MONEY AND THE SRAFFA SYSTEM*

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Many years ago, Keynes asserted the following: "The division of Economics between the Theory of Value and Distribution on the one hand and the Theory of Money on the other hand is, I think, a false division." [10, p. 293] Yet today, over four decades later, this division remains as real as ever. This is despite important developments in both monetary and value theory. In the case of the latter, we refer in particular to the seminal work of Sraffa [20].

In this paper an attempt is made to find a theoretical basis for overcoming this division, on the basis of Sraffa's work. As a result some significant theoretical results shall be obtained, which illustrate the importance of monetary phenomena for both value theory and economics as a whole. However, this paper does not claim to incorporate a comprehensive treatment of monetary theory or phenomena. It is more a 'thought experiment', to indicate a possible line of advance for theoretical research, than a last word on the subject.

I. THE ABSENCE OF MONEY IN THE SRAFFA SYSTEM

The Sraffian model has had an enormous impact on the neoclassical theory of value and distribution. This has, of course, been fully discussed elsewhere, e.g. [6], [7], [9], [16]. Also, Sraffa's work had had important critical consequences for Marxian economics [8], [14], [21]. These results should not be belittled, but, as we shall argue below, they have been confined to a paradigm which excludes money.

This paradigm can be described as one which works within a Classical long-run position. In the case of the Sraffa system it is quite clearly a position where the rate of profit is equalised. Although the Sraffa system is conceptually different from a general equilibrium system of the Walrasian type, or even the von Neumann model [19], these all have one thing in common: they do not include money. Clower [5] has shown that money can never be introduced into a stationary-state, general equilibrium model. Whilst it can be argued that Sraffa's model does not deserve the latter description it has the same characteristic. This is quite easy to demonstrate. The Sraffa system, like many stationary-state general equilibrium models, contains no good which, uniquely, possesses all the important features of money. It is always possible to designate one of the goods in the system as the money commodity, and thus provide the price numéraire for the system. However, apart from having the ascribed status of being the unit of account, such a money commodity is

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indistinguishable from other goods in the system. Its status as the money commodity is simply ascribed, it is not inherent. It is true that the designated money commodity could operate as the medium of exchange, but there is no inherent feature in the system which displays this fact. As a result, any commodity could serve as the medium of exchange in the Sraffian system. Consequently, the good which has the ascribed status of money cannot be money proper. This argument is based on the familiar analysis of money, as the unique medium of exchange, by Clower [3].

It cannot be claimed, in this paper, that all the essential features of money are introduced. In particular, scant attention is given to the process and structure of exchange in a capitalist economy. However, there does not seem to be any reason why the analysis cannot be extended in this direction at a later date. This paper highlights the function of money as a store of wealth. Money, as many writers have pointed out, is a means of dealing with an uncertain future. In the context of this uncertainty, money functions as a special store of wealth, with the unique property that it can be readily exchanged for any other commodity. Marx, in this spirit, describes money as the ‘universal material representative of wealth’ [13, p. 216].

The latter feature of money provides the basis for most of the motives for holding money in a capitalist economy. These well-known reasons for holding money were listed and described by Keynes, particularly in [10]. There is no need to repeat or discuss them here. The formal analysis in this paper presumes simply that there are strong reasons for holding money from one period of production to the next, without attaching itself to one particular explanation of such ‘liquidity preference’. (The latter term is adopted in this paper, but with some reluctance. Its use is not intended to endorse an individualistic or preference-theory explanation of the motives for the holding of money).

As we shall see, the treatment of money in this paper also sees money as a sort of ‘means of production’, i.e., an asset which is necessary for production to take place, within a certain mode of production. The treatment, therefore, differs from exchange-orientated theories of money, which are based on the sphere of exchange. (Typically, this is the neoclassical treatment of money).

