If the great nineteenth-century physicist James Clerk Maxwell were to attend a current meeting of the American Physical Society, he might have serious difficulty keeping track of what was going on. In the field of economics, on the other hand, his contemporary John Stuart Mill would easily pick up the thread of the most advanced arguments among his twentieth-century successors. Physics, applying the method of inductive reasoning from quantitatively observed events, has moved on to entirely new premises. The science of economics, in contrast, remains largely a deductive system resting upon a static set of premises, most of which were familiar to Mill and some of which date back to Adam Smith's *The Wealth of Nations*.

Present-day economists are not universally content with this state of affairs. Some of the greatest recent names in economics—Léon Walras, Vilfredo Pareto, Irving Fisher—are associated with the effort to develop quantitative methods for grappling with the enormous volume of empirical data that is involved in every real economic situation. Yet such methods have so far failed to find favor with the majority of professional economists. It is not only the forbidding rigor of mathematics; the truth is that such methods have seldom produced results significantly superior to those achieved by the traditional procedure. In an empirical science, after all, nothing ultimately counts but results. Most economists therefore continue to rely upon their “professional intuition” and “sound judgment”
to establish the connection between the facts and the theory of economics.

In recent years, however, the output of economic facts and figures by various public and private agencies has increased by leaps and bounds. Most of this information is published for reference purposes and is unrelated to any particular method of analysis. As a result we have in economics today a high concentration of theory without fact on the one hand, and a mounting accumulation of fact without theory on the other. The task of filling the “empty boxes of economic theory” with relevant empirical content becomes every day more urgent and challenging.

This chapter is concerned with a new effort to combine economic facts and theory, known as interindustry or input-output analysis. Essentially it is a method of analysis that takes advantage of the relatively stable pattern of the flow of goods and services among the elements of our economy to bring a much more detailed statistical picture of the system into the range of manipulation by economic theory. As such, the method has had to await the modern high-speed computing machine as well as the present propensity of government and private agencies to accumulate mountains of data. It is now advancing from the phase of academic investigation and experimental trial to a broadening sphere of application in grand-scale problems of national economic policy. The practical possibilities of the method are being carried forward as a cooperative venture of the Bureau of Labor Statistics, the Bureau of Mines, the Department of Commerce, the Bureau of the Budget, the Council of Economic Advisers, and, with particular reference to procurement and logistics, the Air Force. Meanwhile, the development of the technique of input-output analysis continues to interest academic investigators here and abroad. They are hopeful that this method of bringing the facts of economics into closer association with theory may induce some fruitful advances in both.

II

Economic theory seeks to explain the material aspects and operations of our society in terms of interactions among such variables as supply and demand or wages and prices. Economists have generally based their analyses on relatively simple data—such quantities as the gross national product, the interest rate, price and wage levels. But in the real world things are not so simple. Between a shift in wages and the ultimate working out of its impact upon prices there is a complex series of transactions in which actual goods and services
are exchanged among real people. These intervening steps are scarcely suggested by the classical formulation of the relationship between the two variables. It is true, of course, that the individual transactions, like individual atoms and molecules, are far too numerous for observation and description in detail. But it is possible, as with physical particles, to reduce them to some kind of order by classifying and aggregating them into groups. This is the procedure employed by input-output analysis in improving the grasp of economic theory upon the facts with which it is concerned in every real situation.

The essential principles of the method may be most easily comprehended by consulting Table 1-1, which summarizes the transactions that characterized the U.S. economy during 1947. The transactions are grouped into 42 major departments of production, distribution, transportation, and consumption, set up on a matrix of horizontal rows and vertical columns. The horizontal rows of figures show how the output of each sector of the economy is distributed among the others. Conversely, the vertical columns show how each sector obtains from the others its needed inputs of goods and services. Since each figure in any horizontal row is also a figure in a vertical column, the output of each sector is shown to be an input in some other. The double-entry bookkeeping of the input-output table thus reveals the fabric of our economy, woven together by the flow of trade which ultimately links each branch and industry to all others. Such a table may, of course, be developed in as fine or as coarse detail as the available data permit and the purpose requires. The present table summarizes a much more detailed 500-sector master table which had just been completed in 1951 after two years of intensive work by the Interindustry Economics Division of the Bureau of Labor Statistics.

III

For purposes of illustration let us look at the input-output structure of a single sector—the one labeled “primary metals” (sector 14).

1Preliminary data for Table 1-1 were compiled by the Bureau of Labor Statistics. Each number in the body of the table represents billions of 1947 dollars. In the vertical column at left the entire economy is broken down into sectors; in the horizontal row at the top the same breakdown is repeated. When a sector is read horizontally, the numbers indicate what it ships to other sectors. When a sector is read vertically, the numbers show what it consumes from other sectors. The asterisks stand for sums less than $5 million. Totals may not check due to rounding.
Table 1-1

Exchange of goods and services in the U.S. for 1947
<table>
<thead>
<tr>
<th>Purchasing</th>
<th>Durable goods</th>
<th>Non-durable goods</th>
<th>Total</th>
<th>Consumer services</th>
</tr>
</thead>
<tbody>
<tr>
<td>Businesses</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>1.00</td>
<td>1.00</td>
<td>2.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Consumer Goods</td>
<td>0.25</td>
<td>0.25</td>
<td>0.50</td>
<td>0.25</td>
</tr>
<tr>
<td>Services</td>
<td>0.75</td>
<td>0.75</td>
<td>1.50</td>
<td>0.75</td>
</tr>
<tr>
<td>Public Sector</td>
<td>0.50</td>
<td>0.50</td>
<td>1.00</td>
<td>0.50</td>
</tr>
<tr>
<td>Private Sector</td>
<td>0.50</td>
<td>0.50</td>
<td>1.00</td>
<td>0.50</td>
</tr>
<tr>
<td>Total</td>
<td>1.00</td>
<td>1.00</td>
<td>2.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>

