Section 5.2

Part II

8.) 1. (∃x){Px•(y)[(Sy•(∃z)(Pz•Lyz)) → Lyx]}
2. (x)[Sx → (∃z)(Pz•Lxz)] /∴ (∃x)[Px•(y)(Sy → Lyx)]

3.	Px•(y)[(Sy•(∃z)(Pz•Lyz)) → Lyx]	Assume
4.	Sy	Assume
5.	Sy → (∃z)(Pz•Lxz)	2 UI
6.	(∃z)(Pz•Lxz)	4,5 MP
7.	(y)[(Sy•(∃z)(Pz•Lyz)) → Lyx]	3 Simp
8.	[Sy•(∃z)(Pz•Lyz)] → Lyx	7 UI
9.	Sy•(∃z)(Pz•Lyz)	4,6 Conj
10.	Lyx	8,9 MP
11.	Sy → Lyx	4-10 CP
12.	(y)(Sy → Lyx)	11 UG
13.	Px	3 Simp
14.	Px•(y)(Sy → Lyx)	12,13 Conj
15.	(∃x)[Px•(y)(Sy → Lyx)]	14 EG
16.	(∃x)[Px•(y)(Sy → Lyx)]	1, 3-15 EI

10.) 1. (x){Px → [(∃y)Dxyf → (z)Dxzf]} /∴ (∃x)(Pj•~Djxf) → (y)~Djyf

2.	(∃x)(Pj•~Djxf)	Assume
3.	Pj•~Djxf	Assume
4.	Pj → [(∃y)Djyf → (z)Djzf]	1 UI
5.	Pj	3 Simp
6.	(∃y)Djyf → (z)Djzf	4,5 MP
7.	(∃y)Djyf → Djzf	6 UI
8.	~Djyf	3 Simp
9.	~(∃y)Djyf	7,8 MT
10.	(y)~Djyf	9 QN
11.	(y)~Djyf	2, 3-10 EI
12.	(∃x)(Pj•~Djxf) → (y)~Djyf	2-11 CP

11.) 1. (x){[Bx•(y)(Cy → Ayx)] → (w)[(Pw•Lw) → Rwx]}
2. (x){Px → (y)(Rxy → Txy)}
3. (x){Cx → (y)[(Py•Fyx) → (z){(Bz•Wyz) → Axz}]}
/∴ (x){[Px•(y)(Cy → Fxy)] → (z)[(Bz•Wxz) → (w)(Pw•Lw) → Twz]}

4.	Px•(y)(Cy → Fxy)	Assume
5.	Bz•Wxz	Assume
6.	Pw•Lw	Assume
7.	[Bz•(y)(Cy → Ayz)] → (w)[(Pw•Lw) → Rwz]	1 UI
8.	(y)(Cy → Fxy)	4 Simp
9.	Cu	Assume
10.	Cu → (y){(Py•Fyu) → (z)[(Bz•Wyz) → Auz]}	3 UI
11.	(y){(Py•Fyu) → (z)[(Bz•Wyz) → Auz]}	9,10 MP
12.	(Px•Fxu) → (z)[(Bz•Wxz) → Auz]	11 UI
13.	Px	4 Simp
14.	Cu → Fxu	8 UI
15.	Fxu	9,14 MP
15.	Px•Fxu	13,15 Conj
16.	(z)[(Bz•Wxz) → Auz]	12,15 MP
17.	(Bz•Wxz) → Auz	17 UI
18.	Auz	5,17 MP
19.	Cu → Auz	9-18 CP
20.	(y)(Cy → Ayz)	19 UG
21.	Bz	5 Simp
22.	Bz•(y)(Cy → Ayz)	20,21 Conj
23.	(w)[(Pw•Lw) → Rwz]	7,22 MP
24.	(Pw•Lw) → Rwz	23 UI
25.	Rwz	6,24 MP
26.	Pw → (y)(Rwy → Twy)	2 UI
27.	Pw	6 Simp
28.	(y)(Rwy → Twy)	26,27 MP
29.	Rwz → Twz	28 UI
30.	Twz	25,29 MP
31.	(Pw•Lw) → Twz	6-30 CP
32.	(w)[(Pw•Lw) → Twz]	31 UG
33.	(Bz•Wxz) → (w)[(Pw•Lw) → Twz]	5-32 CP
34.	(z){(Bz•Wxz) → (w)[(Pw•Lw) → Twz]}	33 UG
35.	[Px•(y)(Cy → Fxy)] → (z){(Bz•Wxz) → (w)[(Pw•Lw) → Twz]}	4-34 CP
36.	(x){[Px•(y)(Cy → Fxy)] → (z)[(Bz•Wxz) → (w)(Pw•Lw) → Twz]}	35 UG

