FUNDAMENTALS OF REGIONAL MACROECONOMIC MODELING

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FUNDAMENTALS OF REGIONAL MACROECONOMIC MODELING: PART 1

BY

GEORGE I. TREYZ*

Regional macroeconomic models range over a broad spectrum. They include not only economic base and input-output models, but also models with price and wage determination, location responses, flexible production functions and other behavioral elements. These later models are dynamic and include econometrically estimated parameters.

In this two-part article, we approach regional macroeconomic models as tools for making economic forecasts and simulating the total economic effects of a policy change. To do this, we must have some idea of how a regional economy operates. In order to avoid getting lost in the details, we start by abstracting from reality.

The first step in simplifying the economy is to identify essential economic variables. Our next step is to hypothesize key economic relationships. If the model is to be used to describe an actual economy, we must now develop a method to assign numerical values to these simple relationships.

We assume that certain variables, called exogenous variables, are given values determined outside of the regional economy, and all other variables, called endogenous variables, are determined within the economy. We can then develop causal relationships among the variables in the model. To quantify these relationships, we use actual data and use the resulting coefficient estimates for forecasts and simulations.

The way a model is used is shown in the following schematic illustration:

\[\text{Diagram 1}\]

\[
\text{Change in Exogenous Variables} \implies \text{MODEL} \implies \text{Change in Endogenous Variables}
\]

For a simulation, we change one or more of the exogenous variables and then use the model to predict changes in the endogenous variables.

In Part 1 (in this issue), we will develop two Economic Base models (Type A and B). These are one sector aggregate models which describe the total production of a region. One sector models are used to represent basic interrelationships among aggregate variables in the economy without regard to specific industries. In the concluding half of the article, Part 2, we will turn to multi-sector models. The plan of attack is summarized in the accompanying outline.

*This article is taken in part from a longer work entitled Regional Macroeconomic Modeling which is scheduled for completion in 1987. The author wishes to acknowledge extensive editorial assistance from Frederick R. Treyz, and other assistance from Margaret J. Larson, Mario DePillis, Jr., Jack Beleitti, David J. Ehlich and Linda Hillenbrand. The author is Professor of Economics, University of Massachusetts, Amherst and President of Regional Economic Models, Inc.
PART 1 — ONE-SECTOR MODELS

A. ASSUMING INCOME EQUALS OUTPUT

The first step in model building is to define the variables used in the model and to show the accounting relationships among these variables. Our primary regional (i.e. sub-national) unit may be a state economy, a county, or a combination of states or counties. We will start with a simple account for a closed state economy that ignores many of the income flows in the real world. Even though it is unrealistic to think of a self-sufficient state economy, it will simplify our presentation if we use it as a starting point. The accounts for a closed state economy are shown in Table 1.

Table 1

Income and Product Accounts for A
Simplified Closed State Economy

<table>
<thead>
<tr>
<th>A. State Product Account</th>
<th>B. Personal and Local Government Income and Outlay Account</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uses</td>
<td>Sources</td>
</tr>
<tr>
<td>Y (output)</td>
<td>CG (consumption and local government spending)</td>
</tr>
<tr>
<td>IL (local investment)</td>
<td>S (personal savings and local government surplus)</td>
</tr>
<tr>
<td>Y</td>
<td>Y</td>
</tr>
</tbody>
</table>

C. Savings and Investment Account

<table>
<thead>
<tr>
<th>Uses</th>
<th>Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>IL (local investment)</td>
<td>S (personal savings and local government surplus)</td>
</tr>
<tr>
<td>I</td>
<td>S</td>
</tr>
</tbody>
</table>

Y Total state output of goods and services: Gross State Product or Gross Regional Product.

CG Total state use of goods and services for consumption and state and local government spending.

S Local savings by individuals, and state and local government surplus.

IL Residential and non-residential construction, new equipment purchases and inventory changes within the state.*

---

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*Here we are looking at gross investment. Net fixed investment can be negative if new fixed investment is less than depreciation of the fixed capital stock.
In the accounts, we ignore government taxes and transfers, imports and exports, and other economic flows that we will include as we develop our model. The accounts can be represented in equation form. From account A we have:

\[ Y = CG + IL \]  

(1A)

where the income earned in the region \( Y \) comes from sales to the investment sector (IL), and to consumers and local government (CG).

From account B we have:

\[ Y = CG + S \]  

(1B)

where income \( Y \) can be spent by consumers and local government (CG), or can be saved by individuals and local governments (S). Setting account A equal to account B gives us the equation for account C:

\[ IL = S \]  

(1C)

in which all local investment (IL) is equal to savings (S) and all savings is used for local investment. Savings could be negative if individuals and local governments were spending more than was earned in the state (CG > Y). Local investment (IL) would then be negative. This would be possible in a closed economy only if inventory reduction exceeded new fixed investment.

### A.1 AN OPEN ECONOMY

To convert the closed state accounts to an open economy, we include imports and exports in the accounts. Savings and investment can now originate or be used in the rest of the country. We are able to develop an account for the rest of the country that shows the interactions between the state and the outside world.

We can see the difference between a closed and an open economy by comparing the accounts in Tables 1 and 2. Account 1A gave us the state product equation for a closed economy \( Y = CG + IL \) in which exports and imports are equal to 0. In an open economy, a state will import part of its consumption and export part of its production. Thus, account A from Table 2 gives us:

\[ Y = CG + IL + XFG - M \]  

(2A)

Output in the state is equal to the value of locally produced goods and services. Consumer and local government spending (CG), local investment (IL), and sales outside of the state (XFG) represent the final sales of the state. To find the value of locally produced goods and services, we must subtract imports (M) from this amount since part of the goods purchased within the state are produced in another state or country.