II. MONEY AND SHIFTING EQUILIBRIUM

It may be argued that the introduction of money in the Sraffian system must necessarily introduce a degree of dynamism and indeterminacy, and thus it will be impossible to derive relations between prices, wages and profits, as in the long-run Sraffian system. Fortunately, however, this is not the case. It is possible to shift our ground from the Sraffa long-run state to a type of ‘shifting equilibrium’. Such a distinct type of equilibrium which embraces a monetary economy is not new. It is proposed by Keynes in the pregnant, and somewhat neglected, Twenty-First Chapter of his General Theory:

‘we might make our line of division between the theory of stationary equilibrium and the theory of shifting equilibrium — meaning by the latter the theory of a system in which changing views about the future are capable of influencing the present situation.’ [10, p. 293]

Naturally, it is these ‘changing views about the future’ which influence the amounts of
wealth that are held in the form of money. This article will concentrate on some of the effects of such changes. But it is noted that our inclusion of money, and the manner in which it is done, is quite consistent with the notion of 'shifting equilibrium' in Keynes' work as interpreted in an important article by Kregel. According to Kregel's discussion of Keynes' notion of shifting equilibrium:

'entrepreneurs take actions based on expectations regarding an uncertain future. If, by chance, the actions of entrepreneurs are compatible in the aggregate, the economy will be on the warranted or equilibrium path ... If, on the other hand, initial plans are disappointed then equilibrium can be obtained, after a period of historical time, only if it is assumed that despite entrepreneurial realisation of mistakes, the state of short-period expectations is independent of general expectations, so that entrepreneurs persist in their beliefs until equilibrium is established by trial and error. ... If, however, realisation of error alters the state of expectations and shifts the independent behavioural functions, Keynes's model of shifting equilibrium will describe an actual path of an economy over time chasing an ever changing equilibrium — it need never catch it.' [11, pp. 216-7]

The model in this paper can include such a notion in of shifting equilibrium in the following way. For various reasons, capitalists adopt a certain degree of "liquidity preference" relating to their expectations of the future. Production and circulation progress, and capitalists find that their actions do not have the foreseen outcome. Consequently they revise their expectations and change their degree of liquidity preference. The economic system gropes towards an uncertain future. But it never attain equilibrium in the Walrasian sense. (It may be relevant here to note the important works of Benassy [1] and Younes [22] which establish the feasibility of a non-Walrasian equilibria and produce some results which coincide with those in this paper. However, their work is more orientated towards the analysis of exchange, rather than production).

We have demonstrated, therefore, that the inclusion of money relates to the question of expectations of the future and, in turn, to the notion of shifting equilibrium. It is now necessary to discuss certain different types of money. One of the aims of this paper is to show that some different types of money do perform differing roles in the shifting equilibrium state, and thus they have different effects.

We shall assume that, for the purpose of this article, and in order to keep the exposition as simple as possible, that there are just two types of money, account money and cash money. If money is held in the form of cash, either in the form of paper money or as a commodity money such as gold, then the holder obviously does not receive any interest payment. On the other hand, if cash money is deposited in a bank then we part with some of the liquidity associated with the holding of cash money, and this degree of liquidity is transferred to the banker. For this benefit the bank sometimes pays interest, depending on which type of bank account is used. We shall see below that the distinction between interest-bearing account money, and "sterile" cash money, is of great importance in determining the actual equilibrium state.

Another important distinction between cash and account money is that the former is physically produced, and has a tangible physical form, whereas the latter has not. Account money is simply numbers in books or in computer memory-banks. Cash money, on the
other hand, is either bits of printed paper or coins minted with some metal. This physical form is necessary, otherwise cash money could not be fully liquid, nor fulfil its complete function as cash.

III. FORMAL SPECIFICATION OF THE MODEL

We shall now turn to the task of building a simple monetised shifting equilibrium model. It will not embrace all the complex and dynamic features of a monetary economy. But it will serve to illustrate the basic features of the shifting equilibrium state.

For simplicity we shall assume that just two non-money goods exist, $G1$ and $G2$. It is relatively easy, however, to incorporate more non-money goods in the model, with just a little modification. We shall assume that two processes exist to produce these goods, $P1$ and $P2$. The third good is the money commodity, and a third process exists to produce cash money. We shall assume that cash money exists in the form of one denomination only, i.e., it is physically homogeneous. Hence all cash money is produced in the same physical process. As long as this assumption is maintained, cash money could be either homogeneous paper money, or gold. But it would, perhaps, be best to think of cash money as gold, and ignore paper money.