ARE INCLUDED IN HOUSEHOLD BOW

3.75  5.04  1.08  1.20  2.35  5.35  4.14  6.86  3.41  3.39  1.95  0.90  2.17  5.11
14.05 13.57  2.82  3.81  4.84 18.69 10.40 15.25  8.38 14.27  4.00  2.12  4.76  9.21
<table>
<thead>
<tr>
<th>Industry Category</th>
<th>Sector 23</th>
<th>Sector 24</th>
<th>Sector 25</th>
<th>Sector 26</th>
<th>Sector 27</th>
<th>Sector 28</th>
<th>Sector 29</th>
<th>Sector 30</th>
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<tbody>
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<td>Agriculture and Fisheries</td>
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<tr>
<td>Food and Kindred Products</td>
<td>0.08</td>
<td>0.01</td>
<td>0.03</td>
<td>0.07</td>
<td>0.01</td>
<td></td>
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<td></td>
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<tr>
<td>Textile Mill Products</td>
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<td></td>
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<td></td>
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<tr>
<td>Apparel</td>
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<tr>
<td>Lumber and Wood Products</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Furniture and Fixtures</td>
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<td></td>
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</tr>
<tr>
<td>Paper and Allied Products</td>
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<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Printing and Publishing</td>
<td>0.04</td>
<td>0.02</td>
<td>0.10</td>
<td>0.03</td>
<td>0.21</td>
<td>2.45</td>
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<tr>
<td>Chemicals</td>
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<td>0.01</td>
<td>0.02</td>
<td>0.07</td>
<td>0.01</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Products of Petroleum and Coal</td>
<td>0.27</td>
<td>0.09</td>
<td>0.45</td>
<td>0.20</td>
<td>0.01</td>
<td>0.78</td>
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<tr>
<td>Rubber Products</td>
<td>0.13</td>
<td>0.06</td>
<td>0.01</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Leather and Leather Products</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stone, Clay, and Glass Products</td>
<td>0.01</td>
<td>0.04</td>
<td></td>
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<td>Primary Metals</td>
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<tr>
<td>Fabricated Metal Products</td>
<td>0.03</td>
<td>0.01</td>
<td>0.06</td>
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<td></td>
<td></td>
<td></td>
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<tr>
<td>Machinery (Except Electric)</td>
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<td>0.02</td>
<td></td>
<td></td>
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<td></td>
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<tr>
<td>Electrical Machinery</td>
<td>0.04</td>
<td>0.01</td>
<td>0.05</td>
<td>0.01</td>
<td></td>
<td></td>
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<tr>
<td>Motor Vehicles</td>
<td>0.13</td>
<td>0.02</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Other Transportation Equipment</td>
<td>0.04</td>
<td>0.08</td>
<td>0.13</td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Professional and Scientific Equipment</td>
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<tr>
<td>Miscellaneous Manufacturing Industries</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coal, Gas, and Electric Power</td>
<td>0.44</td>
<td>0.09</td>
<td>0.40</td>
<td>0.01</td>
<td>0.06</td>
<td>3.15</td>
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<tr>
<td>Railroad Transportation</td>
<td>0.41</td>
<td>0.06</td>
<td>0.06</td>
<td>0.01</td>
<td>0.42</td>
<td>0.03</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ocean Transportation</td>
<td>0.22</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Other Transportation</td>
<td>0.19</td>
<td>0.04</td>
<td>0.25</td>
<td>0.31</td>
<td>0.13</td>
<td>0.03</td>
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<td></td>
</tr>
<tr>
<td>Trade</td>
<td>0.03</td>
<td>0.01</td>
<td>0.42</td>
<td>0.30</td>
<td>0.01</td>
<td>0.75</td>
<td>0.14</td>
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<td>Communications</td>
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<td>0.04</td>
<td>0.33</td>
<td>0.06</td>
<td>0.09</td>
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<tr>
<td>Finance and Insurance</td>
<td>0.02</td>
<td>0.13</td>
<td>0.30</td>
<td>1.00</td>
<td>1.85</td>
<td>0.56</td>
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<tr>
<td>Real Estate and Rentals</td>
<td>0.02</td>
<td>0.01</td>
<td>0.15</td>
<td>1.96</td>
<td>0.05</td>
<td>0.21</td>
<td>0.21</td>
<td>0.09</td>
</tr>
<tr>
<td>Business Services</td>
<td>0.02</td>
<td>0.03</td>
<td>1.71</td>
<td>0.09</td>
<td>0.14</td>
<td>0.04</td>
<td>0.08</td>
<td></td>
</tr>
<tr>
<td>Personal and Repair Services</td>
<td>0.11</td>
<td>0.01</td>
<td>0.26</td>
<td>1.42</td>
<td>0.02</td>
<td>0.11</td>
<td>0.03</td>
<td>0.07</td>
</tr>
<tr>
<td>Nonprofit Organizations</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
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<td>Amusements</td>
<td>0.04</td>
<td>0.39</td>
<td>0.01</td>
<td>0.11</td>
<td>0.03</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Eating and Drinking Places</td>
<td>0.01</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>New Construction and Maintenance</td>
<td>1.12</td>
<td>0.13</td>
<td>0.18</td>
<td>0.18</td>
<td>0.93</td>
<td>4.08</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Undistributed</td>
<td>0.10</td>
<td>0.04</td>
<td>0.03</td>
<td>3.59</td>
<td>0.01</td>
<td>0.71</td>
<td>0.30</td>
<td>0.31</td>
</tr>
<tr>
<td>Inventory Change (Depletions)</td>
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<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
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<tr>
<td>Foreign Countries (Imports from)</td>
<td>0.04</td>
<td>0.50</td>
<td>0.08</td>
<td>0.03</td>
<td>0.10</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Government</td>
<td>0.91</td>
<td>0.28</td>
<td>0.77</td>
<td>3.30</td>
<td>0.44</td>
<td>1.11</td>
<td>4.00</td>
<td>0.21</td>
</tr>
<tr>
<td>Private Capital Formation (Gross)</td>
<td>5.70</td>
<td>0.90</td>
<td>0.29</td>
<td>1.42</td>
<td>3.15</td>
<td>7.33</td>
<td>14.05</td>
<td>1.68</td>
</tr>
<tr>
<td>Households</td>
<td>9.95</td>
<td>2.29</td>
<td>9.88</td>
<td>41.66</td>
<td>3.17</td>
<td>12.81</td>
<td>28.86</td>
<td>5.10</td>
</tr>
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</table>

Table I-1 (Cont.)
<table>
<thead>
<tr>
<th>Purchasing</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.36</td>
</tr>
<tr>
<td>0.36</td>
</tr>
<tr>
<td>0.36</td>
</tr>
<tr>
<td>0.36</td>
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<tr>
<td>0.36</td>
</tr>
<tr>
<td>0.36</td>
</tr>
<tr>
<td>0.36</td>
</tr>
<tr>
<td>0.36</td>
</tr>
</tbody>
</table>

**Notes:**
- Purchasing: Specific details about purchasing strategies and processes.
The vertical column states the inputs of each of the various goods and services that are required for the production of metals, and the sum of the figures in this column represents the total outlay of the economy for the year's production. Most of the entries in this column are self-explanatory. Thus it is no surprise to find a substantial figure entered against the item "products of petroleum and coal" (sector 10). The design of the table, however, gives a special meaning to some of the sectors. The outlay for "railroad transportation" (sector 23), for example, covers only the cost of hauling raw materials to the mills; the cost of delivering primary metal products to their markets is borne by the industries purchasing them. Another outlay requiring explanation is entered in the trade sector (sector 26). The figures in this sector represent the cost of distribution, stated in terms of the trade margin. The entries against trade in the primary metals column, therefore, cover the middleman's markup on the industry's purchase; trade margins on the sale of primary metal products are charged against the consuming industries. Taxes paid by the industry are entered in the row labeled "government" (sector 40), and all payments to individuals, including wages, salaries, and dividends, are summed up in the row labeled "households" (sector 42). How the output of the metals industry is distributed among the other sectors is shown in row 14. The figures indicate that the industry's principal customers are other industries. "Households" and "government" turn up as direct customers for only a minor portion of the total output, although these two sectors are, of course, the principal consumers of metals after they have been converted into end products by other industries.

Coming out of the interior of the table to the outer row and columns, the reader may soon recognize many of the familiar total figures by which we are accustomed to visualize the condition of the economy. The total outputs at the end of each industry row, for example, are the figures we use to measure the size or the health of an industry. The gross national product, which is designed to state the total of productive activity and is the most commonly cited index for the economy as a whole, may be derived as the grand total of the five columns grouped under the heading of "final demand," but with some adjustments necessary to eliminate the duplication of transactions between the sectors represented by these columns. For example, the total payment to households, at the far right end of row 42, includes salaries paid by government, a figure that duplicates in part the payment of taxes by households included in the total payment to government.
With this brief introduction lay economists are now qualified to turn around and trace their way back into the table via whatever chain of interindustry relationships engages their interest. They will not go far before they find themselves working intuitively with the central concept of input–output analysis. This is the idea that there is a fundamental relationship between the volume of the output of an industry and the size of the inputs going into it. It is obvious, for example, that the purchases of the auto industry (column 18) from the glass industry (row 13) in 1947 were strongly determined by the number of motor vehicles produced that year. Closer inspection will lead to the further realization that every single figure in the chart is dependent upon every other. To take an extreme example, the appropriate series of inputs will show that the auto industry’s purchases of glass are dependent in part upon the demand for motor vehicles arising out of the glass industry’s purchases from the fuel industries.

These relationships reflect the structure of our technology. They are expressed in input-output analysis as the ratios or coefficients of each input to the total output of which it becomes a part. A graph of such ratios in Figure 1-1, computed from a table for the economy as of 1939, shows how much had to be purchased from the steel, glass, paint, rubber, and other industries to produce $1000 worth of automobile that year. Since such expenditures are determined by relatively inflexible engineering considerations or by equally inflexible customs and institutional arrangements, these ratios might be used to estimate the demand for materials induced by auto production in other years. With a table of ratios for the economy as a whole, it is possible in turn to calculate the secondary demand on the output of the industries that supply the auto industry’s suppliers and so on through successive outputs and inputs until the effect of the final demand for automobiles has been traced to its last reverberation in the farthest corner of the economy. In this fashion input-output analysis should prove useful to the auto industry as a means for dealing with cost and supply problems.

The graph of steel consumption ratios (Figure 1-2) suggests, incidentally, how the input–output matrix might be used for the contrasting purpose of market analysis. Since the ultimate markets for steel are ordinarily buried in the cycle of secondary transactions among the metal-fabricating industries, it is useful to learn from this table how many tons of steel at the mill were needed in 1939 to
Figure 1-1

The input to auto industry from other industries per $1000 of auto production was derived from the 1939 interindustry table. Comparing these figures with those for the auto industry in the 1947 table would show changes in input structure of the industry due to changes in prices and technology.
Input-output economics

Figure 1.2

The output of the steel industry depends heavily on what kinds of goods are demanded in the ultimate market. This table shows the amount of steel required to meet each $1000 of the demand for other goods in 1939. The current demand for the top three items is responsible for the steel shortage.

satisfy each $1000 worth of demand for the products of industries that ultimately place steel products at the disposal of the consumer. This graph shows the impressively high ratio of the demand for steel in the construction and consumer durable-goods industries which led the Bureau of Labor Statistics to declare in 1945 that a flourishing postwar economy would require even more steel than the peak of the war effort. Though some industry spokesmen took a contrary position at that time, steel production in 1951 had been exceeding World War II peaks, and the major steel companies were then engaged in a 16-million-ton expansion program which was started even before the outbreak of the war in Korea and the subsequent rearmament.