Net exports (XFG - M), can have a negative value within the accounts. From this account we can see that in an open economy, output \( Y \) can be smaller than \( (C + IL) \), as long as imports \( M \) exceed exports \( XFG \).
### Table 2
Income and Product Accounts for A
Simplified State Economy

<table>
<thead>
<tr>
<th>A. State Product Account</th>
<th>B. Personal and Local Government Income and Outlay Account</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Uses</strong></td>
<td><strong>Sources</strong></td>
</tr>
<tr>
<td>Y (output)</td>
<td>CG (consumption and local government spending)</td>
</tr>
<tr>
<td></td>
<td>IL (local investment)</td>
</tr>
<tr>
<td></td>
<td>XFG (exports including federal government)</td>
</tr>
<tr>
<td></td>
<td>(-M) (imports)</td>
</tr>
<tr>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td></td>
<td>Y</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>C. Savings and Investment Account</th>
<th>D. Rest of Country Account</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Uses</strong></td>
<td><strong>Sources</strong></td>
</tr>
<tr>
<td>IL (local investment)</td>
<td>S (personal savings and local government surplus)</td>
</tr>
<tr>
<td>IR (investment, rest of country)</td>
<td>S</td>
</tr>
<tr>
<td>I</td>
<td>XFG</td>
</tr>
</tbody>
</table>

| XFG                               | Sales outside of the state of goods produced within the state. This includes Federal government spending in the local area. |
| M                                 | Purchases within the state of goods and services produced outside of the state. |
| IR                                | Investment in the rest of the country from the state (negative indicates a net flow of rest of country investment into the state). |
The uses of income in account B are the same in the open economy as the closed economy.

\[ Y = CG + S \]  \hspace{1cm} (1B, 2B)

or

\[ S = Y - CG \]  \hspace{1cm} (1B, 2B)

In this account, all output \((Y)\) is assumed to go to households and local government as income \((Y)\).* It is then spent for personal or local government use \((CG)\), or is saved in the form of personal savings and government surpluses \((S)\). Federal government taxes, transfers and spending, are implicitly assumed to be 0 and will not be considered until our model type B. Meanwhile we define all income \((Y)\) that is not used for personal consumption or local government spending \((CG)\) to be equal to savings \((S)\).

In our closed state economy example, all savings were invested locally. In an open economy, savings can be invested locally or in the rest of the country. By including investment in the rest of the country \((IR)\), the closed economy investment account \((IL = S)\) is converted to the open economy investment account:

\[ S = IL + IR \]  \hspace{1cm} (2C)

or

\[ IL = S - IR \]  \hspace{1cm} (2C)

In an open economy, personal savings and local government surpluses \((S)\) can be invested either locally \((IL)\) or outside of the local area \((IR)\). Local investment \((IL)\) can also exceed local savings\((S)\), if investment in the rest of the country \((IR)\) is negative. In other words, if investment in the locality is larger than the amount of local savings, we know that the rest of the country is funding investment in the locality.

We can now develop an account that shows the interactions the rest of the country has with the region. Account D gives us the equation:

\[ XFG = M + IR \]  \hspace{1cm} (2D)

or

\[ IR = XFG - M \]  \hspace{1cm} (2D)

In this simplified economy, the earnings gained from exports \((XFG)\) can be spent only on imports \((M)\) or investment in the rest of the country \((IR)\). The money used for investment in the rest of the country \((IR)\) is therefore equal to the difference between exports \((XFG)\) and imports \((M)\). Regions with positive net exports \((XFG - M)\) will be experiencing a flow of their savings to the rest of the country to finance investment outside of the state. On the other hand, regions with greater imports than exports will be financed by savings from the rest of the country.**

---

*Output = income \((Y - Y)\) assumed that all value added by local production (wages, profits, rents, etc.) is paid out to local residents and government (through taxes). This assumption is dropped for our Type B model.

**Note that inflows into the state could be used to finance CG rather than IL, if they exceed the value of IL.
A.2 AN EXAMPLE

Before continuing, it is interesting to look at estimated values for the variables in this model for a representative state. Using methods that will be covered later the following accounts were estimated for Michigan.

<table>
<thead>
<tr>
<th>Uses</th>
<th>Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y = 80.8$</td>
<td>$CG = 65.0$</td>
</tr>
<tr>
<td>$IL = 16.2$</td>
<td>$S = 15.8$</td>
</tr>
<tr>
<td>$XFG = 83.7$</td>
<td></td>
</tr>
<tr>
<td>$-M = 84.1$</td>
<td></td>
</tr>
<tr>
<td><strong>80.8</strong></td>
<td><strong>80.8</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Uses</th>
<th>Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>$CG = 65.0$</td>
<td>$Y = 80.8$</td>
</tr>
<tr>
<td>$S = 15.8$</td>
<td></td>
</tr>
<tr>
<td><strong>80.8</strong></td>
<td><strong>80.8</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Uses</th>
<th>Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>$IL = 16.2$</td>
<td>$S = 15.8$</td>
</tr>
<tr>
<td>$IR = -4$</td>
<td></td>
</tr>
<tr>
<td><strong>15.8</strong></td>
<td><strong>15.8</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Uses</th>
<th>Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>$XFG = 83.7$</td>
<td>$M = 84.1$</td>
</tr>
<tr>
<td>$IR = -4$</td>
<td></td>
</tr>
<tr>
<td><strong>83.7</strong></td>
<td><strong>83.7</strong></td>
</tr>
</tbody>
</table>

It may surprise you to find that exports (83.7) are greater than total Gross State Product (80.8). The reason that this is possible is that GSP shows the total value added to production in Michigan. For example when a car is produced the value added in Michigan is net of the materials imported into Michigan to produce the car. On the other hand the exports from Michigan are valued at the total final sales price of the exports.

