategies or of adaptation. The basic question is how does a system which exists at a point in state-space come to know whether or not it is at an optimum point, and when it does believe it is not at an optimum, how does it go about tracking it? We do not yet have an answer to the question of how an economic system moves through state-space because we have hardly acknowledged that a system actually exists at a point in state-space rather than as a ‘potential’ across the field of all possible states. The field assumption is far more analytically tractable, but it is an entirely misleading description of the nature of existence and dynamics of the systems that are the building blocks of the economic process. We return to this matter in the following chapter in the context of the work of Stuart Kauffman on adaptation in complex systems.

To see how this relates to the neoclassical framework, it must be recognized that in that framework, the constructs \( S \) and \( V \) are identical (the set of elements is the system), such that \( S \leftrightarrow \text{field} \), in \( \mathbb{R}^n \). Yet it remains the case that while these two frameworks are radically distinct in their ultimate analytical bearing, both begin with the same logical foundation of a set of elements in a space \( \mathbb{R}^n \). The evolutionary and neoclassical microeconomic frameworks diverge, however, with the next step when we specify the form of interaction between the elements in the space. The essential theoretical difference between these two microeconomic frameworks is, then, that in place of a field the evolutionary framework constructs a set \( (E) \). The set \( E \) is the set of connections in the system, and each possible configuration then defines a state of the system which is a particular topographic geometry of economic space. The domain of possible geometric forms ranges between the null system in \( S \), which occurs when there are no connections in the system (\( E \) is the empty set), and the complete system in \( S \), which is when every element in \( V \) is connected to every other element. So constructed, a complete system in \( S \) is axiomatically equivalent to a field. The evolutionary microeconomic framework, then, encompasses the neoclassical framework as a special case.
3.3.1 Example: A Firm as a System in State-Space

Consider, by way of example, a firm as a system in state-space. Suppose this firm consists of five agents \((a, b, c, d, e)\) with a structure of connection as according to the example in Figure 3.1 above. By inspection it can be seen that element \(a\) is at the centre of this firm and element \(d\) the furthest removed. Consulting the adjacency matrix it can be seen that element \(a\) is of degree three (\(c\) is degree two and \(b, d\) and \(e\) are degree one) and has the largest neighbourhood of all elements in the system.

Other static properties include a relative measure of the centralization of the system. This can be calculated by taking what is termed the moment of an element (Robinson 1966). The method is, for each element sum the number of connections between it and every other element as individual paths. This will calculate \(n\) moments, which we shall denote \(M(V_i)\) as the moment of the \(i\)-th element in \(V\).

\[
M(a) = 5, \quad M(b) = 8, \quad M(c) = 6, \quad M(d) = 9, \quad M(e) = 8
\]

We then take the minimum and maximum moment \((\min = 5, \max = 9)\) and calculate the value of \(m\),

\[
m = \frac{\max - \min}{\min} \quad 0 \leq m < 1 \quad (3.6)
\]

We note that as \(m\) varies between 0 and 1, the system ranges from extreme decentralized to extreme centralized. In the above instance \(m = 0.8\), which is tending towards centralization. Such a statistic is not so valuable in itself, but serves as a metric for comparison between different systems. It is to be noted that the moment \(m\) of a field is 0, (because all elements are adjacent to each other thus \(M(i) = n\) for all \(i\)). That is, a field is by definition a model of extreme decentralization (compare, Stiglitz 1994: ch. 1).

The ratio of the number of connections over the number of elements, which we shall term \(\lambda\), is also an important measure for the analysis of emergent properties in sets of connected elements. If there are \(k\) connections incident to each element, and \(n\) elements in \(V\), then:

\[
\lambda = \frac{k}{n} \quad (3.7)
\]

It is a well established result pertaining to random graphs\(^7\) that for \(\lambda < 0.5\) most elements in a system are isolated, but beyond 0.5 a phase transition contains
most of the points within a single connected component (Erdos and Renyi 1959, 1960). State-cycles and closed loops, emergent complex behaviour, are not expected to occur until $\lambda > 1.0$ (Cohen 1988). In the above case, $\lambda = 0.8$ (coincidentally the same as $m$) and we therefore have a system that is entirely connected, but well short of a phase transition into emergent dynamic behaviour (Kaufman 1993: 307–12).