We shall not assume that any of the three processes are single product processes, unless otherwise specified. The main reason for this is that the joint production framework will allow the introduction of a form of fixed capital, as explored by Sraffa [20, pp. 63-73]. The cost, in terms of added complexity, is very slight indeed. And it is much easier to modify a joint production system algebraically, to turn it back into a single product system, rather than vice-versa.

We shall assume constant returns to scale, at least for the time period in which the 'shifting equilibrium' exists, and for the non-money coefficients in the system. However, our primary concern is not to examine changes in employment and output, but to examine relations between prices, wages, the rate of interest, and the rate of profit.

We shall define the input and output coefficients that are associated with unit labour employment in each process. Hence, with constant returns to scale, it is easy to find actual inputs and outputs by multiplying by the amount of actual labour employment. In $P1$, $a_{11}$ units of $G1$, plus $a_{12}$ units of $G2$, are required as means of production, per unit of labour employed. The output per labour unit employed, in that process, is $b_{11}$ units of $G1$ and $b_{12}$ units of $G2$. Similarly, in $P2$ the inputs are $a_{21}$ and $a_{22}$ units, and the outputs are $b_{21}$ and $b_{22}$ units, of $G1$ and $G2$ respectively, all per unit of labour employed in that process. Clearly, some of these coefficients may be zero, but none may be negative. Hence the non-money aspect of the economic system can be represented by the following schema:

<table>
<thead>
<tr>
<th>INPUTS</th>
<th>OUTPUTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P1: a_{11}$</td>
<td>$b_{11}$</td>
</tr>
<tr>
<td>$a_{12}$</td>
<td>$b_{12}$</td>
</tr>
<tr>
<td>$P2: a_{21}$</td>
<td>$b_{21}$</td>
</tr>
<tr>
<td>$a_{22}$</td>
<td>$b_{22}$</td>
</tr>
</tbody>
</table>

labour
Now we come to the introduction of money in the system. We shall assume that a liquidity preference for $m_1$ units of cash money exists in $P1$, per unit of labour employed in that process, and $m_2$ units in $P2$, per unit of labour employed. Obviously, $m_1$ and $m_2$ cannot be negative. Account money may also exist, but we shall leave the specification of this for a while. Cash money is physically produced in a third process, $PM$. This latter process uses inputs $a_{m1}$ and $a_{m2}$ of $G1$ and $G2$ respectively, per unit of labour employed in that process, and produces outputs of $b_{m1}$ units of $G1$, $b_{m2}$ units of $G2$, and $m_m$ units of money, per unit of labour employed. Cash money $m_m$ is held at the start of the production period in this process. The complete physical input-output schema for the monetised production system is, therefore, as follows:

<table>
<thead>
<tr>
<th></th>
<th>$G1$</th>
<th>$G2$</th>
<th>Cash</th>
<th>Labour</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P1$</td>
<td>$a_{11}$</td>
<td>$a_{12}$</td>
<td>$m_1$</td>
<td>1</td>
</tr>
<tr>
<td>$P2$</td>
<td>$a_{21}$</td>
<td>$a_{22}$</td>
<td>$m_2$</td>
<td>1</td>
</tr>
<tr>
<td>$PM$</td>
<td>$a_{m1}$</td>
<td>$a_{m2}$</td>
<td>$m_m$</td>
<td>1</td>
</tr>
</tbody>
</table>

It must be emphasised, to avoid misunderstanding, that $m_1$, $m_2$ and $m_m$ are variables, rather than fixed constants. They can change from one production period to the next, without difficulty in the model. (Even if they change during production then the stated figure can be regarded as some sort of mean value, and if the production period is short this will not create any great difficulty). Without doubt, these three variables will be affected by, amongst other things, relative and absolute prices. As prices depend, as we shall see, upon the degree of liquidity preference then there is an obvious process of feedback involved. It is certainly not suggested that liquidity preference is independent of prices.