Account B assumes that all GSP is available for consumption by individuals and local government. The difference between this spending and GSP is savings (S). The Savings and Investment Account indicates that a small part of local investment (IL) was financed by a net flow from the rest of the country. In any case these accounts give a simple view of a state economy which we will enhance when we assemble the accounts for our Type B model.
Returning to our accounts we can show the internal consistency of the accounts by substituting (2B) and (2D) into (2C):

\[ IL = Y - CG - XFG + M \]  

(2B')

and rearranging this to give us:

\[ Y = IL + CG + XFG - M \]  

(2A')

or equation (2A).

Our four basic equations are:

\[ Y = CG + IL + XFG - M \]  

(2A)

\[ Y = CG + S \]  

(2B)

\[ S = IL + IR \]  

(2C)

\[ XFG = M + IR \]  

(2D)

We can now express local investment (IL) as planned local investment (ILp) and unplanned investment (ILu) defined as follows:

\[ IL = ILp + ILu \]  

(3)

ILp planned investment, local residential and non-residential construction, new equipment purchases and planned changes in inventory (planned inventory changes are equal to 0 in steady state)

ILu unplanned investment, local; unplanned changes in inventories (usually caused by failure to set output equal to sales)

This distinction will be useful as a way to allow for a difference between output and demand as we develop our model. Our accounting identities can now be shown as:

\[ Y = CG + ILp + ILu + XFG - M \]  

(2AA)

\[ Y = CG + S \]  

(2B)

\[ S = ILp + ILu + IR \]  

(2CC)

\[ XFG = M + IR \]  

(2D)
A.3 AN ECONOMIC BASE MODEL

The accounts give us *definition*al interrelationships among the variables that we are examining. This, however, does not tell us how a change in one variable will affect the other variables. In order to build a model for simulations and forecasts, we must develop *behavioral* relationships between the variables.

First we choose the variables that are determined outside the model, the *exogenous* variables. The remaining variables are *endogenous* variables, and each must be explained by an equation in the model. After we develop the model, we will then be able to make a forecast or carry out a simulation. *A simulation is accomplished by changing the exogenous variables and seeing how they change the endogenous variables.*

Our task now is to develop a model that will include all of the variables in the economy that are defined by our accounts. We will consider (ILp) and (XFG) to be exogenous.* It seems reasonable to assume that planned investment and exports depend on factors determined outside of the local economy, at least in the short run.

The remaining variables in our accounts are the endogenous variables. These are (Y), (CG), (ILu), (M), (S), and (IR). All endogenous variables must be explained within the model, either by a behavioral relationship or by an identity from our accounts.

First, we will assume that unplanned local investment (ILu) is equal to 0 since producers have both a motive and a means to avoid unplanned inventory change. If inventories are growing producers must be producing more than they can sell; if they are decreasing then they must be producing less than they are selling. We will assume that producers will keep their inventories at desired levels by either increasing or decreasing production, rather than changing price levels.

We can now represent account A as a behavioral relationship in which \( ILu = 0 \) due to the behavior of business decision makers.

\[
Y = CG + ILp + XFG - M \tag{4}
\]

Output \( Y \) is set equal to planned local investment \( (ILp) \) plus consumer and local government spending \( (CG) \) plus exports \( (XFG) \) less imports \( (M) \). In other words, producers within the state will produce what they sell, so that production \( Y \) will be equal to net sales \( (ILp + CG + XFG - M) \) and inventories will not change \( (ILu = 0) \).

Next, we assume that imports \( (M) \) are a constant proportion of total final demand. Therefore, our second behavioral relationship is:

\[
M = mCG + mILp + mXFG \tag{5}
\]

In other words, all final demand will be partially satisfied by goods and services provided from outside of the state \( (M) \) and partially satisfied by local value added to production \( (Y) \).

---

*In the traditional economic base model, \( (ILp) \) is combined with \( (CG) \) and considered endogenous. We do not combine them because it is difficult to envision \( ILp \), which includes investment to build new capital stock, to be a fraction of a flow variable such as income. In a footnote below the alternative model is shown.*
From this, a parameter \( r \) is defined as:

\[
r = 1 - m
\]

This is the proportion of final demand that is produced within the state. The coefficient for total final demand is split into that portion provided by imports \( (m) \) and that provided locally \( (r) \), in the equation

\[
r + m = 1
\]

Substituting (5) into (4) imports \( (M) \) can be eliminated from the model:

\[
Y = CG - mCG + XFG - mXFG + ILp - mILp
\]

\[
Y = rCG + rXFG + rILp
\]

Next, we assume that consumer and government spending \( (CG) \) are a constant proportion of total output \( (Y) \), introducing the coefficient \( b \):

\[
CG = bY
\]

The model can now be restated as:

\[
Y = rCG + rXFG + rILp
\]

\[
CG = bY
\]

Returning to our specific example for Michigan from equation (10) we could estimate \( b \) as

\[
b = \frac{CG}{Y} = \frac{65}{80.8} = 0.804
\]

Likewise from equation (11) we could estimate \( r \) as

\[
r = \frac{Y}{CG + XFG + IL} = \frac{80.8}{65 + 83.7 + 16.2} = 0.490
\]

This would enable us to rewrite equations (11) and (12) in explicit form

\[
Y = 0.49 CG + 0.49 XFG + 0.49 ILp
\]

\[
CG = 0.804 Y
\]

Substituting equation (12) into (11) we obtain:

\[
Y = (r b) Y + rXFG + rILp
\]
Where \((tb)Y\) is the fraction of \((Y)\) that is used for locally produced consumption and local government spending. This is obtained when \((b)\), the proportion of income \((Y)\) that is spent, is multiplied by the proportion of spending for locally produced goods \((r)\). Output \((Y)\) is expressed as a function of exports \((rXFG)\), local planned investment \((rILp)\) and the fraction of \((Y)\) that is used for local consumption and local government spending \((tb)Y)\).