But most interesting, I think, is the application of graph theory to the study of adjacent states. Once past the initial identification problem of defining the sets $V$ and $E$, thus analytically constructing a single state, we can systematically map all states that are adjacent. This is done by simply flipping, one at a time, each entry in the upper triangle in $S(A)$, from 0 $\rightarrow$ 1, or 1 $\rightarrow$ 0. A complete system of order $n$ has $M$ connections; that is, there are $M$ places we can place a connection in a system of $n$ elements. By definition (from (3.4) and (3.5)),

$$\text{Number of adjacent states} = \binom{n}{2} - 1$$

$$\text{Number of total states} = 2^n$$

Thus for $n = 5$, there are nine adjacent states (corresponding to the number of elements in the upper triangle of $S(A)$ minus one; we do not count the present state). Thus from the present structure there are nine other states that can be reached by a single change in the connective structure of the system. As we seek to model the dynamics of structure, the first thing we need to define is the set of adjacent possibilities from each point, and, if we assume incremental change, then one will eventuate and also have nine adjacent states. In this way we can start to build up maps of the pathways taken by firms as they explore the state-space (of 32 possible forms between the null system and the complete system).

Of further interest, then, is how the number of adjacent states and the size of the state-space changes with a change in the number of elements. A number of points are immediately apparent. First, growth in the number of adjacent states and the total size of state-space increases in non-linear proportion with respect to growth in the size of $V$. If we add to this a bounded rationality constraint, such that an element can connect to no more than, say, seven other elements (see Miller 1956) then beyond $n = 7$ we expect structural decomposition to occur. That is, the ability of the firm to grow in the standard sense of adding more elements to $V$, is in fact conditional upon the position of the system in state-space. Which is to say that from certain positions the firm will not be able to grow, because all elements will already be of degree 7 (which is the constraint). However, from other regions of state-space the system may be able to grow in many directions. In this sense, the economic problem for the firm is not only the search of state-space by incrementally moving through adjacent states (see Allen 1988, 1993; Leydesdorff 1994) for global optima, but also avoiding becoming
The new evolutionary microeconomics

...ed on local optima (see Arthur 1989, 1994) and retaining enough flexibility that the number of adjacent states is maximized. This problem will be addressed in a more general context in Chapter 4.

... may be objected that the above conception is no more representatively a than the standard model, but I would suggest that two major new extensions have been added. First, a formal way of conceiving the space seen extreme centralization and decentralization has been furnished. We now begin to study the populations that live between these states, including Chandler's (1962) notion of U-form and M-form structures, Kay's (1997) maps corporate synergy, and in particular the creation or emergence of networks as major new forms of business organization (see Teece 1992). This framework provides us with a way of ordering the relationships internal to a firm, and thus viding a structural metric that has been so conspicuously absent from previtical work. Second, and more importantly, marginal difference is given a plural meaning with the concept of adjacency defining a neighbourhood. Moreover, this adapts easily to consideration of non-marginal change, are many connections may be changed simultaneously so that a system may be a 'long-jump' in state-space (Kauffman 1993: 69–76).

1 System-Element Duality and Hyperstructure

...ird foundational component of the evolutionary microeconomics is system-...ment duality, which expresses the concept that once a system emerges as an ity, although itself a complex entity, it can serve as a singular building block of a higher-level system. A system can itself be an element for a higher-level stem and symmetrically, an element may itself be a system at a lower level. In s... way routines build skills, skills build competence, competence builds firms, ms build industries, industries build economies and so on. There are many...els of systems in an economic system and the elements of each system S—st...-is, the elements of the set V—will themselves be entire systems at a lower el...el. It would be highly convenient from a mathematical and logical perspective to abstract from this complex stratified reality, which is what a field-theory-based approach does, but the cost of doing so is that we forgo all insight to the nature of emergence and hierarchy. These are the twin principles by which systems are constructed into higher-level systems (emergence) and the sultant structure (hierarchy). More than anyone else, Herbert Simon has deavoured to introduce these principles to economists (see Loasby 1989). This conception is wholly distinct from the paradigm either of atomism or of a field. Following Baas (1994, 1997), I propose to call this geometric conception hyperstructure', which he explains as 'higher order or cumulative structure'. A hyperstructure is a power set: that is, a set of sets, or, as in this case, a system of...
Economic systems

The geometry of economic space is defined not as an integral field but as a topographical hyperstructure.

The evolutionary microeconomics combines the notions of emergence and hierarchy into a single construct: hyperstructure. A hyperstructure is a system of systems, or a system of systems of systems, and so forth where we may then speak of an n-level hyperstructure as the number of layers of emergence within. I shall deal only with 3-level hyperstructure as this allows us to conceptualize a primary system (a human agent) which is both made of systems (routines and skills, for example) and also an element in higher-level systems (such as a firm or institution). Thus, hyperstructure is the geometry created by synthesizing the concept of emergence and hierarchy, and system-element duality is recognition that the definition of an element or a system depends entirely upon the level at which it is viewed. The concept of an element or a system depends upon our frame of reference.