Note that, in the above schema, the monetised production system would be a joint production system even if the non-money aspect of the system produced single products. (But in that case the monetised production system would be an all-productive system, to use Schefold's terminology, and it would share many of the features of a pure single product system (see [18])). Note also that $m_1$ and $m_2$ appear as both an 'input' and an 'output' in both $P1$ and $P2$. This may appear strange, as cash money is neither used up nor produced in these two processes. But its placement in the input-output schema is closely analogous to Sraffa's treatment of fixed capital, where a machine may appear as both an input, and an output, in addition to another physically produced product.

It is also interesting to note that according to Sraffa's formal definition of a basic good [20, pp. 49-55], money may be regarded as a basic good. This may suggest a modification of Sraffa's formal definition.

We now come to another important feature of the model that distinguishes it from a stationary-state equilibrium model. Cash money is produced in the economy, so the question may be asked, where does it go? The other processes do not use up cash money, per unit of labour employment, so if the economy were at a stationary level then no more
cash money need be produced. It is an important feature of our shifting equilibrium model that we must assume that total employment and output in the non-money aspect of the system are expanding over time. Hence there is a need for more and more cash money in the system, at least as long as \( m_1 \) and \( m_2 \) remain constant. This insures the need for production actually to take place in the money-producing process \( PM \). Hence, in contrast to Sraffa's long-run model, the actual level of output and employment are, to some extent, involved in the specification of a shifting equilibrium model.

It may be argued that it is illegitimate to include money in the production system, particularly in the form of an input and an output in a production process. To argue against this point we must first repeat our previous assertion that money is a necessary part of a capitalist economic system. To remove money would mean a radical change in the whole functioning of the system. But it may still be argued that money is unlike technology, in that the latter can be considered independently of the economic system, so it may be illegitimate to smuggle in the strange substance of money. Of course it is true that, to some degree, technology can be considered as an autonomous entity, but that does not allow us to regard it as an asocial thing, independent of the social relations of the economic system. The labour process, for example, which is involved in all non-automated technical processes of manufacture, always bears the stamp of the social system and social relations that pervade the whole economy. So our first counter-argument is that technology, like money, is not independent of the type of economic and social system that is involved. (See [17], pp. 76-7 and [12], pp. 136-9).

The reader may still not be convinced that it is legitimate to include money in an input and output table. We have already argued that cash money is a physically produced commodity. But we can give additional arguments. The work of von Neumann [15] and Sraffa [20] includes a way of treating fixed capital as an input and an output in a production process, so that fixed capital not only appears as a required input but also as a 'produced' output. The fixed capital could endure a physical change in the production process, by either being physically worn, or being run-in, but that is not necessarily the case. Hence, like money, the fixed capital does not necessarily experience a physical metamorphosis. Neither is it the intentional aim of production. Yet Sraffa and von Neumann regard fixed capital as both a product and an input, and they derive some important theoretical results by doing this. We can, therefore, cite their work as a precedent for including money as an input and an output in processes which do not actually 'produce' money. The reader will have to judge, from our exposition below, whether or not this approach is actually fruitful from the theoretical point of view. But at this stage we can certainly claim that there is nothing actually illegitimate in our treatment of money.

IV. SOME PRELIMINARY IMPLICATIONS

We can now explore some theoretical consequences of the introduction of money in the system. The first of these is the effect on embodied labour values, as utilised by Ricardo and Marx. The embodied labour value of a commodity is defined such that the total embodied labour value of the gross output of a process equals the embodied labour value of all the inputs plus the amount of socially necessary living labour employed.
Therefore, in the monetised production system, we get the following equations, one for each process:

\[ a_{11}l_1 + a_{12}l_2 + m_1l_m + 1 = b_{11}l_1 + b_{12}l_2 + m_1l_m \]
\[ a_{21}l_1 + a_{22}l_2 + m_2l_m + 1 = b_{21}l_1 + b_{22}l_2 + m_2l_m \]
\[ a_{m1}l_1 + a_{m2}l_2 + m_ml_m + 1 = b_{m1}l_1 + b_{m2}l_2 + m_ml_m + b_ml_m \]
where \( l_1 \), \( l_2 \) and \( l_m \) are the embodied labour values of one unit of GI, G2 and cash money, respectively, according to the above definition.