12.) 1. (x){[Wx•(∃y)(Sy•Txy)] → (z)(Pz → Uzx)}
2. (∃x)[Rx•Wx•(∃w)(Aw•Cwx)]
3. (x)[(Rx•Wx) → (∃y)(Iy•Sy•Txy)]
/∴ (∃x)[Px•(y)(Axy → ~Uxy)] → (∃z)[(∃w)(Aw•Cwz)•(∃v)(Pv•~Avz)]

4.	(∃x)[Px•(y)(Axy → ~Uxy)]	Assume
5.	Pu•(Auy → ~Uuy)	Assume
6.	Rx•Wx•(∃w)(Aw•Cwx)	Assume
7.	Rx•Wx	6 Simp
8.	(Rx•Wx) → (∃y)(Iy•Sy•Txy)	3 UI
9.	(∃y)(Iy•Sy•Txy)	7,8 MP
10.	Iy•Sy•Txy	Assume
11.	Sy•Txy	10 Simp
12.	(∃y)(Sy•Txy)	11 EG
13.	(∃y)(Sy•Txy)	9, 10-12 EI
14.	Wx	7 Simp
15.	Wx•(∃y)(Sy•Txy)	13,14 Conj
16.	[Wx•(∃y)(Sy•Txy)] → (z)(Pz → Uzx)	1 UI
17.	(z)(Pz → Uzx)	15,16 MP
18.	Pu → Uux	17 UI
19.	Pu	5 Simp
20.	Uux	18,19 MP
21.	(y)(Auy → ~Uuy)	5 Simp
22.	Aux → ~Uux	21 UI
23.	~Aux	20,22 MT
24.	Pu•~Aux	19,23 Conj
25.	(∃v)(Pv•~Avx)	24 EG
26.	(∃w)(Aw•Cwx)	6 Simp
27.	(∃w)(Aw•Cwx)•(∃v)(Pv•~Avx)	25,26 Conj
28.	(∃z)[(∃w)(Aw•Cwz)•(∃v)(Pv•~Avz)]	27 EG
29.	(∃z)[(∃w)(Aw•Cwz)•(∃v)(Pv•~Avz)]	2, 6-28 EI
30.	(∃z)[(∃w)(Aw•Cwz)•(∃v)(Pv•~Avz)]	4, 5-29 EI
31.	(∃x)[Px•(y)(Axy → ~Uxy)] → (∃z)[(∃w)(Aw•Cwz)•(∃v)(Pv•~Avz)]	4-30 CP

Section 5.3

1.)	1.	(x)[Cx → (y)(Ly → Mxy)]	/∴ (x)(Cx → ~Lx)
	2.	(x)~Mxx	Add Premiss
	3.	Cx	Assume
	4.	Cx → (y)(Ly → Mxy)	1 UI
	5.	(y)(Ly → Mxy)	3,4 MP
	6.	Lx → Mxx	5 UI
	7.	~Mxx	2 UI
	8.	~Lx	6,7 MT
	9.	Cx → ~Lx	3-8 CP
	10.	(x)(Cx → ~Lx)	9 UG
2.)	1.	Mab	
	2.	Mbc	/∴ (x)(Lcx → Mxc) → ~Lca
	3.	(x)(y)(z)[(Mxy•Myz) → ~Mxz]	Add Premiss
	4.	(x)(Lcx → Mxc)	Assume
	5.	(y)(z)[(Mxy•Myz) → ~Mxz]	3 UI
	6.	(z)[(Mxy•Myz) → ~Mxz]	5 UI
	7.	(Mab•Mbc) → ~Mac	6 UI
	8.	Lca → Mac	4 UI
	9.	~Mac → ~Lca	8 Trans
	10.	(Mab•Mbc) → ~Lca	7,9 HS
	11.	Mab•Mbc	1,2 Conj
	12.	~Lca	10,11 MP
	13.	(x)(Lcx → Mxc) → ~Lca	4-12 CP