The following definitions apply:

1) Superscripts denote emergent levels of hyperstructure. We define a three level hyperstructure with integer superscripts 0, 1, 2. Thus \( S^0 \), \( S^1 \) and \( S^2 \) are respectively the zero-th, first and second level systems. So if \( S^1 \) is an agent, then \( S^0 \) would be systems interior to the agent, such as decision heuristics, and \( S^2 \) would be systems exterior to the agent in which the agent is an element, such as a firm.

2) Subscripts denote coordinate enumeration at a given level of hyperstructure. For example, \( P^1_{13} \) is element 1 in system level 1. \( P^1_{21} \) is element 2 in the same system. For the sets of connections, \( E^1_{13} \) denotes the connection, if it exists, between \( P^1_{13} \) and \( P^1_{21} \). Symmetrically, \( E^1_{13} = E^1_{13} \).

A three-level hyperstructure equates the following nest of equations.

\[
S^0 = (V^0, E^0) \\
S^1 = (V^1, E^1) \\
S^2 = (V^2, E^2)
\]

\[
S^0 = [(V^0, E^0)] \\
S^1 = [(V^1, E^1)] \\
S^2 = [(V^2, E^2)]
\]

Thus it follows that if \( S^1 = (V^1, E^1) \) then, \( S^0 = (S^0, E^0) \). Similarly, if \( S^0 = (V^0, E^0) \), then \( S^1 = (S^1, E^1) \). The duality condition (system-element duality) then is:

\[
V^2 = S^1 \\
V^1 = S^0
\]

In general, system-element duality is defined by the identity:
The new evolutionary microeconomics

\[ V^n = S^{n-1} \]  \hspace{1cm} (3.9)

**Figure 3.5** A three-level hyperstructure

Figure 3.5 represents a system \((S^2)\) embedded in a system \((S')\) embedded in a system \((S^1)\). It is a three-level hyperstructure, as might be agents \((S^0 \equiv V)\) in a firm \((S' \equiv V')\) in an industry \((S^1 \equiv V^1)\).

To summarize, the concept of hyperstructure is simply the concept of a system extended hierarchically. In this respect it is not a new idea, and has been fully presaged by early systems theorists (for example, Bertalanffy 1962, Koestler 1969). Herbert Simon (1962, 1981) has also presaged the scheme of this concept in his discussions of modular decomposable systems. The point I emphasize, however, is not so much the mathematical form (which has been suitably defined by others, Baas 1997) but its ontological aspect, namely system-element duality. The conceptual and mathematical framework of hyperstructure will certainly have analytical applications (see, for example, Kirman 1997b, Padgett 1997) but our present concern extends only so far as first principles. In short, system-element duality makes the identity of an element or of a system arbitrary. In the realm of the economic, there are neither absolute systems nor absolute elements. Whether something is inferred as a system or an element depends entirely upon one's level of analysis. This problem does not occur in the context of a field. To begin to make sense of this analytical paradigm we now turn to discuss the concept of building blocks of economic systems.
3.5 The Building Blocks of Economic Systems

Markets, hierarchies and networks are pieces of a larger puzzle that is the economy. The properties of the parts of this system are defined by the kinds of interactions that take place among them.

(Walter Powell 1991: 270)

3.5.1 On the Representation of Existence

In introducing and defining the framework of graph theory I have employed the terms element, set and system as if such terms were *prima facie* self-evident and unproblematic. Yet on closer inspection these concepts prove to be somewhat slippery. As it transpires, however, these concepts have always been vague of meaning, fudged by the concept of a field. Because an evolutionary microeconomics rests squarely upon such concepts, rather than obliquely, the measure of abstraction which proves acceptable for the orthodox analysis is wholly unacceptable for the heterodox framework. We shall thus unpack the concept of categorical existence in an attempt to make clear the nature of the building blocks of the economic process.

A system is made of elements and an element is defined as a member of a set, such that \( v_i \in V \) if \( v_i \) is a member of (belongs to) the set \( V \). By the term 'set' I mean a collection of objects, in which given any object it can be determined whether or not it is in the collection. Epistemically, either \( v_i \in V \) or \( v_i \notin V \) must be true for any \( v_i \) if it is the case that we can define \( v_i \) as an element and \( V \) as a set. Mathematically, these definitions are simple and self-evident.