The ratios shown in Figures 1.1 and 1.2 are largely fixed by technology. Others in the complete matrix of the economy, especially in the trade, services, and households sectors, are established by custom and other institutional factors. All, of course, are subject to
Input-output economics

modification by such forces as progress in technology and changes in public taste. But whether they vary more or less rapidly over the years, these relationships are subject to dependable measurement at any given time.

Here we have our bridge between theory and facts in economics. It is a bridge in a very literal sense. Action at a distance does not happen in economics any more than it does in physics. The effect of an event at any one point is transmitted to the rest of the economy step by step via the chain of transactions that links the whole system together. A table of ratios for the entire economy gives us, in as much detail as we require, a quantitatively determined picture of the internal structure of the system. This makes it possible to calculate in detail the consequences that result from the introduction into the system of changes suggested by the theoretical or practical problem at hand.

In the case of a particular industry, we can easily compute the complete table of its input requirements at any given level of output, provided we know its input ratios. By the same token, with somewhat more involved computation, we can construct synthetically a complete input-output table for the entire economy. We need only a known "bill of final demand" to convert the table of ratios into a table of magnitudes. The 1945 estimate of postwar steel requirements, for example, was incidental to a study of the complete economy based upon a bill of demand that assumed full employment in 1950. This bill of demand was inserted into the total columns of a table of ratios based on the year 1939. By arithmetical procedures the ratios were then translated into dollar figures, among which was the figure for steel, which showed a need for an absolute minimum of 98 million ingot tons. Actual production in 1950, at the limit of capacity, was 96.8 million tons.

V

Though its application is simple, the construction of an input-output table is a highly complex and laborious operation. The first step, and one that has little appeal to the theoretical imagination, is the gathering and ordering of an immense volume of quantitative information. Given the inevitable lag between the accumulation and the collection of data for any given year, the input-output table will always be a historical document. The first input-output tables, prepared by the author and his associates at Harvard University in the early 1930s, were based upon 1919 and 1929 figures. The 1939 table was
Input-output economics

not completed until 1944. Looking to the future, a table for 1953, which is now under consideration, could not be made available until 1957. For practical purposes the original figures in the table must be regarded as a base, subject to refinement and correction in accord with subsequent trends. For example, the 1945 projection of the 1950 economy on the basis of the 1939 table made suitable adjustments in the coal and oil input ratios of the transportation industries on the assumption that the trend from steam to diesel locomotives would continue throughout the period.

The basic information for the table and its continuing revision comes from the Bureau of the Census and other specialized statistical agencies. As the industrial breakdown becomes more detailed, however, engineering and technical information plays a more important part in determining the data. A perfectly good way to determine how much coke is needed to produce a ton of pig-iron, in addition to dividing the output of the blast furnace industry into its input of coke, is to ask an ironmaster. In principle there is no reason why the input-output coefficients should not be entirely derived from “below,” from engineering data on process design and operating practice. Thus, in certain studies of the German economy made by the Bureau of Labor Statistics following World War II, the input structures of key industries were set up on the basis of U.S. experience. The model of a disarmed but self-supporting Germany developed in these studies showed a steel requirement of 11 million ingot tons, toward which actual output is now moving. Completely hypothetical input structures, representing industries not now operating, have been introduced into tables of the existing U.S. economy in studies conducted by Air Force economists.

VI

This brings us to the problem of computation. Since the production level required of each industry is ultimately dependent upon levels in all others, it is clear that we have a problem involving simultaneous equations. Though the solution of such equations may involve no very high order of mathematics, the sheer labor of computation

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4In November 1964 the U.S. Department of Commerce published a preliminary version and in September 1965 the final version of the newly compiled input-output table of the U.S. economy for the census year 1958. It contained 86 producing and 5 final demand sectors. In releasing these figures the Secretary of Commerce announced that in the future up-to-date input-output tables would be published “as an integral part of national income and product accounts.”
can be immense. The number of equations to be solved is always equal to the number of sectors into which the system is divided. Depending upon whether a specific or a general solution of the system is desired, the volume of computation will vary as the square or the cube of the number of sectors involved. A typical general solution of a 42-sector table for 1939 required 56 hours on the Harvard Mark II computer. Thanks to this investment in computation, the conversion of any stipulated bill of demand into the various industrial production levels involves nothing more than simple arithmetic. The method cannot be used, however, in the solution of problems that call for changes in the input-output ratios, since each change requires a whole new solution of the matrix. For the larger number of more interesting problems that require such changes, special solutions are the rule. However, even a special solution on a reasonably detailed 200-sector table might require some 200,000 multiplications and a greater number of additions. For this reason it is likely that the typical nongovernmental user will be limited to condensed general solutions periodically computed and published by special-purpose groups working in the field. With these the average industrial analyst will be able to enjoy many of the advantages of the large and flexible machinery required for government analyses relating to the entire economy.

A demonstration of input-output analysis applied to a typical economic problem is presented in Figure 1-3, which shows the price increases that would result from a general 10 percent increase in the wage scale of industry. Here the value of the matrix distinguishing between direct and indirect effects is of the utmost importance. If wages constituted the only ultimate cost in the economy, a general 10 percent rise in all money wages would obviously lead to an equal increase in all prices. Since wages are only one cost and since labor costs vary from industry to industry, it can be seen in the chart that a 10 percent increase in wages would have decidedly different effects upon various parts of the economy. The construction industry shows the greatest upward price change, as it actually did in recent decades. For each industry group the chart separates the different effect of increases in its own wage bill from the indirect effects of the wage increases in other industries from which it purchases its inputs. Giving effect to both direct and indirect increases, the average increase in the cost of living is shown in the chart to be only 3.7 percent. The 10 percent money-wage increase thus yields a 6.3 percent increase in real wage rates. It should be noted, however, that the economic forces that bring increases in wages tend to
Figure 1-3

The price increases that would be caused by a 10 percent increase in wages were computed from the 1939 interindustry table. The increases include the direct effect of the rise in each industry's own wage bill (black bars) and the indirect effect of price increases on purchases from others (shading).

bring increases in other costs as well. The advantage of the input-output analysis is that it permits the disentanglement and accurate measurement of the indirect effects. Analyses similar to this one for wages can be carried through for profits, taxes, and other ultimate components of prices.
In such examples changes in the economy over periods of time are measured by comparing before and after pictures. Each is a static model, a cross-section in time. The next step in input-output analysis is the development of dynamic models of the economy to bring the approximations of the method that much closer to the actual processes of economics. This requires accounting for stocks as well as flows of goods, for inventories of goods in process and in finished form, for capital equipment, for buildings, and, last but not least, for dwellings and household stocks of durable consumer goods. The dynamic input-output analysis requires more advanced mathematical methods; instead of ordinary linear equations it leads to systems of linear differential equations.

Among the questions the dynamic system should make it possible to answer, one could mention the determination of the changing pattern of outputs and inventories or investments and capacities that would attend a given pattern of growth in final demand projected over a five- or ten-year period. Within such broad projections, for example, we would be able to estimate approximately not only how much aluminum should be produced but how much additional aluminum-producing capacity would be required and the rate at which such capacity should be installed. The computational task becomes more formidable, but it does not seem to exceed the capacity of the latest electronic computers. Here, as in the case of the static system, the most laborious problem is the assembly of the necessary factual information. However, a complete set of stock or capital ratios, paralleling the flow ratios of all of the productive sectors of the U.S. economy for the year 1939, has now been completed.

This table of capital ratios shows that in addition to the flow of raw pig-iron, scrap, coal, labor, and so on, the steel works and rolling mills industry—when operating to full capacity—required $1800 of fixed investment for each $1000 worth of output. This would include $336 worth of tools, $331 worth of iron and steel foundry production, and so on, down to $26 worth of electrical equipment. This means that in order to expand its capacity so as to be able to increase its output by $1 million worth of finished products annually, the steel works and rolling mills industry would have to install $336,000 worth of tools and spend corresponding amounts on all other types of new fixed installations. This investment demand constitutes, of course, additional input requirements for the product of the corresponding capital goods industries, input requirements that are automatically taken into account in the solution of an appropriate system of dynamic input-output equations.
Input-output analysis is a method of systematically quantifying the mutual interrelationships among the various sectors of a complex economic system. In practical terms, the economic system to which it is applied may be as large as a nation or even the entire world economy, or as small as the economy of a metropolitan area or even a single enterprise.

In all instances the approach is essentially the same. The structure of each sector's production process is represented by an appropriately defined vector of structural coefficients that describes in quantitative terms the relationship between the inputs it absorbs and the output it produces. The interdependence among the sectors of the given economy is described by a set of linear equations expressing the balances between the total input and the aggregate output of each commodity and service produced and used in the course of one or several periods of time.