Note that \((Y)\) will change if the value of \((rXFG)\) or \((rILp)\) changes. When \((Y)\) increases, \((tb)Y)\) will also increase, further increasing \((Y)\), increasing \((Y)\) again, etc. This is called the induced demand, or the demand created by the respending of income gained due to changes in output.

This effect can be shown mathematically by taking the total differential of (15):

\[
dY = d(tbY) + d(rXFG) + d(rILp) \tag{16}
\]

This equation separates exogenous change in \((Y)\) from the change in \((Y)\) caused by induced demand. In the equation, the change in induced demand \((d(tbY))\) is added to the exogenous variable changes \((d(rXFG)\) or \((rILp)\)) to calculate the change in output \((dY)\).

The equilibrium output, in which supply is equal to demand, is at the intersection of the 45 degree line \((D)\) and the demand function, \((tb)Y + rXFG + rILp\), at the equilibrium \(Y^*\). Since we are only at equilibrium when expenditure is equal to output \((Y = rCG + rXFG + rILp)\), the 45 degree line is used to graphically

\[A.4\] RESTATEMENT VIA THE 45 DEGREE LINE DIAGRAM

The above model is represented in Diagram 2.

\[Diagram 2\]

Income Determination for a Region

\[(\text{Demand for local output})

Y = \text{Expenditure}

D

(tb) Y + rXFG + rILp

\(e\)

\(a\)

\(b\)

\(Y^{**}\)

\(Y^*\)

Local Output
represent all the points in which the value along the vertical axis is equal to the value along the horizontal axis. The equilibrium is the rate of output at which producers will have no reason to increase or decrease their output. Since it is exactly equal to what they can sell (i.e. \( I_{L_2} = 0 \)), this must be along this 45 degree line.

The equilibrium must also be on the expenditure line defined by \((rb)Y + rXFG + rILp\) from (15). The points along this line represent the theoretical expenditure that would take place at given levels of output \(Y\).

The intersection of these two lines gives the actual equilibrium of expenditure for goods and services produced in the locality \((rb)Y + rXFG + rILp\) and local output \(Y\). This intersection shows the actual amount of production and expenditure that will occur in this model when all of the actors are following the behavior postulated for them.

To illustrate how this equilibrium is achieved, suppose that demand were at point \(a\) and output at \(Y^{**}\). Point \(b\) would be the level at which output equaled expenditure, leading to excess demand \(a - b\) at an annual rate. Initially, this will cause decreasing inventories \(IL_2 < 0\), leading to an increase in production. Production will be increased by business to stem the loss of inventory. This will move output to an equilibrium on the 45 degree line where inventory no longer changes and producers will have no incentive to increase or decrease production.

Now, suppose that one of the exogenous factors were to change. For example, assume that \(XFG\) increases. The change in \(Y\) from (16) \(dY = d(rb)Y + d(rXFG) + d(rILp)\) can be demonstrated graphically in Diagram 3:

![Diagram 3](image)

\[ F' = (rb)Y + rILp + rXFG' \]

\[ F = (rb)Y + rILp + rXFG \]

*Any points off the 45 degree line imply that output exceeds expenditure, or expenditure exceeds output. If this were the case we would observe unplanned inventory increase (or decrease) which would lead business to lower (or raise) output until output reached the equilibrium point.*
In this diagram, the demand curve shifts up from (F) to (F') due to an increase in the exogenous variable (XFG). The equilibrium expenditure and output, determined by the intersection of the demand curve and the 45 degree line, moves from (g) to (d). The total equilibrium demand moves from H* (g) to H*** (d), corresponding to the equal rise in output from Y* to Y**. Although exports only increased the distance H** - H* (d - e), expenditure increased H*** - H* (d - f), significantly larger than just the exogenous change in exports. The difference H*** - H** (e - f) is equal to the induced demand created by the spending caused by increases in Y (d(rbY)). If (rb) were equal to 0, the distance H*** to H*** (e - f) would be equal to 0.

### A.5 THE MULTIPLIER

Equation (15) is in the form:

\[ Y = (rb)Y + rXFG + rILp \]  \hspace{1cm} (15)

in which the endogenous variable Y is on both sides of the equation.

We arrange Equation (15) so that:

\[ Y = \left(\frac{1}{1 - rb}\right)(rXFG + rILp) \]  \hspace{1cm} (17)

This is a reduced form equation. It shows the endogenous variable (Y) as a function of model parameters (r and b, which would be estimated values in an actual model) and the model's exogenous variables (XFG) and (ILp). By using the reduced form equation, we can estimate the effects on output (Y) of a change in the exogenous variables (XFG) and (ILp).

We can use 1977 values from Michigan to develop a Michigan equation:

\[ Y = \left(\frac{1}{1 - .49}\right)(.49XFG + .49ILp) = 1.65 (.49XFG + .49ILp) \]  \hspace{1cm} (18)

Equation (15) can also be expressed in the form:

\[ \frac{Y}{rXFG + rILp} = \frac{1}{1 - (rb)} \]  \hspace{1cm} (19)

When the derivative of (Y) is taken with respect to the derivative of (rXFG), (rILp) and (rXFG + rILp) we have:

\[ \frac{dY}{dX} = \frac{dY}{drXFG + rILp} = \frac{Y}{rXFG + rILp} = \frac{1}{1 - (rb)} = K \]  \hspace{1cm} (20)

which gives us the multiplier K. This proportional form is used for our economic base impact models.