Clearly in the above equations the terms \( m_ml_m \), \( m_1l_m \) and \( m_2l_m \) cancel out completely. Hence \( m_1 \), \( m_2 \), \( m_m \) will not appear in the solution for \( l_1 \), \( l_2 \) and \( l_m \). We may conclude that the degree of liquidity preference, i.e., \( m_1 \), \( m_2 \), \( m_m \), does not affect the embodied labour values in the system. Obviously this is because money cash holdings do not add to or detract from the amount of embodied labour value in each process as production takes place.

The first step towards a definition of the Sraffian standard commodity, in the monetised production system, is to find a row vector \( x \) such that

\[(1 + R)x = \begin{bmatrix}
  a_{11} & a_{12} & m_1 \\
  a_{21} & a_{22} & m_2 \\
  a_{m1} & a_{m2} & m_m
\end{bmatrix} x = \begin{bmatrix}
  b_{11} & b_{12} & m_1 \\
  b_{21} & b_{22} & m_2 \\
  b_{m1} & b_{m2} & m_m + b_m
\end{bmatrix} x\]

where \( R \) is the maximum rate of profit, as defined by Sraffa. Terms containing \( Rm_1 \) and \( Rm_2 \) will remain after cancellation. Hence the \( x \) vector depends on liquidity preference for cash money, i.e., \( m_1 \), \( m_2 \), \( m_m \). This result also follows from the fact that, according to Sraffa's definition, cash money is a basic good. The standard commodity itself is any scalar multiple of either the right or the left hand side of the above matrix equation.

Consequently, in general, cash money will appear as part of the standard commodity, and the relative amount of cash money that appears will depend on \( m_1 \), \( m_2 \), \( m_m \). In other words the composition of the standard commodity is significantly dependent on the degree of liquidity preference for cash money.

If the latter were regarded as variable, whilst the \( a_{ij} \) and \( b_{ij} \) technological coefficients are constant, then the standard commodity can no longer serve as an invariable standard as it does in Sraffa. A constant wage-profit function, where wages are measured in terms of the standard commodity, is unobtainable in these circumstances.

However, the above results indicate a profound difference between the approach to the theory of value in Sraffa's work, and that in Ricardo and Marx. We have shown that Sraffa's standard commodity does take monetary conditions into account, when it is re-defined in the context of a monetised system, but the Ricardo-Marx embodied labour values are not affected by liquidity preference for cash money.
V. PRICES, PROFITS AND THE RATE OF INTEREST

In the above discussion of embodied labour values and the standard commodity we have concentrated exclusively on the physical aspect of the production system. It has been convenient, therefore, to leave the discussion of non-physical, account money, until now. None of our previous results need to be altered by this tardy introduction of account money.

From now on we must consider the monetary and financial system in more detail. As we have shown, cash money is produced in $PM$. We shall now assume that it is produced under capitalist conditions. The capitalists in $PM$ produce money, and trade it with the other capitalists for amounts of $G1$ and $G2$ which they require for further production of money. Cash money is thus continually injected into the non-money aspect of the system, and the cash money is required in ever-increasing amounts to cope with the increasing liquidity requirements as the economy expands.

It is assumed that the function of banking is carried out by the industrial capitalists themselves. Cash money is lent and borrowed between capitalists, and accounts are kept of debts and credits. Interest is paid on all this money in the accounts, which we shall call account money, and this clearly distinguishes it from cash money, which is not interest-bearing.

We shall assume that the rate of interest is uniform, on all account money, in all debts and credits, at a uniform rate $i$. Someone's debt is always someone else's credit. Hence the algebraic sum of all debts and credits (debts as negative quantities) must, of course, be zero.

We shall assume that no physical or labour inputs are required to administrate the account money banking system. Hence account money is not produced in the production system, like cash money. Also, we repeat, it has no physical presence. Account money, therefore, unlike cash money, cannot be considered as an input or output in the production system. We shall simply bear in mind that the account money that is held in each process is defined as $m_{a1}$, $m_{a2}$, and $m_{am}$ in $P1$, $P2$, and $PM$ respectively. Overdrafts are represented by negative quantities.