The technical structure of the entire system can accordingly be represented concisely by the matrix of technical input-output coefficients of all its sectors. It constitutes at the same time the set of parameters on which the balance equations are based.

I. Input-output tables

An input-output table describes the flow of goods and services between all the individual sectors of a national economy over a
Table 2-1  
Simplified input-output table for a three-sector economy

<table>
<thead>
<tr>
<th>from</th>
<th>into</th>
<th>Sector 1: Agriculture</th>
<th>Sector 2: Manufacture</th>
<th>Sector 3: Households</th>
<th>Total Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sector 1:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Agriculture</td>
<td></td>
<td>25</td>
<td>20</td>
<td>55</td>
<td>100 bushels of wheat</td>
</tr>
<tr>
<td>Sector 2:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Manufacture</td>
<td></td>
<td>14</td>
<td>6</td>
<td>30</td>
<td>50 yards of cloth</td>
</tr>
<tr>
<td>Sector 3:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Households</td>
<td></td>
<td>80</td>
<td>180</td>
<td>40</td>
<td>300 man-years of labor</td>
</tr>
</tbody>
</table>

stated period of time, say, a year. A simplified example of an input-output table depicting a three-sector economy is shown in Table 2-1. The three sectors are agriculture, whose total annual output amounts to 100 bushels of wheat; manufacture, which produced 50 yards of cloth; and households, which supplied 300 man-years of labor. The nine \((3 \times 3)\) entries inside the main body of the table show the intersectoral flows. Of the 100 bushels of farm products turned out by the agriculture, 25 bushels were used up within the agricultural sector itself, 20 were delivered to and absorbed as one of its inputs by manufacture, and 55 were taken by the households sector. The second and third rows of the table describe in the same way the allocation of outputs of the two other sectors.

The figures entered in each column of the table thus describe the input structure of the corresponding sector. To produce the 100 bushels of its total output, agriculture absorbed 25 bushels of its own products, 14 yards of manufactured goods, and 80 man-years of labor received from the households. The manufacturing sector to be able to produce the 50 yards of its total output, had to receive and use up 20 bushels of agricultural—and 6 yards of its own (i.e., of manufactured)—products as well as 180 man-years of labor from households. In their turn, the households have spent the incomes—which they have received for supplying 300 man-years of labor—to pay for the consumption of 55 bushels of agricultural and 30 yards of manufactured commodities and 40 man-years of direct services of labor.

All entries in Table 2-1 are supposed to represent quantities, or at least physical indices of the quantities, of specific goods or services. A less aggregative, more detailed input-output table describing the same national economy in terms of not 3 but of 50, 100, or even 1000 different sectors, would permit a more specific qualita-
Input-output analysis

tive identification of all the individual entries. In a larger table, manu-
ufacturing would, for example, be represented not by one but by
many distinct industrial sectors; its output—and consequently also
the inputs of the other sectors—would be described in terms of
“yards of cotton cloth,” “tons of paper products,” or even “yards
of percale,” “yards of heavy cotton cloth,” as well as “tons of news-
print” and “tons of writing paper.”

Input-output tables and national income accounts

Although in principle the intersectoral flows as represented in an
input-output table can be thought of as being measured in physical
units, in practice most input-output tables are constructed in value
terms. Table 2-2 represents a translation of Table 2-1 into value
terms on the assumption that the price of agricultural products is $2
per bushel, the price of manufactured goods is $5 per yard, and the
price of services supplied by the household sector is $1 per man-
year. Thus the values of total outputs of the agricultural, the manu-
facturing, and the households sectors are shown in the new trans-
lated table as being equal to, respectively, $200 (= 100 × 2), $250
(= 50 × 5) and $300 (= 300 × 1). The last row shows the com-
bined value of all outputs absorbed by each of the three sectors.
Such column totals could not have been shown in Table 2-1 since
the physical quantities of different inputs absorbed by each sector
cannot be meaningfully added.

The input-output table expressed in value terms can be inter-
preted as a system of national accounts. The $300 showing the value
of services rendered by the households over the period of the years
obviously represents the annual national income. It equals the sum

*Table 2-2*

<table>
<thead>
<tr>
<th>from</th>
<th>into</th>
<th>Sector 1: Agriculture ($)</th>
<th>Sector 2: Manufacture ($)</th>
<th>Sector 3: Households ($)</th>
<th>Total Output ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sector 1:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Agriculture</td>
<td>50</td>
<td>40</td>
<td>110</td>
<td>200</td>
<td></td>
</tr>
<tr>
<td>Sector 2:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Manufacture</td>
<td>70</td>
<td>30</td>
<td>150</td>
<td>250</td>
<td></td>
</tr>
<tr>
<td>Sector 3:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Households</td>
<td>80</td>
<td>180</td>
<td>40</td>
<td>300</td>
<td></td>
</tr>
<tr>
<td>Total Input ($)</td>
<td>220</td>
<td>250</td>
<td>300</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
total of the income payments—shown in row 3—received by the households for services rendered to each sector; it also equals the combined value of goods and services—as shown in column 3—purchased by the households from themselves and from the other sectors. To the extent to which the column entries (showing the input structure of each productive sector) cover the current expenditures but not purchases made on capital account, the latter—being paid out of the net income—should be entered in the households column.

All figures in Table 2.2—except the column sums shown in the bottom row—can also be interpreted as representing physical quantities of the goods or services to which they refer. This only requires that the physical unit in which the entries in each row are measured be redefined as being equal to that amount of output of that particular sector that can be purchased for $1 at prices that prevailed during the interval of time for which the table was constructed.

National input-output tables are now constructed in some 80 countries. Many regional and metropolitan input-output tables have also been compiled. The number of sectors that describe the economic system has increased dramatically in recent years. Some of the more detailed tables describe a national economy in terms of 500 or 600 separate sectors.

II. Technical coefficients

Let the national economy be subdivided into \( n + 1 \) sectors; \( n \) industries, that is, producing sectors and the \( n + 1 \)th final demand sector, represented in input-output Tables 2-1 and 2-2 by the households. For purposes of mathematical manipulation, the physical output of sector \( i \) is usually represented by \( x_i \) while the symbol \( x_q \) stands for the amount of the product of sector \( i \) absorbed—as its input—by sector \( j \). The quantity of the product of sector \( i \) delivered to the final demand sector \( x_{n+1} \) is usually identified in short as \( y_i \).

The quantity of the output of sector \( i \) absorbed by sector \( j \) per unit of its total output \( j \) is described by the symbol \( a_{ij} \) and is called the input coefficient of product of sector \( i \) into sector \( j \).

\[
(2-1) \quad a_{ij} = \frac{x_{ij}}{x_j}
\]

A complete set of the input coefficients of all sectors of a given economy arranged in the form of a rectangular table—corresponding to the input-output table of the same economy—is called the struc-
Table 2-3
Simplified structural coefficient matrix of three-sector economy

<table>
<thead>
<tr>
<th>from</th>
<th>into</th>
<th>Sector 1: Agriculture</th>
<th>Sector 2: Manufacture</th>
<th>Sector 3: Households</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sector 1:</td>
<td></td>
<td>0.25</td>
<td>0.40</td>
<td>0.133</td>
</tr>
<tr>
<td>Agriculture</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sector 2:</td>
<td></td>
<td>0.14</td>
<td>0.12</td>
<td>0.100</td>
</tr>
<tr>
<td>Manufacture</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sector 3:</td>
<td></td>
<td>0.80</td>
<td>3.60</td>
<td>0.133</td>
</tr>
<tr>
<td>Households</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Structural matrix of that economy. Table 2-3 represents the structural matrix of the economy whose flow matrix is shown on Table 2-1. The flow matrix constitutes the usual—although not necessarily the only possible—source of empirical information in the input structure of the various sectors of an economy. The entries in Table 2-3 are computed according to formula (2-1) from figures presented in Table 2-2:

\[
(2-1) \quad a_{11} = \frac{25}{100} = 0.25 \quad a_{14} = \frac{20}{50} = 0.40 \quad \text{etc.}
\]

In practice the structural matrices are usually computed from input-output tables described in value terms, such as Table 2-2. In any case, the input coefficients—for analytical purposes described below—must be interpreted as ratios of two quantities measured in physical units. To emphasize this fact, we derived the structural matrix (Table 2-3) in this example from Table 2-1, not Table 2-2.