---

*I This model makes the assumption that local government spending can be included with consumption demand. Equation (12) implies that local gross output is the main determinant of local consumption and government demand. No allowance is made here for possible effects on demand of earnings by state residents out of state, of federal taxes that state residents might be subject to, of federal government spending, or of transfer payments, dividends, interest and rent that residents might receive from out of state. We will deal with these other factors later in economic base model B.
Using the Michigan (1977) case as an example,

\[
\frac{dY}{d(rXFG)} = \frac{dY}{d(rILp)} \left( 1 - \left( \frac{.49}{.804} \right) \right) = 1.65
\]  

(21)

While this multiplier shows the change that is required in equilibrium output in response to a change in an exogenous variable it is also possible to use a period by period approach to calculating the change in Y that will occur in response to an exogenous change in rXFG or rILp. We can do this by imagining that a short time response lag is involved and writing our explicit equations as

\[
B_t = .49 XFG_t + .49 ILp_t
\]

(22)

\[
Y_t = .49 CG_t + B_t
\]

(23)

\[
CG_t = .804 \cdot Y_{t-1}
\]

(24)

Now suppose we start in period 0 and in all previous periods with \( B = 0 \) (i.e. \( XFG = ILp = 0 \)) and in period 1 and for all future periods we allow \( XFG = 2.04 \) which implies that \( B \) will equal one. The sequential steps can be shown on the following table.

### The Period Multiplier Approach to Estimating K

<table>
<thead>
<tr>
<th>Period</th>
<th>( Y_t )</th>
<th>( .49 \cdot CG_t )</th>
<th>( CG_t )</th>
<th>( B_t )</th>
<th>( XFG_t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>.39</td>
<td>.80</td>
<td>1</td>
<td>2.04</td>
</tr>
<tr>
<td>2</td>
<td>1.39</td>
<td>.39</td>
<td>.80</td>
<td>1</td>
<td>2.04</td>
</tr>
<tr>
<td>3</td>
<td>1.54</td>
<td>.54</td>
<td>1.12</td>
<td>1</td>
<td>2.04</td>
</tr>
<tr>
<td>4</td>
<td>1.61</td>
<td>.61</td>
<td>1.24</td>
<td>1</td>
<td>2.04</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>n</td>
<td>1.65</td>
<td>.65</td>
<td>1.33</td>
<td>1</td>
<td>2.04</td>
</tr>
</tbody>
</table>

In period 0 equilibrium output \( Y = 0 \). Then in period 1, XFG is increased permanently by 2.04. By multiplying this increase in exports by the coefficient \( r = .49 \) we can calculate the value added by local production. Thus \( B_t \) is increased by 1.00 \((.49 \times 2.04)\) which leads to an increase in \( Y \) by 1.00. This increase then is assumed to lead to an increase in the income of local residents and local governments. Since we have estimated that .804 of this increased income is respent by local consumers and government this leads to an increase in \( CG \) of .80. Of this .80, .49 percent will be provided locally \((.49 \times .8 = .39)\). Thus, .39 of new local \( Y \) is shown in period 2. In period 3 the \( CG \) value is .804 of the .39 of \( Y \) in period 2. The process continues period after period. We can see that by period 4, \( Y \) has already increased to 1.61. By continuing only a few more periods the changes from period to period would become very small. Thus we can use this period by period approach as a way to find the new equilibrium value. However, in our case we have already calculated the exact equilibrium value from our reduced form equation

\[
\Delta Y = \frac{1}{1 - (.49) (.8)} (.49 \Delta XFG) = 1.65
\]

(25)
We can confirm that this will be the eventual equilibrium by calculating the induced output for CG in the last period. The output induced by the increase in XFG to 2.04 is equal to

\[ r_{CG} = (0.49)(1.33) = 0.65 \]  

(26)

The final amount of \( Y \) in period \( n \) equals the exogenous output plus the induced output or

\[ Y_n = 1 + 0.65 = 1.65 \]  

(27)

Thus, the new equilibrium values will simultaneously solve all of the equations in the system.*

**A.6 CONVERTING OUTPUT (Y) TO EMPLOYMENT (E)**

Regional economic models are used to make predictions of the changes in economic activity in a local area for the purpose of planning and public policy making. Employment data is the primary source of data for sub-national areas. Employment is also the greatest concern in public policy making. In order to predict changes in employment, the model is converted from dollar terms to employment terms.

Four new definitions are presented in order to do this:

\[ E = \text{total employment} \]
\[ EX = \text{employment dependent on exports including the Federal Government} \]
\[ EI = \text{employment dependent on local investment} \]
\[ EL = \text{employment dependent on local consumer and government spending} \]

Algebraically, the terms are defined:

\[ E = EL + EX + EI \]  

(28)

*As an alternative model we could collapse the CG and IL variables into a variable labelled CGIL.

CGIL = consumption, government spending and investment.

Instead of using the coefficient \( b \), the proportion of \( Y \) used for consumer and local and government spending, we will introduce the coefficient \( f \) as the proportion of \( Y \) going to investment, local government spending and consumption.

\[ CGIL = FY \]

and

\[ Y = rCGIL + rXFG \]

or the reduced form:

\[ Y = \frac{rXFG}{(1-rf)} \]

The value of \( K \) changes from \((E/(EX + EI))\) to:

\[ K = \frac{E}{EX} = \frac{1}{1-rf} = \frac{Y}{rXFG} \]
To convert the dollar amounts to employment, we must make an assumption about the relationship of value added to employment. We assume that employees per unit of value added in producing $Y$ is equal to the average number of employees per unit of value added. The employees per unit of value added ($E/Y$) is called (e), producing:

\[ e = \frac{E}{Y} \quad (29) \]

or

\[ Y = \frac{E}{e} \quad (29') \]

Thus, the number of employees per unit of output is assumed to be fixed and the same for all production.