The following additional notation is adopted. The prices of $G1$ and $G2$ are $p_{1}$ and $p_{2}$ respectively. Money itself is obviously chosen as numéraire, and its price is unity. We shall let the wage rate be $w$, and assume that all wages are paid at the end of the production period. The rate of profit, which we shall assume to be the same in all industries, as we are still considering an equilibrium situation, is $r$. However, despite the fact that the system is in a shifting equilibrium, there may be a case for regarding the rate of profit as being different in magnitude from the rate of interest. This would be because of the uncertainty that is inherent in a shifting equilibrium system. A greater degree of uncertainty would apply in industrial production, due to the spectres of supply interruptions, technical change, and so on, than would apply to bank deposits of account money that yield interest. It would be reasonable to assume the following, therefore;

$$0 < i \leq r.$$  

Investment in industrial production can be thus rewarded with a higher rate of return,
due to the higher risks and uncertainty involved. However, although it seems reasonable to make this assumption, it is by no means crucial, and it is not an essential feature of the model in this paper.

As noted above, the amounts of account money gain interest at the rate \( i \), and as a result they are multiplied by a factor of \((1 + i)\). This is also true of overdrafts, which carry a negative sign. Interest, of course, is part of the gross profits of each industry. The rate of profit is calculated in the usual manner; profits are divided by the price of the total advanced capital. Let us take process \( P1 \) as an example. The price of all outputs, including cash money, and account money with interest, is as follows:

\[
b_{11}p_1 + b_{12}p_2 + m_1 + m_{a1}(1+i)
\]

Profits are

\[
(b_{11}p_1 + b_{12}p_2 + m_1 + m_{a1}(1+i)) - (a_{11}p_1 + a_{12}p_2 + m_1 + m_{a1} + w).
\]

The total invested capital in \( P1 \) is

\[
a_{11}p_1 + a_{12}p_2 + m_1 + m_{a1}.
\]

The rate of profit, therefore, is given by the following equation:

\[
r = \frac{(b_{11}p_1 + b_{12}p_2 + m_1 + m_{a1}(1+i)) - (a_{11}p_1 + a_{12}p_2 + m_1 + m_{a1} + w)}{a_{11}p_1 + a_{12}p_2 + m_1 + m_{a1}}.
\]

It is evident that we may derive three equations for the three processes, from the three expressions for the rate of profit like the one above:

\[
(a_{11}p_1 + a_{12}p_2 + m_1 + m_{a1}(1+r) + w = b_{11}p_1 + b_{12}p_2 + m_1 + m_{a1}(1+i)
\]

\[
(a_{21}p_1 + a_{22}p_2 + m_2 + m_{a2}(1+r) + w = b_{21}p_1 + b_{22}p_2 + m_2 + m_{a2}(1+i)
\]

\[
(a_{m1}p_1 + a_{m2}p_2 + m_m + m_{am}(1+r) + w = b_{m1}p_1 + b_{m2}p_2 + b_m + m_m + m_{am}(1+i)
\]

Note that as long as the rate of profit is positive, liquidity preference for cash money will not cancel out from the above equations. Account money will disappear, however, if the rate of profit is equal to the rate of interest.

Using matrix algebra, the solution for the price vector is found along the following lines:

\[
\begin{bmatrix}
p_1 \\
p_2 \\
1
\end{bmatrix}
= w
\begin{bmatrix}
b_{11} - (1+r)a_{11} & b_{12} - (1+r)a_{12} & -rm_1 - (r-i)m_{a1}^-1 & 1 \\

b_{21} - (1+r)a_{21} & b_{22} - (1+r)a_{22} & -rm_2 - (r-i)m_{a2}^-1 & 1 \\

b_{m1} - (1+r)a_{m1} & b_{m2} - (1+r)a_{m2} & b_m - rm_m - (r-i)m_{am}^-1 & 1
\end{bmatrix}
\]

There is no need to pursue the algebra. It is clear that wages and prices depend on the degree of liquidity preference for cash money (as long as the rate of profit is not zero), and the amounts of account money (as long as the rate of interest is not equal to the rate of profit), and the rate of interest, as well as the rate of profit. The wage-profit frontier can be found by generating the third of the above equations, which does not contain \( p_1 \).
and \( p_2 \). It is clear that this wage function includes the rate of profit and the rate of interest.