Static input-output system

The balance between the total output and the combined inputs of the product of each sector, as shown in our example in Tables 2-1 and 2-2, can be described by the following set of \( n \) equations:

\[
(2-2) \quad \begin{align*}
(x_1 - x_{11}) - x_{12} - \ldots - x_{1n} &= y_1 \\
-x_{21} + (x_2 - x_{22}) - \ldots - x_{2n} &= y_2 \\
\ldots & \ldots \\
-x_{n1} - x_{n2} - \ldots + (x_n - x_{nn}) &= y_n
\end{align*}
\]

A substitution of equations (2-1) in (2-2) yields \( n \) general equilibrium relationships between the total outputs, \( x_1, x_2, \ldots x_n \), of all pro-
ducing sectors and the final bill of goods, \( y_1, y_2, \ldots, y_n \), absorbed by households, government, and other final users:

\[
\begin{align*}
(1 - a_{11})x_1 - a_{12}x_2 - \cdots - a_{1n}x_n &= y_1 \\
-a_{21}x_1 + (1 - a_{22})x_2 - \cdots - a_{2n}x_n &= y_2 \\
\cdots & \cdots \cdots \cdots \cdots \cdots \cdots \cdots \\
-a_{n1}x_1 - a_{n2}x_2 - \cdots + (1 - a_{nn})x_n &= y_n
\end{align*}
\]

(2-3)

If the final demand, \( y_1, y_2, \ldots, y_n \), that is, the quantities of all the different kinds of goods absorbed by households and all other sectors whose outputs are not represented by the variables appearing on the left-hand side of equation (2-3), are assumed to be given, the system can be solved for the \( n \) total outputs, \( x_1, x_2, \ldots, x_n \).

The general solution of these equilibrium equations for the "unknown" \( x \)'s in terms of the given \( y \)'s can be presented in the following form:

\[
\begin{align*}
x_1 &= A_{11}y_1 + A_{12}y_2 + \cdots + A_{1n}y_n \\
x_2 &= A_{21}y_1 + A_{22}y_2 + \cdots + A_{2n}y_n \\
\cdots & \cdots \cdots \cdots \cdots \\
x_n &= A_{n1}y_1 + A_{n2}y_2 + \cdots + A_{nn}y_n
\end{align*}
\]

(2-4)

The constant \( A_{ij} \) indicates by how much the output \( x_i \) of the \( i \)th sector would increase if \( y_j \), that is, the quantity of good \( j \) absorbed by households (or any other final users), had been increased by one unit. Such an increase would affect sector \( i \) directly (and also indirectly) if \( i = j \), but when \( i \neq j \) the output \( x_i \) is affected only indirectly, since sector \( i \) has to provide additional inputs to all other sectors which in their turn—directly or indirectly—must contribute to the increase in the delivery \( y_j \) made by sector \( j \) to the final users. From the computational point of view, that means that the magnitude of each coefficient \( A \) in the "solution" (2-4) in general depends on all the input coefficients \( a \) appearing on the left-hand side of the system of equilibrium equations (2-3).

In mathematical language, the matrix

\[
\begin{bmatrix}
A_{11} & A_{12} & \cdots & A_{1n} \\
A_{21} & A_{22} & \cdots & A_{2n} \\
\cdots & \cdots & \cdots & \cdots \\
A_{n1} & A_{n2} & \cdots & A_{nn}
\end{bmatrix}
\]

of constants appearing on the right-hand side of the solution (2-4) is identified as the inverse of the matrix

\[
\begin{bmatrix}
(1 - a_{11}) & -a_{12} & \cdots & -a_{1n} \\
-a_{21} & (1 - a_{22}) & \cdots & -a_{2n} \\
\cdots & \cdots & \cdots & \cdots \\
-a_{n1} & -a_{n2} & \cdots & (1 - a_{nn})
\end{bmatrix}
\]
Input-output analysis

of constants appearing on the left-hand side of equations (2-3). The computation involved in finding such a solution is called the inversion of the coefficient matrix of these original equations. The inverse of the matrix

\[
\begin{bmatrix}
1 & -0.25 \\
-0.14 & 1 - 0.12
\end{bmatrix}
\]

based on Table 2-3 is

\[
\begin{bmatrix}
1.457 & 0.6623 \\
0.2318 & 1.2417
\end{bmatrix}
\]

Inserted in solution (2-4), this yields two equations:

\[
\begin{align*}
x_1 &= 1.457y_1 + 0.6623y_2 \\
x_2 &= 0.2318y_1 + 1.2417y_2
\end{align*}
\]

which permits us to determine what total outputs of agricultural and manufacturing sectors, \(x_1\) and \(x_2\), would correspond to any given combination and the deliveries of their respective products, \(y_1\) and \(y_2\), to the exogenous sector, households. To verify this result by comparing them with the corresponding entries in Table 2-1, set \(y_1 = 55\) and \(y_2 = 30\) on the right-hand sides of the two equations, and they will yield \(x_1 = 100\) and \(x_2 = 50\) on the left.

Only if all elements \(A_0\) of the inverted matrix are nonnegative will there necessarily exist for any given set of final deliveries, \(y_1, y_2, \ldots, y_n\), a combination of positive total outputs, \(x_1, x_2, \ldots, x_n\), capable of satisfying it. A sufficient condition for this is that the determinant of the matrix,

\[
\begin{bmatrix}
a_{11} & a_{12} & \cdots & a_{1n} \\
a_{21} & a_{22} & \cdots & a_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
a_{n1} & a_{n2} & \cdots & a_{nn}
\end{bmatrix}
\]

and of all its principal submatrices,

\[
1 - a_{11} > 0 \left[ \frac{1 - a_{11}}{1 - a_{21}} \right] > 0 \ldots
\]

\[
\begin{bmatrix}
a_{11} & a_{12} & \cdots & a_{1n} \\
a_{21} & a_{22} & \cdots & a_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
a_{n1} & a_{n2} & \cdots & a_{nn}
\end{bmatrix}
\]

should be positive. If this so-called Hawkins-Simon condition is satisfied for one arbitrarily numbered sequence of sectors, it is neces-
necessarily satisfied for any other sequence too. The material inter-
pretation of that condition is that if an economic system in which each
sector functions by absorbing output of other sectors directly and
indirectly is to be able not only to sustain itself but also to make
positive delivery to final demand, each one of the smaller and
smaller subsystems contained in it must necessarily be capable of
doing so too. If even one of them cannot pass that test, it is bound
to cause a leak that will destroy the sustainability of the entire
system.

A simpler sufficient, but not necessary, condition of sustainability
of an economy is that the sum of the coefficients of each column of
its structural matrix should be less than or equal to 1, with at least
one of the column sums strictly less than 1.

In most cases in which the structural matrix of a national economy
has been derived from a set of actually observed value flows such as
are represented, for example, in Table 2-2, the condition stated
above will be satisfied.

Since in an open input-output system the households are usually
treated as a final, that is, an exogenous sector, its total output, \( x_{n+1} \),
that is, the total employment, usually does not appear as an
unknown variable on the left-hand side of system (2-3) and on the
right-hand side of its solution (2-4). After the outputs of the endog-
enous sectors, \( x_1, x_2, \ldots, x_n \), have been determined, the total
employment can be computed from the following equation:

\[
(2-6) \quad x_{n+1} = a_{n+1,1}x_1 + a_{n+1,2}x_2 + \ldots + a_{n+1,n}x_n + y_{n+1}
\]

The technical coefficients, \( a_{n+1,1}, a_{n+1,2}, \ldots, a_{n+1,n} \) represent the
inputs of labor absorbed by various industries (sectors) per unit of
their respective output; \( y_{n+1} \) is the total amount of labor directly
absorbed by households and other exogenous sectors. Such an
employment equation constructed for the three sector system with
the structural matrix shown in Table 2-3 is:

\[
(2-7) \quad x_3 = 0.80x_1 + 3.60x_2 + y_3
\]

Households must not necessarily be considered to be part of the
exogenous sectors as they are in the example used above. In dealing
with problems of income generation in its relation to employment,
the quantities of consumer goods and services absorbed by house-
holds can be considered to be structurally dependent on the total
level of employment in the same way as the quantities of coke and
ore absorbed by blast furnaces are considered to be structurally
related to the amount of pig iron produced by them. With house-
Input-output analysis

Holds shifted to the left-hand side of equations (2-2) and (2-4), the exogenous final demand appearing on their right-hand side will comprise only such items as governmental purchases and exports and, in any case, additions to or reductions in stocks of goods, that is, real investment or disinvestment.

When all sectors and all purchases are considered to be endogenous, the input-output system is called closed. A static system cannot be truly closed, since endogenous explanation of investment or disinvestment requires consideration of structural relationships between inputs and outputs that occur in different periods of time (see "Dynamic Input-Output Analysis," below).

Exports and imports

In an input-output table of a country or region that trades across its borders, exports can be entered as positive and imports as negative components of final demand. If the economy described in Table 2-1 ceased to be self-sufficient and started to import, say, 20 bushels of wheat and to export 8 yards of cloth—while letting the households consume the same amounts of both products as before—a new balance between all inputs and outputs would be established, as described in Table 2-4.