This means that (e) is assumed to be the average employment per dollar of local value added in the export sector ($EX/rXFG$) and in the investment sector ($EIp/(rILp)$), producing the equalities:

\[ EX = e \cdot rXFG ; EIp = e \cdot rILp \quad (30) \]

or

\[ rXFG = EX/e ; rILp = EIp/e \quad (30') \]

Export employment ($EX$) and local investment employment ($EIp$) are determined by the production in the respective sectors multiplied by the average number of employees per dollar of value added in all sectors (e).

Using equation (15):

\[ Y = (rb)Y + rXFG + rILp \quad (15) \]

we can substitute for $Y$, $rXFG$ and $rILp$, obtaining:

\[ \frac{E}{e} = rb \cdot \frac{E}{e} + \frac{EX}{e} + \frac{EIp}{e} \quad (31) \]

To obtain the reduced form, we find it directly from (31) or we substitute (29') and (30') into equation (17) to obtain:

\[ \frac{E}{e} = \frac{1}{1 - rb} \left[ \frac{EX}{e} + \frac{EIp}{e} \right] \quad (32) \]

\[ E = \frac{1}{1 - rb} \left[ \frac{(EX + EIp)e}{e} \right] \quad (33) \]
To estimate a new multiplier in employment terms called $K'$ we can divide equation (33) by $(EX + ELp)$ to obtain

\[
K' = \frac{E}{EX + ELp} = \left(\frac{1}{1 - rb}\right) \frac{E}{e} = K', \tag{34}
\]

In order to predict the change in total employment ($E$) based on the change in exogenous factors ($EX + ELp$), we take the derivative of ($E$) with respect to ($EX + ELp$):

\[
\frac{d(E)}{d(EX + ELp)} = K \tag{35}
\]

Note that since we are taking the derivative of a linear relationship, we have the same multiplier $K$ as before.

Now that we are using employee units of measurement we must still find a way to distinguish between export -employees and employees dependent on local use. Three of the standard ways of doing this, are (1) judgmental, (2) location quotient, and (3) minimum requirements.

Each way of estimating ($EX$) will require us to examine individual industries in the local area. For each of the ($i$) industries, we would then have to find the exports ($X(i)$) for that industry in that state for each time period. The value of ($EX$) must then be obtained by adding up all export values of all industries ($i$) for each time period.

To estimate and use an economic base model, we must estimate the economic base (i.e. the export dependent employment and output in the area). But we defer our discussion of this until later.

**B. RECOGNIZING THAT REGIONAL OUTPUT AND INCOME DIFFER**

In economic base model A, we considered income to be equal to output in a state. Yet the income generated by the production in a state may often go to residents outside of the state, and, conversely, the income of a state's residents may come from outside of the state.

In making an economic prediction or carrying out a simulation for a region, it is vital that we take this distinction into account.

For example, many people earn income in cities yet live in suburbs in a different state or region. Thus, when the economy of the city is stimulated, a high proportion of the generated income will be spent in the suburbs and will not lead to further induced spending in the city. In the suburbs, induced demand using economic base model A would overestimate induced spending because in fact a high proportion of spending depends on income earned outside of the region and is thus exogenous income.
How much economic activity will be created may be overestimated because we have the wrong model. In economic base model B, we will separate income by place of work (YP) from income by place of residence (YP) recognizing for the following three reasons for the discrepancy:

2. Income earned from capital invested outside of place of residence.
3. Income earned by working outside of place of residence.

The new K values that we will generate will be based on data that is available for states and counties assuming that an estimate of exports (XFG) has been obtained.

**B.1 ACCOUNTS**

The accounts for Economic Base Model B build on Economic Base Model A, with place-of-work income separated from place-of-residence income and with some additional flow variables added. In the accounts for Model B, account A is defined on a place-of-work basis, while account B is defined on a place of residence basis. We start with the simple account A for Economic Base Model A, and then show the parallel account for Economic Base Model B:

<table>
<thead>
<tr>
<th>Account for Model A</th>
<th>Account for Model B</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. State Product Account</td>
<td>A. State Product Account</td>
</tr>
<tr>
<td>Uses</td>
<td>Uses</td>
</tr>
<tr>
<td>Y (output)</td>
<td>YLPL (local earnings by local residents)</td>
</tr>
<tr>
<td>CG (consumption and local government spending)</td>
<td>YLPU (earnings locally by non-residents)</td>
</tr>
<tr>
<td>IL (local investment)</td>
<td>IL (local investment)</td>
</tr>
<tr>
<td>XFG (exports, including federal government)</td>
<td>XFG (exports, including federal government)</td>
</tr>
<tr>
<td>− M (imports)</td>
<td>−M (imports)</td>
</tr>
</tbody>
</table>

| Y | Y | Y (Gross State Product) | Y (Gross State Product) |

| YLPL | Labor and proprietors income earned in the state that stays in the state |
| YLPU | Labor and proprietors income earned in the state by non-residents of the state |
| H | Profits earned in the state |
The source of local output (Y), which is often called Gross State Product, is consumer and local government spending (CG) plus local investment (IL) plus exports including federal government spending (XFG) minus imports (M). The use of local income is now divided into laborers' and proprietors' income, earned by residents (YLPL) and non-residents (YLPU) of the state, and the residual, profits (H = Y - YLPL - YLPU).