It could be said, therefore, that prices, wages and profits all depend, to some extent, on variables that pertain to the monetary side of the economy, as well as the technical coefficients in the non-money aspect. In addition, they are all affected by the technical conditions of production in the money-producing sector \( PM \). However, without further information on all the variables and coefficients, we cannot say in which direction wages, prices, and profits will be affected, nor what will be the extent of the change. Unlike the general equilibrium model of the stationary state type, our shifting equilibrium model includes monetary information in the derivation of prices and wages, in addition to the orthodox technical input-output coefficients.

VI. A SIMPLE EXAMPLE

The effect of these monetary phenomena can be illustrated with a simple model that is cast in the mould of the above analysis. Consider the following monetised shifting equilibrium system:

<table>
<thead>
<tr>
<th></th>
<th>Inputs</th>
<th>Outputs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( G1 )</td>
<td>( G1 )</td>
</tr>
<tr>
<td></td>
<td>( G2 )</td>
<td>( G2 )</td>
</tr>
<tr>
<td>( P1 )</td>
<td>1 ( \rightarrow )</td>
<td>2 ( \rightarrow )</td>
</tr>
<tr>
<td>( P2 )</td>
<td>1 ( \rightarrow )</td>
<td>1 ( \rightarrow )</td>
</tr>
<tr>
<td>( PM )</td>
<td>1 ( \rightarrow )</td>
<td>1 ( \rightarrow )</td>
</tr>
<tr>
<td></td>
<td>Cash Money</td>
<td>Cash Money</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>( m )</td>
<td>( m )</td>
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<td></td>
<td>( m )</td>
<td>( m )</td>
</tr>
</tbody>
</table>

Note that we have assumed that \( m_1 = m_2 = m \), and \( m_m = 0 \). We shall also assume that \( m_{a1} = m_{a2} = 2m \), and \( m_{am} = 0 \). Finally, we shall assume that the rate of interest is half the rate of profit; \( i = \frac{1}{2}r \).

Using the former algebraic analysis of prices, wages and profits, we may derive the following equations:

\[
\begin{align*}
\omega &= 1 - r - 2m r^2 \\
p_1 &= 1 + 2mr \\
p_2 &= 1/(1-r)
\end{align*}
\]

The maximum rate of profit is \( (\sqrt{1 + 8m} - 1) / 4m \).

A number of important facts emerge from this example. First, relative prices, as well as some absolute prices, are dependent on liquidity preference \( (m) \). Secondly, the maximum rate of profit is also dependent on liquidity preference, and it decreases as \( m \) increases. If \( m \) is small the maximum rate of profit will be close to unity. Third, wages fall if \( m \) increases. If \( G2 \) is the consumption good upon which all the workers' wages are spent, then the real wage is also dependent upon \( m \), and will also decrease if \( m \) increases.

It is clear, therefore, that in a general monetised shifting equilibrium system, both the relative and absolute magnitudes of all the variables that are measured in price terms are
likely to be dependent on liquidity preference for money. The introduction of money does not simply fix the numéraire for the system, leaving the relative magnitudes untouched; relative prices are altered as well, and the maximum rate of profit is reduced.

The above point can be emphasised if the non-money aspect of the previous example is analysed in the normal manner, which excludes money. The following functions are obtained:

\[ p_1 = \frac{w}{1-r} \quad p_2 = \frac{w}{(1-r)^2} \]

Whatever numéraire we choose, the above functions are different to the ones that were obtained in the monetised shifting equilibrium system, as long as \( m \) is positive. However, the above functions are close to the previous ones if \( m \) is sufficiently small. Hence, the ordinary price, wage and profit functions that are obtained in the Sraffa system without money are a limiting case of the general, monetised Sraffa equations. It is easy to demonstrate the generality of the above proposition, by development of the algebra in the previous section.