The input coefficients of the endogenous sectors, and consequently also the structural matrix of the system and its inverse, remain the same as before. To form the new column of final demand, we have to add to the quantity of each good absorbed by the households the amount that was exported and subtract the amount that was imported (i.e., imports can be treated as negative exports):

\[
y_1 = x_1 \cdot n + 1 + e_1, \quad y_2 = x_2 \cdot n + 1 + e_2, \ldots \quad y_n = x_{n+1} + e_n
\]


| Table 2-4 |
|-------------------------|-------------------------|-------------------------|-------------------------|-------------------------|
| **Final demand**        |                         |                         |                         |                         |
| from                     | into                    | Sector 1: Agriculture   | Sector 2: Manufacture   | Sector 3: Households    | Exports (+) or Imports  | Total Final Demand | Total Output       |
| Sector 1: Agriculture    | 19.04                   | 22.12                   | 55                      | -20                     | 35                      | 76.16 bushels     |
| Sector 2: Manufacture    | 10.66                   | 6.64                    | 30                      | + 8                     | 38                      | 55.30 yards       |
| Sector 3: Households     | 60.93                   | 199.07                  | 40                      |                         | 40                      | 300 man-years     |
The corresponding sectoral outputs can then be derived (see above) from the general solution (2-4). For the present numerical example, we can use directly equations (2-5). The total labor requirement of the economy—300 man-years—remains in this particular case unchanged after it enters foreign trade, because the total direct plus indirect labor content of the 20 bushels of imported wheat happens to be equal to the labor content of the 8 yards of exported cloth.

If the imports of good \( i \)—that is, the negative \( e_i \)—happens to exceed the final domestic consumption of that good \( x_{ii} \), the corresponding "net" final demand \( y_i \) will turn out to be negative. As \( y_i \) diminishes, the total output of all sectors and in particular the total output \( x_i \) must (ceteris paribus) diminish. At some point, that output will be reduced to zero, which means that the entire direct and indirect demand for that particular commodity will be covered by imports. The corresponding domestic industry will be automatically eliminated from the endogenous part of the input-output table. The imports of such goods are called noncompeting, particularly when—as in the case of coffee and certain minerals—even a large increase in demand does not call forth their domestic production. The magnitude of total domestic demand for noncompeting imports can be computed in the same way as the total demand for labor can be derived from equation (2-6).

**Prices in an open static input-output system**

Prices are determined in an open input-output system from a set of equations which states that the price that each productive sector of the economy receives per unit of its output must equal the total outlays incurred in the course of its production. These outlays comprise not only payments for inputs purchased from the same and from the other industries but also the value added, which essentially represents payments made to the exogenous sectors:

\[
(2-9) \quad (1 - a_{11})p_1 = a_{12}p_2 - \ldots - a_{1n}p_n = v_1 \\
- a_{12}p_1 + (1 - a_{22})p_2 - \ldots - a_{2n}p_n = v_2 \\
\vdots \\
- a_{1n}p_1 - a_{n2}p_2 - \ldots + (1 - a_{nn})p_n = v_n
\]

Each equation describes the balance between the price received and payments made by each endogenous sector per unit of its product; \( p_i \) represents the payments made by sector \( i \)—per unit of its product—to all exogenous (i.e., the final demand) sectors. These
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Usually comprise wages, interest on capital and entrepreneurial revenues credited to households, taxes paid to the government, and other final demand sectors.

In analogy to the solution (2.4) of output equations (2.3), the solution of the price equation (2.9) permits the determination of prices of all products from the given values added (per unit of output) in each sector:

\[ p_1 = A_{11}v_1 + A_{12}v_2 + \ldots + A_{1n}v_n \]
\[ p_2 = A_{21}v_1 + A_{22}v_2 + \ldots + A_{2n}v_n \]
\[ \ldots \]
\[ p_n = A_{n1}v_1 + A_{n2}v_2 + \ldots + A_{nn}v_n \]

The constant \( A_{ij} \) measures the dependence of the price \( p_i \) of the product of sector \( j \) on the value added, \( v_i \), earned per unit of its output in sector \( i \).

Each row of the \( a_i \) coefficients appearing in the output equations (2.3) makes up the corresponding column of coefficients appearing in price equations (2.9); the \( A_{ij} \) coefficients appearing in each row of the output solution equations (2.4) make up the corresponding coefficient column in the price solution equations (2.10).

Thus, inserting the inverse computed in the example used above in solution (2.10) of the price equation, we have:

\[ p_1 = 1.457v_1 + 0.2318v_2 \]
\[ p_2 = 0.6623v_1 + 1.2417v_2 \]

From Tables 2-2 and 2-3 we can see that in our example the values added paid out (i.e., the wages) by agriculture and in manufacture per unit at their respective outputs amounted to $0.8 and $3.6. According to the two equations above, this yields \( p_1 = $2 \), \( p_2 = $5 \), which are the prices of agricultural and manufactured products used in deriving the value figures presented in Table 2-2 from Table 2-1, which described the input-output flows only in physical units.

The internal consistency of the price and the quantity relationships within an open input-output system is confirmed by the following identity derived from equations (2.4) and (2.9):

\[ x_1v_1 + x_2v_2 + \ldots + x_nv_n = y_1p_1 + y_2p_2 + \ldots + y_np_n \]

On the left-hand side stands the sum total of value added paid out by the endogenous to the exogenous sectors of the system; on the right-hand side are the combined values (quantities times prices) of their respective products delivered by all endogenous sectors to the final (exogenous) demand. This identity confirms, in other words,
the accounting identity between the national income received and
the national income spent, as shown in Table 2-2.

For the purposes of more detailed price analysis, the technical
"cooking recipe" for producing, say, one ton of bread not only has
to specify the requisite amounts of current inputs such as flour, milk,
and yeast, but also has to list needed pots and pans and other kinds
of capital goods required for that purpose. Thus the matrix $A$ of
technical flow coefficients has to be supplemented by a correspond-
ing matrix of capital stock coefficients, $B$:

$$
B = \begin{bmatrix}
    b_{11} & b_{12} & \cdots & b_{1n} \\
    b_{21} & b_{22} & \cdots & b_{2n} \\
    \vdots & \vdots & \ddots & \vdots \\
    b_{n1} & b_{n2} & \cdots & b_{nn}
\end{bmatrix}
$$

A capital coefficient $b_{ij}$ represents the technologically determined
stock of the particular kind of goods—machine tools, industrial
buildings, "working inventories" of primary or intermediate mate-
rials—produced by industry $i$ that industry $j$ has to employ per unit
of its output. In other words, each column of matrix $B$ describes the
physical capital requirements (per unit of its total output) of a par-
ticular industry, in the same way that the corresponding column of
matrix $A$ describes its "current inputs" requirements.

The price analysis outlined above can be advanced one step fur-
ther by splitting each of the values added, appearing on the right-
hand side of each equation in (2-9), into two parts: the returns on
capital invested in buildings, machinery, and other stocks of goods
required for production of the output in question on the one hand,
and wages on the other. The return on capital can be represented
as the value (price times quantity) of all productive stocks (used per
unit of its output) in each industry multiplied by the given rate of
return.

The relationship between wage rates, the rate of return on capital
(i.e., the "price" of capital), and the price of different goods and
services takes on the following form:

$$
(2-13) \quad P = (1 - A' - rB')^{-1} W
$$

where $W$ is a column vector of wage costs paid by different indus-
tries per unit of their respective outputs.

Insertion in equation (2-13) of the numerical values of the flow
coefficient matrix $A'$ as given in Table 2-3 and of capital coefficient
matrix $B$ as given above and inverting the bracketed expression on
the right-hand side yields an explicit solution of that equation for
various values of the rate of return on capital, $r$. For instance,
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\[
\begin{align*}
\text{if } r = 10\% & \quad \text{if } r = 20\% \\
p_1 &= 1.55w + 0.26w_2 & p_1 &= 1.65w + 0.30w_2 \\
p_2 &= 0.76w_1 + 1.34w_2 & p_2 &= 0.87w + 1.45w_2
\end{align*}
\]

The magnitudes of all numerical parameters increase as the value of \( r \) is raised from 10 to 20 percent. That means that, with given wage costs \( w_1 \) and \( w_2 \), a rise in the rate of return, that is, in the costs of capital, must obviously result in higher prices \( p_1 \) and \( p_2 \). With higher prices, the purchasing power of money wages (i.e., the real wages) must necessarily fall.

III. Dynamic input-output analysis

The following set of linear difference equations represents dynamic input-output relationships employed in description and analysis of the process of economic growth.

\[
\text{(2-14)} \quad X(t) - AX(t) - B[X(t + 1) - X(t)] = Y(t)
\]

The column vectors \( X(t) \) and \( X(t + 1) \) represent the output levels of different industries in time periods \( t \) and \( t + 1 \), while the column vector \( Y(t) \) represents the amounts of various goods and services delivered in year \( t \) by these producing sectors to households and other final users. \( A \) is the matrix of input coefficients referred to above, while \( B \) is the matrix of capital coefficients described above.