<table>
<thead>
<tr>
<th>Account for Model A</th>
<th>Account for Model B</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>B. Personal and Local Government Income and Outlay Account</strong></td>
<td><strong>B. Personal and Local Government Income and Outlay Account (Place of residence)</strong></td>
</tr>
<tr>
<td><strong>Uses</strong></td>
<td><strong>Uses</strong></td>
</tr>
<tr>
<td>CG (consumer and local government spending)</td>
<td>Y (output)</td>
</tr>
<tr>
<td>S (personal savings and local government surplus)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Y</td>
<td>Y</td>
</tr>
</tbody>
</table>

Table 5 Account B for Model B is a place of residence account and both sides of the account are equal to the income of residents of the state. Output (Y) from our simple account is replaced by all sources of income for residents of the region: laborers' and proprietors' income earned within the state (YLPL) and outside of the state (UYLP), dividends, interest and rent received by residents of the state (DIR) and government transfers to residents of the state (V). The uses for the income are, consumer and regional government spending (CG) and the residual, which includes federal taxes and local savings (SETC). The sum of both sides of the equation gives us the personal income of the residents of the state (YP).
<table>
<thead>
<tr>
<th>XFG (exports including federal government)</th>
<th>Uses</th>
<th>Rest of Country</th>
</tr>
</thead>
<tbody>
<tr>
<td>M (Imports)</td>
<td>IR (investment, rest of country)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sources</th>
<th>Uses</th>
<th>IR (investment, rest of country)</th>
</tr>
</thead>
<tbody>
<tr>
<td>S (personal savings and local government)</td>
<td>C. Savings and Investment Account</td>
<td>A. Federal Government</td>
</tr>
<tr>
<td>XFG (exports excluding federal government)</td>
<td>S (personal savings and local government)</td>
<td>B. Federal Reserve</td>
</tr>
<tr>
<td>M (Imports)</td>
<td>V (government transfers)</td>
<td>D. Residual Account</td>
</tr>
<tr>
<td>- DIR (dividends)</td>
<td>- IT (interest income)</td>
<td>E. Residual Account</td>
</tr>
<tr>
<td>- E (profits earned by foreigners)</td>
<td>F. Residual Account</td>
<td></td>
</tr>
</tbody>
</table>

**Table 6: Comparison of Account CD for Model B to Account C and D for Model A**

In account CD, we combine accounts C and D for Model A into a residual account for Model B. This allows the use and sources of funds for the rest of the country and the Federal government. The main reason for this account, which will not enter into our model directly, is to complete the accounts.
The complete accounts for economic base model B can be shown in Table 7:

<table>
<thead>
<tr>
<th>Uses</th>
<th>Sources</th>
<th>Uses</th>
<th>Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>YLPL</td>
<td>CG</td>
<td>CG</td>
<td>YLPL</td>
</tr>
<tr>
<td>YLPU</td>
<td>IL</td>
<td>SETC</td>
<td>UYLP</td>
</tr>
<tr>
<td>H</td>
<td>XFG</td>
<td>DIR</td>
<td>V</td>
</tr>
<tr>
<td></td>
<td>M</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Y</td>
<td></td>
<td>YP</td>
<td>YP</td>
</tr>
<tr>
<td>Y</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

CD. Residual Account

<table>
<thead>
<tr>
<th>Uses</th>
<th>Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>XFG</td>
<td>M</td>
</tr>
<tr>
<td>UYLP</td>
<td>YLPU</td>
</tr>
<tr>
<td>DIR</td>
<td>H</td>
</tr>
<tr>
<td>V</td>
<td>SETC</td>
</tr>
<tr>
<td>IL</td>
<td></td>
</tr>
</tbody>
</table>

B.2 THE NEW EQUATIONS

The equations behind these accounts can be used as part of the basis for building Economic Base Model, Type B, of a regional economy. In order to assign values to the equations for Economic Base Model B, however, we must use the available data, which is in the form:

YLP total regional income: the amount of income earned by residents and workers in a region; and
RA residence adjustment: the net amount of the excess of earnings by local residence outside of the local area and earnings of out of area residents in the local area.
Algebraically, these are:

\[ YLP = YLPL + YLPU \]  \hspace{1cm} (36)

and

\[ RA = UYLP - YLPU \]  \hspace{1cm} (37)

Substituting equation (36) into account A, then adding equations (36) and (37) we obtain

\[ YLP + RA = YLPL + UYLP \]  \hspace{1cm} (38)

which we substitute in account B. By also substituting equation (37) into account CD, we have a slightly altered set of accounts so that we can use the existing data. These accounts and their measured values for Michigan in 1977 are shown in Table 8.

| Table 8 |
|-----------------|-----------------|
| Social Accounts for any Region with Specific Estimates for Michigan in 1977 |

A. State Product Account  
(Place of Work)

<table>
<thead>
<tr>
<th>Uses</th>
<th>Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>YLP</td>
<td>CG = 65.0</td>
</tr>
<tr>
<td></td>
<td>IL = 16.2</td>
</tr>
<tr>
<td>H</td>
<td>XFG = 83.7</td>
</tr>
<tr>
<td></td>
<td>- M = 84.1</td>
</tr>
<tr>
<td>Y</td>
<td>= 80.8</td>
</tr>
</tbody>
</table>

B. Personal Income and Outlay Account  
(Place of Residence)

<table>
<thead>
<tr>
<th>Uses</th>
<th>Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>CG</td>
<td>YLP = 55.0</td>
</tr>
<tr>
<td>RA</td>
<td>= 0.3</td>
</tr>
<tr>
<td>DIR</td>
<td>= 8.1</td>
</tr>
<tr>
<td>V</td>
<td>= 5.8</td>
</tr>
<tr>
<td>YP</td>
<td>= 69.2</td>
</tr>
</tbody>
</table>