VII. SUMMARY AND CONCLUSIONS

An attempt has been made to introduce money into the physical input-output system of Sraffa. The method used is to first introduce produced cash money in the input-output framework, and then adjoin account money to this physical cash base. Account money has a distinct interest rate in this system.

However, the monetised Sraffa model should not be conceived as either a simple amendment, or a fundamental critique, of Sraffa's original long-run model. With the introduction of money we no longer have a 'golden age' with perfect knowledge and certainty of the future. With a monetised model changing expectations of the future are likely to influence current economic behaviour, through, in part, changes in liquidity preference. This monetised system can be interpreted as a "shifting equilibrium" system as described by Keynes, where "changing views about the future are capable of influencing the present situation".

We have developed a simple monetised model, both in general algebraic terms, and in terms of a simple arithmetic model. It has been demonstrated that, in general, monetary elements have a relative, absolute, and structural effect on prices, wages, and profits. In other words, unlike Sraffa's long-run model, prices, wages and profits are not just determined by technological conditions and the real wage. However, it has been noted that the standard long-run Sraffa model can be regarded as a limiting case of the "shifting equilibrium" system. Consequently, the original Sraffa system is not invalidated, more it is complemented, by the approach adopted in this paper.
REFERENCES

18. B. Schefold, *Mr. Sraffa and Joint Production* (unpublished mimeo).

AFTERWORD

Since the completion of the above paper it has occurred to me that the paper may have a wider application than that first implied by its approach. The model developed above relies on the assumption that cash money is a produced commodity (such as gold), produced, that is, under capitalist conditions. A version of the model can survive even if that assumption is dropped.

Assume, instead, that cash money takes the form of notes or coin, produced, not under capitalist conditions, but by the state. Cash money is no longer homogeneous and it is not produced in an industry subject to the general rate of profit. A standard Sraffian equation will not apply.
The third and final equation for the three processes will have to be replaced by another equation, and the coefficients on the third row of the square matrix will have to be likewise replaced. Most suitably, the replacement equation would include $w$.

Two suitable replacement equations come to mind. The first is derived from an assumption that the real wage in the economy is constant. This could result from government-imposed indexation of wages so that the increase in the price of the real wage is matched exactly by a proportionate increase in money wages. This matching proportionate increase, resulting in a constant real wage, could, instead, be seen as a result of trade union pressure.

In each case we get the following equation:

$$c_1 p_1 + c_2 p_2 = w$$

where $c_1$ and $c_2$ are the components of the real wage.

The second alternative replacement equation results from the assumption that there is a government-imposed wage freeze, the wage being frozen in money terms. This would give

$$m_w = w.$$ 

Where $m_w$ is the magnitude of the frozen money wage.

These two different equations can be combined, to give a generalised third equation:

$$c_1 p_1 + c_2 p_2 + m_w = w$$

This can be made even more general by stating that $c_1$ and $c_2$ need not even be components of the real wage; they could correspond to any bundle of commodities. But the two alternative assumptions about the wage can be derived from the latter equation by, in the first case regarding $c_1$ and $c_2$ as the real wage and setting $m_w$ as zero, or in the second case setting both $c_1$ and $c_2$ as zero.

Using the general equation, prices and profits can be calculated from the following matrix equation:

$$\begin{bmatrix} p_1 \\ p_2 \\ 1 \end{bmatrix} = w \begin{bmatrix} b_{11} - (1+r)g_{11} & b_{12} - (1+r)g_{12} & -r_m - (r - i)m_{a_1} \\ b_{21} - (1+r)g_{21} & b_{22} - (1+r)g_{22} & -r_m - (r - i)m_{a_2} \\ c_1 & c_2 & m_w \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

As before, relative prices, absolute prices, and profits depend upon monetary phenomena. What is new with this approach is not the method of fixing $w$, this has been done often before, but the inclusion of such monetary phenomena in the Sraffa equations. In the latter case the conditions of production in the cash producing industry have no effect, unlike the model in the main part of this paper.

The model in this Afterword may have some application to studies of the effects of various types of incomes policy, in different capitalist economies.