The balance relationship described by equation (2-14) is based on the assumption that a good added to the capital stock in year \( t \) is put to use in the year \( t + 1 \).

In a closed version of this dynamic system, the final demand sectors are treated as if they were absorbing, like ordinary industries, inputs originating in other sectors and producing outputs—for instance, labor services—that they, in turn, deliver to other sectors. The flow and the capital coefficients reflecting the structure of households, government, and other final demand sectors appear in a closed input-output model on the left-hand side of the dynamic balance equation, side by side with all other industries, so that the column vector \( Y(t) \) becomes zero on the right-hand side, its contents having been transferred to the left-hand side.

By setting the determinant of the characteristic matrix \( |1 - A - B| \) of the resulting homogeneous system of linear difference equations equal to zero, we can determine the values of its \( n \) characteristic roots, \( \lambda_1, \lambda_2, \ldots, \lambda_n \). By the so-called Frobenius theorem, the largest of these roots is necessarily simple and positive, and so are all elements of its characteristic vector.

The reciprocal of that root, \( 1/\lambda_{\text{max}} \), represents the rate at which the
closed economy described by dynamic equations will expand, while the relative magnitudes of the elements of the characteristic vector corresponding to that root represent the relative levels of sectoral outputs (including the output of labor produced by households) that have to expand evenly from year to year.

The set of difference equations stating the price relationships corresponding to the physical relationships, described by equation (2-14), has to include, among other cost elements, interest payments on the stock of capital invested in each industry. The "real" rate of interest, that is, the money rate adjusted for the change in the general price level, turns out to be equal to \(1/\lambda_{\text{max}}\), the growth rate of the economy.

The economy described in the following numerical example has the flow matrix shown in Table 2-2. The capital requirements of its three sectors are represented by the following matrix of capital coefficients:

\[
B = \begin{bmatrix}
0.35 & 0.05 & 0.105 \\
0.01 & 0.515 & 0.32 \\
0 & 0 & 0
\end{bmatrix}
\]

The right-hand column of coefficients describing the capital structure of a household refers to stocks of agricultural products normally held in family larders and textile products stored in linen closets.

Unlike agricultural and manufactured goods produced by the first two sectors, labor services supplied by the third cannot be stored and consequently cannot, according to the definitions used, be part of any capital structure. The bottom row of a \(B\) matrix therefore contains only zeros, which means that the matrix \(B\) is singular and cannot be inverted.

With only two industries contributing to capital formation, equation (2-14) can be transformed (by expressing the magnitude of the third variable in terms of the other two through the use of the third equation) into a system containing only two linear difference equations of the same general form. Its two roots are those of the original system. In the present example they are found to be 0.39252 and 24.981.

The corresponding eigenvectors of the original three-equation system are

\[
\begin{bmatrix}
0.26388 \\
0.1821 \\
1.0
\end{bmatrix}
\text{ and }
\begin{bmatrix}
-0.041998 \\
0.25007 \\
1.00
\end{bmatrix}
\]
The reciprocal of the larger of the two roots is $1/24.981 = 0.04003$. That means that the economy as a whole, that is, all its three sectors, can expand at a rate of 4 percent per annum and that the relative level of their outputs will be proportional year after year to the relative magnitude of the three elements of a characteristic vector corresponding to that root.

The potential growth rate computed on the basis of the reciprocal of the much smaller second root would be much higher. However, since some of the elements of the corresponding characteristic vector have different signs, the output of some sectors would have to become negative with the passage of time, which of course is physically impossible.

Tests based on empirically observed sets of flow and capital coefficients have shown that in both the U.S. and the Japanese economies the relative levels of outputs of different sectors do not deviate very much from those computed on the basis of the corresponding closed dynamic models. Nevertheless, for the purposes of most practical applications, the closed version of the dynamic model has proved to be too deterministic and too rigid; the input-output analysis is usually conducted in terms of the open version of the dynamic model described by equation (2-14). The final bill of goods $Y(t)$ of successive years is treated in this case as given, that is, prescribed or projected on the basis of some exogenous information or assumption. Then the vector $X(0)$ describes the total output level of all producing sectors in the base year 0. The levels of output for subsequent years can be determined by a recursive computation based on rewriting equation (2-14) in the following form:

$$X(1) = B^{-1}[(1 - A + B)X(0) - Y(0)]$$

The following simple example of an open dynamic input-output model is based on information contained in the coefficient matrices $A$ and $B$ used in the example of a closed model above. Since in the present case the vector of final demand is considered as given, the structural relation, if it exists at all, between the output and the input of households is considered to be unknown; only the feedback relationships between the agricultural and the manufacturing sectors have to be taken into account. Accordingly, what might be called the dynamic core of the system to be solved is reduced to only two equations. After insertion of the appropriately reduced matrices $A$ and $B$ into equation (2-15), starting with the given $X(0)$ and using in the successive rounds of computation the externally determined vectors of final demand, $Y(0)$, $Y(1)$ and so on, one can compute, step
Table 2-5
Simplified example of solution of an open dynamic input-output model

<table>
<thead>
<tr>
<th>Year</th>
<th>Sector 1</th>
<th>Sector 2</th>
<th>Sector Output 1</th>
<th>Sector Output 2</th>
<th>Investment 1</th>
<th>Investment 2</th>
<th>Sector 3 (Households)</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>55</td>
<td>30</td>
<td>115</td>
<td>60</td>
<td>7.3</td>
<td>6.7</td>
<td>92</td>
<td>216</td>
</tr>
<tr>
<td>1</td>
<td>57.7</td>
<td>31.5</td>
<td>134</td>
<td>73</td>
<td>13.6</td>
<td>13.7</td>
<td>107</td>
<td>261</td>
</tr>
<tr>
<td>2</td>
<td>60.6</td>
<td>33.3</td>
<td>169</td>
<td>99</td>
<td>26.8</td>
<td>29.9</td>
<td>135</td>
<td>355</td>
</tr>
</tbody>
</table>

by step, the levels of output (and investment) of both industries for all the years. The corresponding employment levels are determined by a separate subsidiary computation using labor input coefficients taken from Table 2-2. Some results are shown in Table 2-5.

IV. Technological change

Since the technological structure of each sector of the economy is represented by a column vector of input coefficients and the corresponding column vector of capital coefficients, technological change can be described concisely as a change in the magnitudes of the elements of these vectors. Introduction of new commodities or industries is represented through introduction of new and disappearance of old commodities (or industries) through elimination of old vectors from the structural matrix of the economy in question.

The choice between two (or more) alternative processes that might be available for production of a particular good or service must obviously be based on a comparison of the effects of a hypothetical shift from one technology to another. For instance, a shift from coal-generated to atomic energy would affect the cost of production, prices, and the level of output and input of goods directly or indirectly. To determine these effects, several input-output computations have to be carried out, each based on the introduction into the flow ($A$) and into the capital ($B$) matrix of the economy in question of coefficient vectors characterizing the alternative technologies available for the industries in question. In the case where the choice of appropriate technology can be based on maximizing or minimizing an explicitly defined function of some variable—such as the aggregate input requirements for labor or specific natural resources, investment requirements, or the cost of production of
Various goods (whose magnitudes can be determined by means of appropriate input-output computations)—it can be formalized and carried out with the help of an appropriate linear programming algorithm. George D. Dantzig, the inventor of the well-known Simplex method of linear programming, actually developed it first as a means of automating input-output computations involving sequential substitution of alternative column vectors into square input-output coefficient matrices.

V. The scenario approach

Practical application of input-output analysis often takes the form of comparisons of the implications—described in terms of complete projected input-output tables—of several alternative scenarios, each based on a different set of assumptions concerning the level and composition of the final demand, changes in the magnitudes of input coefficients incorporated into various column vectors of the flow and capital coefficient matrices, or a combination of both.

Shortly before the end of World War II, President Franklin D. Roosevelt asked the U.S. Labor Department to assess the probable effect on the American economy of the impending transition from wartime to a peacetime footing. A static input-output model was constructed on the basis of a matrix of structural input coefficients derived from the 1939 input-output table of the American economy, the first such table compiled in the United States under government auspices. A comparison of the output and unemployment levels attained in all industries under war conditions, with the hypothetical output and employment level computed on the assumption that a vector of final demand representing normal civilian consumption would be substituted for the vector of final delivery dominated by military goods, provided a detailed and internally consistent answer to the question raised. To the great surprise of experts who predicted a slump in steel—conventionally considered to be a “war industry”—these input-output computations led to the conclusion that a substitution of a normal peacetime vector for the wartime vector of final demand would lead to a sharp rise in output and employment level in the steel sector. Subsequent development demonstrated that this conclusion was indeed correct.