Yet mathematical definitions are only logical and not ontological: they describe relationship but not existence. Still, a purely concrete definition does not necessarily fare better. For instance, as the first Polish encyclopaedia (of 1834) insists: 'What a horse is, is evident to everyone'. This may well be, yet economic abstractions regarding elements and sets are not generally in the same league of self-evidence as Polish horses. The problem, however, is that they are typically treated as such, but without nearly as much candour.

In its atomistic mode, the neowalrasian microtheory defines the rudimentary units of an economic system—as factor inputs, commodity outputs and agents—in a way that is credited with being self-evident according to the form of the set definitions above. A commodity, for instance, is abstracted to an element in the context of a set, both as an individual entity \( c_i \) (a particular banana in the set of all bananas) or as an entire class \( c \) (the element bananas in the set of all commodities \( c \)). The typical textbook treatment of commodities is no more insightful than the encyclopaedic treatment of Polish horses, in that it acknowledges existence but provides no indication of the nature of that existence. Similarly, a firm is conceived as an element in the set context of an
industry (or supply function), as is a consumer in the set context of a market (or demand function). Consumer and firm endowments are sets, as described by the term 'bundle'. These are presumed to be the building blocks of an economic system, in the sense of being the basic elements of the categorical sets.

The notion of categorical sets derives from the heritage of political economy, where factor based theories of value explained the emergence of value in terms of the mixing of basic elements. John Stuart Mill, characteristic of the Classical theorists, makes 'labour and natural agents the primary and universal requisites of production' (Political Economy, Book 1, ch. 4 § 1). Similarly, Böhm-Bawerk argues 'all production is the result of two and only two elementary agents of production, nature and labour' (Kapital and Kapitalins, 1884 pt. ii, p. 83). This doctrine accounted for both the emergence of value and surplus, and was (and remains) in tidy accord with the tripartite of economic returns: wages, interest and rent. These notionally different income streams were and are thus associated with notionally different productive wellsprings. Thus the construct of categorical sets was well entrenched prior to the emergence of the neowalrasian microtheory and the associated shift of analytical emphasis from production to exchange.

The mathematical requirements of the nascent microtheory pivoted about the methods of the differential calculus, engendering, as Kirman (1987a: 493) notes, 'an implicit acceptance of the perfect divisibility of goods'. Kirman continues with this ontological point, observing that 'if we consider the most elaborate extension of the basic Arrow-Debreu model it is one in which there is a continuum of agents and a continuum of goods' (idem). In this context, it makes no sense whatsoever to conceive of the economic elements as anything other than abstract points in the context of a one-dimensional field (a continuum, $R^1$). The concept of a categorical set $(v_i \in V)$ has been collapsed to a representative element $(v_i^*)$ which has been interpreted as a point on a continuum $(v_i^* \in R^1)$. In this way, the atomistic mode $(v_i \in V)$ has transmogrified into the field mode $(v_i^* \in R^1)$.

What has occurred, gradually and seemingly imperceptibly, is that as the spotlight of analytical focus has swung from production to exchange, switching attention from the study of the emergence of value to the distribution of value, and therefore from combinatorial methods to differential methods of analysis, the necessary requirement of integrability has taken root largely unnoticed and unquestioned (see further, Mirowski 1991a, Carlson 1997). Integrability, as the assumption of a continuously differentiable function, carries the implication that commodities, and indeed all other elements, are credited with possessing elementary point-like existence. As such, if a more general mathematical foundation will allow non-integrability then it remains for us yet to explore the sense in which economic elements are elementary.
Empirically considered, almost all formally conceived elements of an economic system are complex entities or structures. If we consider the class of consumer goods, it is obvious that such things as cars, computers or clothing are complex entities, each a specific combination of many and perhaps thousands of component parts (and therefore technologies, Ziman 1998). We also recognize that 'negative combinations' belong equally to the same abstract category. In this way, standard commodities such as sugar may also be regarded as complex entities, in that a specific set of negative combinations (refining, milling, and so on) were required to manifest the product. In other words, all economic elements, metaphorically speaking, are molecules. They are composite systems of particular combinations. From this perspective, commodities are systems. In the orthodox microtheory, however, such concern is entirely irrelevant because a single parameter (price) suffices to locate the commodity in the market field (commodity space), thus collapsing the subject entity back from a system to a point on a continuum. All further information relating to the state of existence of the commodity is extraneous because in the context of a field, by definition, an entity cannot exist other than as a point. Elementary complexity, heterogeneity, modularity, decomposability and other such aspects of composite structural existence are logically excluded from a field-theory framework.