CD. Residual Account

<table>
<thead>
<tr>
<th>Uses</th>
<th>Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>XFG</td>
<td>M = 84.1</td>
</tr>
<tr>
<td>RA</td>
<td>H = 25.8</td>
</tr>
<tr>
<td>DIR</td>
<td>SETC = 4.2</td>
</tr>
<tr>
<td>V</td>
<td>= 5.8</td>
</tr>
<tr>
<td>IL</td>
<td>= 16.2</td>
</tr>
<tr>
<td></td>
<td>114.1</td>
</tr>
<tr>
<td>YLP</td>
<td>114.1</td>
</tr>
</tbody>
</table>

YLP  
Labor and proprietor's income earned in the state.

RA  
Residential adjustment (UYLP - YLPU): The net of income earned in the rest of the country by state residents and the earnings in the state of non-residents.
Using these accounts we can now develop model B as an alternative to model A. We first recall that the equations for model A were

\[ B = rXFG + rILp \]  \hspace{1cm} (39)  
\[ Y = rCG + B = 0.49CG + B \]  \hspace{1cm} (40)  
\[ CG = bY = 0.804Y \]  \hspace{1cm} (41)  

For our type B model the equations for \( B \) and \( Y \) will remain the same. However, the equation for \( CG \) has an obvious problem since spending locally depends on local output not on the local income available for spending. It is much more appropriate to make local spending on personal consumption and local government (CG) depend on local personal income \( YP \) than on local output. Thus for model B we use the relationship

\[ CG = cYP \]  \hspace{1cm} (42)  

The equation for \( YP \) can be written from account B as

\[ YP = YLP + RA + DIR + V = YLP + RDV \]  \hspace{1cm} (43)  

where

\[ RDV = RA + DIR + V \]  \hspace{1cm} (44)  

In order to obtain the equation for \( YLP \), we must assume that the ratio of \( YLP + Y = p \) remains constant (i.e., that profits, interest payments, etc., remain at a constant proportion of \( Y \)). Thus,

\[ YLP = pY \]  \hspace{1cm} (45)  

is the equation that closes the model if RDV is assumed to be exogenous. The major exception to the simplifying assumption that the components of RDV in equation (44) are exogenous occurs if the residential adjustment (RA) is negative. In that case changes in \( Y \) would be expected to lead to changes in earnings by out-of-state residents. Therefore, when RA is negative, it should be removed from (44) and included with \( YLP \) in equation (45).

### B.3 THE NEW MULTIPLIER

By substitution we can now write

\[ B = rXFG + rILp \]  \hspace{1cm} (46)  
\[ Y = rCG + B \]  \hspace{1cm} (47)  
\[ CG = c(pY + RDV) \]  \hspace{1cm} (48)  

Thus,

\[ Y = r(cY + RDV) + B \]  \hspace{1cm} (49)  
\[ Y = \frac{1}{1 - rcp}(rcRDV + B) \]  \hspace{1cm} (50)
where
\[ r = \frac{Y}{CG + IL + XFG} = 0.49 \text{ as before} \quad (51) \]
\[ c = \frac{CG}{YP} = \frac{65}{69.2} = 0.94 \quad (52) \]
\[ p = \frac{YLP}{Y} = \frac{55}{80.8} = 0.68 \quad (53) \]

We can now determine the Economic Base Model B multiplier:
\[ KB = \frac{1}{1 - rcp} = \frac{1}{1 - 0.313} = \frac{1}{0.686} = 1.46 \quad (54) \]

which compares with our earlier multiplier in Model A of \( K = 1.65 \).

The new multiplier is smaller than the old multiplier. This recognizes that some local spending is supported by income that is not related to local output. This lower multiplier also reflects that a portion of the value added that is generated locally will go to outside owners of local capital. The shareholders of General Motors who live outside of Michigan own part of that State’s capital.

Even when we don’t have the values in the accounts in Table 8 we can still estimate the KB value in terms of the data available for every state and county in the U.S. We can do this by using the K value and the ratio RDV to YLP.

It can be shown that* 
\[ KB = \frac{(1 + RDV/YP)K}{(RDV/YP)K + 1} \quad (55) \]

In the case of Michigan where RDV/YLP = 0.258 and \( K = 1.65 \). This yields
\[ KB = \frac{(1.258)(1.65)}{(.258)(1.65) + 1} = 1.46 \quad (56) \]

which confirms our result from direct calculation in (54).

Thus, even for simple economic base studies by calculating RDV/YLP from data that is publicly available from the Bureau of Economic Analysis (BEA) the accuracy of the multiplier can be improved. Note that only in the case where RDV/YLP equals zero will \( K \) and \( KB \) be identical.

---

*The derivation is available from the author. If RA is negative, exclude it from RDV and add its negative value to YLP in equation 55.
In Part I, two major differences between national and regional models have been shown. The first is the major role of imports into and exports from the region and the second is the effect of differences between local output and local income. Both of these differences affect the multiplier. In this part we have also shown the need to relate the models to employment and personal income variables for which regional data is available. In the process of building models for local regions we have followed a framework that is parallel to the simple Keynesian models developed for national economies. Since there is free capital flow among regions, the absence of monetary policy variables and of interest rate determination may be more justified in regional analysis than it would be in national model building.

In Part II our regional models will be extended to take account of different industries and their interdependencies. Then we will show how these core models can be made more flexible and how new behavioral relationships can be added. Finally we will describe how data requirements can be met.