DETERMINATIONS OF THE SIMPLE MULTIPLIER

The following shows two ways to find the money multiplier of \( M_1 \) under the special assumptions used in class. The first one is more direct and simpler but it hides the multiplicative process involved.

Notations:
- \( M \): The money supply as defined by \( M_1 \).
- \( C \): Currency in circulation.
- \( D \): Demand deposits.
- \( R = RR + ER \): Reserves are the sum of required reserves (\( RR \)) and excess reserves (\( ER \)).
- \( B \): Monetary base.
- \( r_d = R/D \): required reserve ratio on demand deposits (\( 0 \leq r_d \leq 1 \)).
- \( c = C/D \): Currency-deposit ratio (\( 0 \leq c \leq 1 \)).
- \( m = M/B \): \( M_1 \)-multiplier.

Method 1: Using definitions

We know that \( M_1 \) is approximately equal to the sum of currency in circulation (\( C \)) and demand deposits (\( D \)). We also know that the monetary base (\( B \)) is defined as the sum of currency in circulation and reserves (\( R \)):

\[
M \equiv C + D \\
B \equiv C + R
\]

Dividing the first identity by the second one, we have:

\[
\frac{M}{B} \equiv \frac{c+1}{c + r_d}
\]

Therefore, taking the definition of the money-multiplier:

\[
M \equiv m \cdot B \quad \text{with} \quad m \equiv \frac{c+1}{c + r_d}
\]

Knowing that \( B \equiv C + R \) then \( \Delta B \equiv \Delta C + \Delta R \). Assuming that there are no excess reserves in the banking system, if the central bank suddenly increases the quantity of reserves (\( \Delta R > 0 \)), this creates an excess reserve in the banking system and we have:

\[
\Delta M \equiv m \cdot \Delta R \quad \text{or} \quad \Delta M \equiv m \cdot ER
\]

Money is multiplied by the initial amount of excess reserve and the multiplicative process stops when \( ER = 0 \).
Method 2: Using logic

Let us assume that $r_d = 10\% = 0.1$. We have seen that an initial creation of reserves by the Central Bank leads to the following process under special hypotheses:

<table>
<thead>
<tr>
<th>Bank</th>
<th>$\Delta DD$</th>
<th>$\Delta R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bank A</td>
<td>+100</td>
<td>$\Delta R(1 - r_d)$</td>
</tr>
<tr>
<td>Bank A</td>
<td>+90</td>
<td>$\Delta R(1 - r_d)^2$</td>
</tr>
<tr>
<td>Bank B</td>
<td>+81</td>
<td>$\Delta R(1 - r_d)^3$</td>
</tr>
<tr>
<td>Bank C</td>
<td>+72.9</td>
<td>$\Delta R(1 - r_d)^4$</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>Bank Z</td>
<td>+0</td>
<td>$\Delta R(1 - r_d)^n$</td>
</tr>
<tr>
<td>Sum</td>
<td>Banking system</td>
<td>Sum $\Delta DD = +1000$ = $S_n$</td>
</tr>
</tbody>
</table>

The problem is to find the mathematical formula that leads to $S = 1000$. This will allow determining the analytical formulation of the multiplier. We know that the sum ($S_n$) is:

$$S_n = \Delta R + \Delta R(1 - r_d) + \Delta R(1 - r_d)^2 + \Delta R(1 - r_d)^3 + \ldots + \Delta R(1 - r_d)^n$$

In order to solve this sum we, first, multiply each side by $(1 - r_d)$:

$$(1 - r_d)S_n = \Delta R(1 - r_d) + \Delta R(1 - r_d)^2 + \Delta R(1 - r_d)^3 + \Delta R(1 - r_d)^4 + \ldots + \Delta R(1 - r_d)^{n+1}$$

Then, we take the difference between the two equations:

$$S_n - (1 - r_d)S_n = \Delta R - \Delta R(1 - r_d)^{n+1}$$

By rearranging we have:

$$S_n = \Delta R \left[ 1 - \frac{(1 - r_d)^{n+1}}{r_d} \right]$$

If $n \to \infty$ and if $r_d < 1$ then the sum converges. Indeed:

$$S_n = \frac{\Delta R}{r_d} - \frac{\Delta R(1 - r_d)^{n+1}}{r_d} \quad \text{and} \quad \lim_{n\to\infty} \left[ \frac{\Delta R(1 - r_d)^{n+1}}{r_d} \right] = 0$$

So:

$$S = \frac{\Delta R}{r_d}$$

Thus, an initial variation of reserve ($\Delta R$) will lead to a variation of $M$ ($\Delta M$) by a multiple amount $m = 1/r_d$:

$$\Delta M = m\Delta R \quad \text{with} \quad m = \frac{1}{r_d}$$

Thus if the central bank can control $\Delta R$, it can control $\Delta M_1$. 