A small number of economists have advanced the (critical) observation of intrinsic commodity complexity towards theory—Houthakker (1952), Lancaster (1966) and Earl (1986) in the context of demand; Sraffa (1960), Robinson (1962) and Langlois and Robertson (1995) in the context of supply. Lancaster argues that it is not commodities per se that consumers value, but the characteristics they embody. Therein, a commodity is to be understood as a set of characteristics. Earl integrates this notion with the algorithmic form of the consumer decision process, recognizing that if there are manifold points of contact with each commodity (rather than only price) then the way these points relate to one another can systematically affect the outcome of the decision process. Drawing attention to the work of Simon (1959) and Kelly (1963), Brian Loasby (1991) has emphasized that the way agents mentally construct the world about them is of crucial significance to understanding the behaviour of the agents. For Loasby to make this bridge into the growth of knowledge as the ultimate basis of the economic process, it is implicit in his schema that the basic elements (of the economic process) are irreducibly complex. When theorizing about the behaviour of economic agents, we must recognize that this behaviour is with respect to something. The something will be variously other agents, commodities or, generally speaking, other building blocks. If these building blocks are complex, then our microtheory cannot logically be defined in an integral space, but requires a domain in which we can conceivably map the structure of interactions or relations between elementary units. Yet we have practically no systematic understanding of the multifarious relations internal to
the set of all commodities in terms of object properties. Nor, in the same sense, do we have a map, as it were, of the typical patterns of interactions between agents constituting an economic system (although see Kay 1997). As has been sometimes lamented, the economic science contains a conspicuous gap in its empirical knowledge of the characteristic structural forms of the various subsystems of the economy.

Along these lines Leibenstein (1979) long ago proposed that 'a branch of economics is missing: micro–micro theory'. What Leibenstein means by micro–micro theory—as the study of what goes on inside the black box', and as associated also with the works of Scitovsky, Baumol, Marris, Williamson, Simon, Cyert, March and Day, among others—is essentially concerned with the study of individuals as complex entities. He observes that 'microeconomics has avoided the study of individuals but that is no reason why we should continue to do so' (Leibenstein 1979: 497), and moreover he suggests that the study of micro–micro theory may constitute the first clear sense of a research frontier in economics since the macroeconomic revolution instigated by Keynes. That was in 1979 and prior to the Nelson and Winter (1982) spurred re-emergence of evolutionary economics. I venture to suggest that Leibenstein's proposal equates now not as micro–micro-theory but as evolutionary microtheory.

Leibenstein, however, remained somewhat unclear about the ultimate nature of the micro–micro theory, and in particular, I think, failed to distinguish carefully between a theory of elementary units and the units of elementary theory. The distinction is crucial and revealed by the construct of a complex system as a theory of elementary units in which the elementary units are themselves lower-level systems. Once this principle is recognized, it becomes clear that the micro–micro theory in fact transcends the microtheory in generality. It is significant that every subject that the domain of Leibenstein's micro–micro theory addresses is of the abstract form of a complex system. The micro–micro theory is the study of a complex of elements and thus fits within the rubric of an evolutionary microeconomics. Evolutionary microeconomics, then, constructs in part the microtheory of microeconomics, addressing itself to the logic by which elements interact to form systems. But this, we may now recognize, overlaps with macroeconomics when the elements we identify are, say, entire production sectors or aggregate market demand (Leijonhufvud 1993). And herein arises the difference between Leibenstein's conception of micro–micro theory and the definition of evolutionary microeconomics that I propose. Essentially, what is meant by an element in the evolutionary microeconomics is entirely arbitrary with respect to the level at which this is identified. Yet we must simultaneously recognize that if both the part and the whole are both in the same sense an element then there must also be a referent system of taxonomy descriptive of the nested hierarchy of sets of elements forming higher-level elements.
3.5.2 Species of System

A taxonomy of sorts can be outlined in terms of five distinct species of system that seem central to the study of economic evolution. These are: (1) organizations as systems; (2) commodities and capital as systems; (3) cognitive and skilled agents as systems; (4) technology as systems; and (5) institutions (including markets) as systems.

It is not difficult to conceive of a firm as a system, in the sense that it is a set of component parts interacting in specific ways. Within any firm, individual actions are guided by the systematic specification of subordinate and superordinate relations, as pertaining to departments, teams, divisions and so forth, and by other forms of relation external to the firm, such as interactions with customers, suppliers or contractors. A firm, as a species of organization, is obviously a species of system. To be clear, the nest of definitions from a firm to an organization to a system is one of increasing abstraction along the line of structural form, such that the epistemic classification 'all firms are systems' recognizes that all firms are organic composites of separable parts.