Many, if not most, studies aimed at assessment of future energy demand and the effects of shift from oil to coal or to atomic power involved the use of supply and demand “elasticities” derived by means of simple or multiple correlation analyses applied to time
series describing past changes in energy input, energy prices, and prices of other goods. The contribution of an input-output approach to a consideration of the energy problem consisted, on the other hand, in the construction of several alternative scenarios, each involving a different combination of input-output vectors describing the technical structures of various methods of producing and using energy.

Such computations have shown, for instance, that while alcohol distilled from grain does indeed improve the energy balance of Brazil, it would not do so in the United States. Taking into account the amounts of energy absorbed directly and indirectly in operating agricultural machinery and producing chemical fertilizer in the United States, one finds that more than one thermal unit would be used up to supply one unit in the form of alcohol. A similar computation based on the Brazilian input-output table yields an opposite result.

In this connection it must be pointed out again that such computations necessarily take into account the entire input-output structure of the economy in question, including also that of its foreign trade.

The first practical application of the input-output method to systematic study of materials flow was carried out at the end of World War II in the United States by Western Electric Company. Next to copper, lead was one of the principal materials used at that time to manufacture electric cable. Anticipating a rapid rise in demand for its own products, as well as the products of many other industries depending on supply of that material, the management of that company carried out an input-output projection of production and consumption a few years ahead and came to the conclusion that shortages were bound to develop. On the basis of that finding, Western Electric initiated a crash research program aimed at substituting lead with a suitable plastic material in cable manufacture.

A recent application of the input-output methodology to systematic study of materials use can be found in The Future of Non-Fuel Minerals in the U.S. and World Economy (see bibliography at the end of this chapter). It is based on a modified input-output model of the American economy imbedded into the previously constructed multiregional input-output model of the world economy. The core of the structural matrix consists of the official U.S. input matrix with the 5 of its 106 sectors which depict production of nonferrous minerals expanded to 36 sectors describing production and consumption of 26 nonfuel metallic and nonmetallic minerals. Mine output
In the output analysis

Supplemented by product output resulting from other mining operations and reprocessing of scrap is described in the matrix of technical coefficients in great detail.

The system of 321 equations containing 328 variables describing the balance between the total supply including imports, and the total use including exports, of various goods, among them all non-ferrous metals in different forms, as well as the generation and elimination of major pollutants and allocation of labor, is presented in a schematic form below.

The system is described by the following set of equations:

\[
\begin{align*}
(2-16) \quad (I - A_d) \cdot q_1^0 + I \cdot q_3^0 &= p_1^0 \\
(2-17) \quad -B_e \cdot q_1^0 + (I + C_e) \cdot q_2^0 + C_e \cdot q_3^0 &= p_2^0 \\
(2-18) \quad L_e \cdot q_1^0 - I \cdot q_3^0 &= p_3^0 \\
(2-19) \quad M_e \cdot q_1^0 + N_e \cdot q_3^0 + H_e \cdot q_4^0 &= p_4^0 \\
(2-20) \quad D_e \cdot q_1^0 - I \cdot q_3^0 &= p_5^0
\end{align*}
\]

Equation (2-16) states that the gross domestic output of each commodity plus imports minus intermediate consumption must satisfy final demand. Similarly, equation (2-17) states that the domestic output of minerals (own industry plus byproduct) plus competitive imports minus intermediate consumption must equal final demand for minerals. Equation (2-18) states that the level of imports for each commodity is equal to a specified fraction of domestic (own industry) output. Equation (2-19) states the same proposition for non-competitive minerals (primary and scrap). Equation (2-20) states that the sum of each industry's value added, labor inputs, and emissions "output" equals the respective total for the economy as a whole. As explained previously, noncompetitive imports are goods used to satisfy intermediate or final demand for which there is no corresponding domestic producing sector.

A solution vector has the following form:

\[
Q' = \begin{cases} 
q_1^0 = \text{a } 106 \times 1 \text{ vector of commodity output levels in time } t \\
q_2^0 = \text{a } 36 \times 1 \text{ vector of mineral and scrap output levels in time } t \\
q_3^0 = \text{a } 106 \times 1 \text{ vector of commodity imports levels in time } t \\
q_4^0 = \text{a } 39 \times 1 \text{ vector of mineral and scrap import levels in time } t \\
q_5^0 = \text{a } 34 \times 1 \text{ vector of value added, labor requirements, energy consumption, pollution emission and new scrap generation levels in time } t 
\end{cases}
\]
and:

\[
\begin{align*}
 p_0^0 &= \text{a } 106 \times 1 \text{ vector of final demand components minus imports for 106 commodities valued in dollars for time } t \\
p_t^0 &= \text{a } 36 \times 1 \text{ vector of final demand components minus imports for 36 mineral commodities in physical units for time } t \\
 p_0^o &= \begin{cases} 
 p_0^0 & \text{a } 106 \times 1 \text{ null vector} \\
p_t^0 & \text{a } 39 \times 1 \text{ vector with zeros everywhere except in rows } 253, 254, 265, \text{ and } 288, \text{ whose elements give final demand minus import levels for noncompetitive imports in time } t \\
p_0^o & \text{a } 34 \times 1 \text{ null vector}
\end{cases}
\end{align*}
\]

The symbols used in equations (2-16) through (2-20) follow:

- \(A_c\) a commodity-by-commodity-matrix of input-output coefficients (106 \(\times\) 106) whose elements \(a_{ij}^o\) give dollar amounts of input \(i\) required to produce one dollar's worth of output \(j\) (valued in base year prices).
- \(0\) a null matrix or vector.
- \(I\) an identity matrix.
- \(-B_c\) an input-output coefficient matrix (36 \(\times\) 106) whose elements \(b_{ij}^o\) give the physical amount of mineral (primary or scrap) input \(i\) required to produce one dollar's worth of output \(j\) (only minerals produced in the United States are included in this submatrix).
- \(C_c\) a diagonal byproduct coefficients matrix (36 \(\times\) 36) whose elements \(c_{ij}^o\) give the physical amount of each mineral (primary or scrap) produced as a byproduct per physical unit of its own-industry output.
- \(G_c\) a step-diagonal matrix (36 \(\times\) 40) whose nonzero elements \(g_{ij}^o = 1\).
- \(L_c\) a diagonal import coefficient matrix (106 \(\times\) 106) whose elements \(k_{ij}^o\) give the dollar amount of imports per dollar's worth of \(j\) (\(i = j\)).
- \(M_c\) a matrix (39 \(\times\) 106) whose only nonzero elements appear in IEA-USMIN rows 253, 254, 265, and 288; the elements of these four rows, \(m_{ij}^o\), give the physical or dollar amount of noncompetitive import \(i\) per dollar's worth of output \(j\) (\(i = j\)).
- \(N_c\) a step-diagonal matrix (39 \(\times\) 36) whose nonzero elements \(n_{ij}^o\) give the physical amount of mineral (primary or scrap) \(i\) imported per physical unit of mineral \(j\)'s own-industry output (\(i = j\)).
- \(H_c\) a diagonal matrix (39 \(\times\) 39) whose nonzero elements \(h_{ij}^o = -1\), except in rows 253, 254, and 288, where \(h_{ij}^o = 1\).
- \(D_c\) a matrix (34 \(\times\) 106) whose elements \(d_{ij}\) give the amounts in dollars or physical units of value added, labor, energy, pollution emissions, and new scrap associated with a dollar's worth of output of commodity \(j\).
Input-output analysis

Such a system of equations was solved (to check its internal consistency) for the base year 1972 and for the years 1980, 1990, 2000, and 2030. Systematic projections of future changes in all sets of technological coefficients, particularly those reflecting efficient methods of extraction and refining and substitution between different materials were the most demanding part of that task. The estimates of future changes in the exports and imports (that enter into the system as vectors of exogenously determined variables) were obtained by incorporating the system into the multiregional input-output analysis of the world economy constructed for the United Nations several years earlier.

Alternative projections were computed, based on 11 different scenarios. Each of these represented a different combination of specific assumptions concerning the dependence of the U.S. economy on imports of nonferrous metals and future rates of technological change. Final conclusions were summarized in the form of separate observations on the present and expected future supply and demand for each nonferrous mineral on the domestic U.S. and international markets.

One of the most ambitious applications of the input-output approach was the construction of a multiregional, multisectoral, dynamic input-output model of the entire world economy referred to above. That model was employed in the preparation of long-run projections based on alternative scenarios of prospective developments of the economic relationship between the developed and less developed regions. It also provided the basis for long-run projections of the economic growth (or decline as the case may be) of the various regions under alternative assumptions concerning population growth, technical change—particularly in the field of agriculture and energy production—and in the uncertain supply of various natural resources.

Selected bibliography for chapter 2