In recognizing that a firm is a form of organization and therefore a species of system it follows that all instances of organization are systems. Evidently, then, we may regard commodities and capital, as well as durable goods and services as systems in the sense that they are all specific combinations of elements. It is easy to conceive of such goods as computers or automotive machinery as systems, in that they are complexes of engineered componentry, but it is perhaps more of a stretch to conceive of such primitive household items as toothpaste and paperclips, for instance, as systems. Yet all stocks and services are systems in that they are complex wholes. A paperclip is a definite refinement of wire into a particular form that exhibits a set of engineered qualities. That the virus is a more primitive organic system than the elephant does not diminish its status as an organic system, so too this principle extends to the set of systems traded and procured within an economic system.

The human agent, as a cognitive and skilled being, is also to be understood as a system in the economic context. The agent is a decision-making system in the sense of being a complex of rules for filtering and evaluating information. In a wider sense the agent is a system of constructs, and organic and social reflex (skills and routines) forming a behavioural complex that we may regard as a system. This perspective is well founded in the Behavioural school of economics, and, increasingly, in the algorithmic and computational approaches to rationality being developed within the Santa Fe school (see Morowitz and Singer 1995).

Technologies are systems by definition. A technology is the know-how of combination to form a complex outcome, with the elements of technologies themselves being other technologies. When we understand institutions as system
we are recognizing that they are complexes composed of elements of social behaviour. The abstract concept of a system applies widely over the phenomenal domain of an economic system, such that we may identify the components of an economic system as systems in their own right forming populations of different species of system.

The simple point I make is that a single abstract concept—system—can represent a very wide domain of subject phenomena. By centring upon the concept of a system we bring institutions, technology and cognitive processes all under the same analytical rubric and therefore place them on the same conceptual basis as the more orthodox material components. By making our basic analytic object-abstraction a complex system we attain a great generality of analytical scope and clear identity of subject. This is essential if we are to hope to understand the phenomenon of economic evolution, which presents itself incessantly in qualitative form with the flux of technologies, institutions, skills and organizational dynamics in a penumbra about the more quantitative dynamics of linear growth. By defining a wide domain of systems we may effectively place the analysis of otherwise highly qualitative dynamics on a basis which is, in principle, well defined in respect of what is changing in an evolutionary process.

3.6 The Arithmetic of Evolutionary Systems

One of the great opportunities... for the next few decades is the development of a mathematics which is suitable to social systems, which the sort of 18th-century mathematics which we mostly use is not. The world is topological rather than numerical. We need a non-Cartesian algebra as we need a non-Euclidean geometry, where minus minus is not always plus, and where the bottom line is often an illusion. So there is a great deal to be done.

(Kenneth Boulding 1991: 3)

The scheme of the argument thus far has been essentially geometrical (specifically combinatorial). But a geometry also implies an arithmetic. In the spirit of first principles completeness, I now submit some remarks on the relationship between arithmetic and geometry in the context of an evolutionary framework. In the quotation above, which is from Boulding's introductory remarks to launch the Journal of Evolutionary Economics, he suggests that 'we need a non-Cartesian algebra'. But what is such a thing, and what is meant by the suggestion that we need one?

What Boulding is saying, it would seem, is that we need a way of making sense of the observation that the whole is not necessarily equal to the sum of the parts. This is an observation that many heterodox economists have, directly or indirectly, struggled to make analytical sense of. A system $S = (V, E)$ is a
Economic systems

geometric object in which the parts are the elements \( V \) but the system itself \( S \) is more than the aggregate of these elements. The set of connections also makes up the totality of the system. But this concept cannot be expressed in arithmetic, at least not in arithmetic as we know it.

Nevertheless, here is a way of thinking about it. Consider the algebraic equation \( 1x + 1x = 2x \), and the underlying arithmetic equality \( 1 + 1 = 2 \). We note that the cardinality of the operations (the \(' + '\)) does not systematically affect the logic of the equality. For instance, in the equality \( 1 + 1 + 1 = 2 + 1 = 3 \) the fact that there are two explicit operations of addition in the first part, one in the second and none in the third has absolutely no bearing on the status of the equality. But what if these operations were not neutral? That is, what if we could fold arithmetic back upon itself. This would be an algorithm that after completing the set of operations tallies the number of distinct operations involved, say, as an integer variable \( \lambda \), and then multiplies this by a scalar \( k \). The product \( \lambda k \) would then correct the first-order sum (itself by an arithmetical process involving a further operator) to obtain a second-order sum, representing the equality. In this framework an epistemic equality can be drawn between, say, \( 1 + 1 \) and any real number we choose as proportional to the parameter \( k \). Note that when \( k = 0 \) the correction \( \lambda k = 0 \) and the second component of the algorithm makes zero contribution such that the system is effectively 'ordinary'. (This conception is not the same as a finite arithmetic, where a counting system periodically cycles back upon itself.) When \( \lambda k > 0 \) we have an arithmetical phenomenon of superadditivity. When \( \lambda k < 0 \) we have the phenomenon of subadditivity, where the whole adds up to less than the sum of the parts. This is not an argument that arithmetic is in a general sense arbitrary, such that the sum of any two numbers can be anything at all (I am not plotting to unhinge arithmetic!) but rather an exploration of the effect of treating the arithmetic operators as part of the same. This is, analogously, what is meant by considering connections (as explicit operators) as distinct from the concept of a field (which embeds operators).

Boulding has not been the only one to notice that arithmetic fails to account for basic features of social and organizational processes. This was a persistent leitmotif for the early systems theorists (for example, Bertalanffy 1962, Simon 1965, Koestler 1978; Koestler and Smythies 1969, Doyle 1976, Boulding 1978). Ansoff (1968) associated such non-summation with the concept of synergy, or economies derivable from shared resources. Kay (1982, 1984) further identified synergy with activities undertaken by firms, and in particular with firms that are involved in multiple activities. Synergy then becomes a story of why firms engage in multiple activities, form strategic alliances, and other such agglomerate behaviours. It is sometimes known as economies of scope (see Baumol et al. 1982, Langlois 1999). The primary observation, though, is that certain complex wholes, such as the collective activities of a firm, the behaviour
of teams or the phenomenon of consciousness for example, are not easily reduced to the sum of the component parts. The parts seem to be only part of the story. The concept of synergy has arisen, as such, to express this seeming observation. But it does not in itself make sense of it, which was precisely Boulding's point. According to the extant mathematical logic, we should not witness this phenomenon. But to the extent that we do observe 'arithmetic violations', we may explain these in one of two ways: either (1) that we have failed to account for some part; or (2) that the underlying mathematical logic is inapplicable to the circumstance (and therefore not of universal application). The Cartesian paradigm always responds in the first way. The second way suggests, in effect, that there may be other forces at work rather than unaccounted elements. Synergy, then, is not an explanation of a phenomenon (it is not a theory) but a recognition that a phenomenon has occurred which lies beyond the theory. Synergy is to the theory of the firm what, for instance, phyletic punctuations in the fossil record were to palaeobiology: it is a recognition that the theory is, in a fundamental way, incomplete.

It is my argument that the framework of an evolutionary microeconomics can furnish the scheme of a theoretical explanation. To do so, however, we must translate this arithmetic problem into a geometric equivalent. Algebra is the branch of mathematics in which the procedures of arithmetic are generalized and applied to variable quantities, as well as numbers. In the abstract sense, algebra is the study of mathematical structures in which there are operations that have the properties of addition and multiplication (that is, groups, rings, integral domains and fields). Algebra also refers to the study of algebraic geometry, which interlinks the study of geometric objects, such as a conic section, with an algebraic description, such as a polynomial equation. In this way, arithmetic is the base of mathematics which consists of numbers and relations among numbers and algebra, then, is the generalization of arithmetic such that numbers are generalized to include variables, and relations are generalized to universal operations, as transformations and mappings.

It is evident that the arithmetic operations that relate variables to one another have no systematic affect on the variables or numbers themselves. This is a rudiment of algebra and carries through to algebraic geometry with the concept of a field in $\mathbb{R}^n$. However, we may interpret the argument of the previous chapter, in which a non-integral framework was a statement about the geometry of the space, in terms of arithmetic by noting that the same non-integral conception can be arrived at if the arithmetic operations themselves are not complete. In this case, synergy, or suchlike, can be given a geometric interpretation to the effect that the connection between the elements combined exists. The simple message is that in so far as our representation of an evolutionary system requires a new framework of geometry, then, by association, arithmetic will also be affected. But our recourse to explain
perceived arithmetic incongruities must come from geometric argument. The arithmetic of an evolutionary system is simply an expression of the geometry of an evolutionary system.

3.7 Conclusion

In economics, we have gotten the relationship between the system and its elements—that is, between the economy and its individual agents—backwards.

(Axel Leijonhufvud 1993: 3)

At this point we now have before us the foundations of an evolutionary microeconomics set in the ontological dimension. Which is to say that we have now given analytical form to the way in which the economic process exists as elements which are variously connected such that they form systems that then become elements for higher-level systems, and so on. In this analytical framework, the concept of a set of connections replaces the concept of a field. In the neoclassical framework there was only one component of existence—the set of elements—plus the construct of a field, which stood in for all issues of information, knowledge and structure. An evolutionary system of microeconomics reinstates these aspects with two components to existence—the set of elements and the set of connections—that then gives rise to a third mode of existence with the concept of a system. We may define this as a third entry in the hard core:

E-HC3: There exist systems. Systems are the basic objects of theory and units of analysis.

The most significant aspect of this conception, from the perspective of heterodox theorizing at least, is that (like Einstein's reinterpretation of Newtonian space-time) space and time are interior constructs of the framework, and, moreover, they are non-integral. Evolutionary microeconomics rests squarely on the idea of essential incompleteness over the 'space-time' of connections. In this way, it is the existence or not of these connections that effectively defines the spatial and temporal dimension of an economic system. These dimensions do not exist a priori, but are created in the processes of economic coordination. By looking forward into the future, economic agents thereby create that future.

Many connections in the economic system are predominantly temporal, in that they extend most manifestly in the temporal dimension. Contracts are an obvious example, but so too are implicit contracts, goodwill, tacit understanding, repeated trading relationships, trust, familiarity, brand names, marketing
mechanisms and other such phenomena. Skills, competence, strategies, habits and institutions are irrepressibly temporal in their mode of existence. When we speak of knowledge, uncertainty, expectations and suchlike we are projecting the domain of what we perceive to be the economic system forward into the future. Yet the framework of orthodoxy does not allow us to go there, rather it requires that the future be collapsed back into the present. Despite its temporal dimension, all neoclassical theory denies time its primary reality: time is change. But where does such change have its effect? Certainly it moves and displaces the elements within the system, and the neoclassical theory describes this well. But change also occurs within the connections between the elements, and as such upon the nature of the systems such connections then build. This is the phenomenon of economic evolution, and it is human behaviour that causes this phenomenon to occur. In many respects we are free to choose connections, in other respects many are chosen for us; there is also the phenomenon of self-organization, whereby systems of interacting elements tend to be connected in characteristic ways. This will be the subject of the next chapter. The point is that economic behaviour fully extends over the realm of connections and systems and there is a primary economic problem associated with this phenomenon: how do we, both as individuals and as collectives, make good choices of the connections that build systems?

There are many symptoms of this overly narrow circumscription of the economic problem to exclude the choice of connections, and thereby denying the existence of systems, and as such of higher-level structure. Broadly these come under the rubric of the so-called crisis in economic theory that has surfaced sporadically over the past few decades, but for any particular economist it is perhaps the sometimes suspicion that there is something fundamental missing from the picture. Whatever these may be in the particular, they can be gathered under a common theoretical form: connections, and therefore systems. And as such, the first principles of a new departure in microeconomic theory proceed from this ontological reassessment of the status of connections.

Notes

1 As such, a more compact description might be: HC1: There exist economic agents. HC2: There exists a field. The following concepts are embedded within the field: (i) preferences, (ii) optimization, (iii) markets, (iv) knowledge, (v) coordination (Also technology.)
3 Although it does not follow that connections necessarily exhibit the same epistemic or material bearing. Navigation of this aspect is the task of the particular model, or, moreover, the particular assumptions which render the model tractable and epistemically valid. The ontological assertion
Economic systems

does nothing more than instate the logic into the theory (a model of course being a particular interpretation of a theory).

It should be noted that graph theory is not entirely unknown in economic theory (see Kirman 1983, 1987b). For instance, Mirowski (1991a) employs this mathematical form to describe the structure of a closed loop of bilateral exchange and Kirman (1997b: 496) has described the Walrasian system of exchange as a complete graph.

As Wilson (1985: 10) remarks, 'the language of graph theory is decidedly non-standard—every author has his own terminology'. What I call an element, graph theorists have variously termed a vertex, point, node, dot, cell, 0-simplex or junction. It seems to me that graph theorists use the terms 'system' and 'connection' when they want to be understood, and other terms (such as arc, join, loop, and so on) when they want to be precise.

This involves the concept of the centre of a graph, which is a vertex v with the property that the maximum of the distances between v and other vertices of S is as small as possible. It is a theorem that every connected graph has either one centre or two adjacent centres. The lowest moment occurs at the centre of a graph, the highest moment at a terminal vertex.

A random graph is a graph where the vertices (connections) are assigned in some random way (Kaufman 1993: 205, 307). The main question of interest in the study of random graphs is when percolation thresholds, as large connected webs of elements, will form. These have been investigated as corresponding to emergence behaviour (for example, crystalization or superfluidity).

Holland (1998) has recently advanced the proposition that emergence is a separate study in itself, distinct, for instance, from dynamics.

See, for instance, Sklar (1974: 42–54) for discussion of the relation between geometry, algebra and space.