4.)	1.	(x){(Bx•Px) → (y)[(Gy•Py) → Dxy]}	/∴ (y){(Gy•Py) → (x)[(Bx•Px) → Dyx]}
	2.	(x)(y)(Dxy → Dyx)	Add Premiss
	3.	Gy•Py	Assume
	4.	Bx•Px	Assume
	5.	(Bx•Px) → (y)[(Gy•Py) → Dxy]	1 UI
	6.	(y)[(Gy•Py) → Dxy]	4,5 MP
	7.	(Gy•Py) → Dxy	6 UI
	8.	Dxy	3,7 MP
	9.	(x)(Dxy → Dyx)	2 UI
	10.	Dxy → Dyx	9 UI
	11.	Dyx	8, 10 MP
	12.	(Bx•Px) → Dyx	4-11 CP
	13.	(x)[(Bx•Px) → Dyx]	12 UG
	14.	(Gy•Py) → (x)[(Bx•Px) → Dyx]	3-13 CP
	15.	(y){(Gy•Py) → (x)[(Bx•Px) → Dyx]}	14 UG

6.) 1. (x)[(Wx•(y)Byxk) → Mxs]

2.	(x)[(y)(Fy•Mxy) → Tx]	
3.	(x){Tx → (y)[(z)(Byxz → Cxy)]}	/∴ (y){(x)[(Wx•Byxk) → Cxy]}
4.	Fs	Add Premiss
5.	Wx•Byxk	Assume
6.	(Wx•(y)Byxk) → Mxs	1 UI
7.	(Wx•Byxk) → Mxs	6 UI
8.	Mxs	5,7 MP
9.	Fs•Mxs	4,8 Conj
10.	(y)(Fy•Mxy) → Tx	2 UI
11.	(Fs•Mxs) → Tx	10 UI
12.	Tx	9,11 MP
13.	Tx → (y)[(z)(Byxz → Cxy)]	3 UI
14.	(y)[(z)(Byxz → Cxy)]	12,13 MP
15.	(z)(Byxz → Cxy)	14 UI
16.	(Byxk → Cxy)	15 UI
17.	Byxk	5 Simp
18.	Cxy	16,17 MP
19.	(Wx•Byxk) → Cxy	5-18 CP
20.	(x)[(Wx•Byxk) → Cxy]	19 UG
21.	(y){(x)[(Wx•Byxk) → Cxy]}	20 UG

8.)

1.	(x){Ax → (∃y)[Uy•(∃z)(Gz•Bz•Hyz)•Wxy]} /∴ (∃x){Nx•(∃y)[Cy•(∃z)(Mz•Bz•Hyz)•Wxy]}	
2.	(x)(Gx → Mx)	Add Premiss
3.	(x)(Ux → Cx)	Add Premiss
4.	(x)(Ax → Nx)	Add Premiss
5.	(∃x)Ax	Add Premiss
6.	Ax	Assume
7.	Ax → (∃y)[Uy•(∃z)(Gz•Bz•Hyz)•Wxy]	1 UI
8.	(∃y)[Uy•(∃z)(Gz•Bz•Hyz)•Wxy]	6,7 MP
9.	Uy•(∃z)(Gz•Bz•Hyz)•Wxy	Assume
10.	Uy•Gz•Bz•Hyz•Wxy	Assume
11.	Gz	10 Simp
12.	Gz → Mz	2 UI
13.	Mz	11,12 MP
14.	Bz•Hyz	10 Simp
15.	Mz•Bz•Hyz	13,14 Conj
16.	(∃z)(Mz•Bz•Hyz)	15 EG
17.	Uy → Cy	3 UI
18.	Uy	10 Simp
19.	Cy	17,18 MP
20.	Wxy	10 Simp
21.	Cy•(∃z)(Mz•Bz•Hyz)	16,19 Conj
22.	Cy•(∃z)(Mz•Bz•Hyz)•Wxy	20,21 Conj
23.	(∃y)[Cy•(∃z)(Mz•Bz•Hyz)•Wxy]	22 EG
24.	Ax → Nx	4 UI
25.	Nx	6,24 MP
26.	Nx•(∃y)[Cy•(∃z)(Mz•Bz•Hyz)•Wxy]	23,25 Conj
27.	(∃x){Nx•(∃y)[Cy•(∃z)(Mz•Bz•Hyz)•Wxy]}	26 EG
28.	(∃x){Nx•(∃y)[Cy•(∃z)(Mz•Bz•Hyz)•Wxy]}	9, 10-27 EI
29.	(∃x){Nx•(∃y)[Cy•(∃z)(Mz•Bz•Hyz)•Wxy]}	8, 9-28 EI
30.	(∃x){Nx•(∃y)[Cy•(∃z)(Mz•Bz•Hyz)•Wxy]}	5, 6-29 EI

9.)

1.	(x){(Tx•Bx) → (∃y)[Ty•Ay•Pyx]}	
2.	(x){(Tx•Mx) → (∃y)[Ty•Dy•Pxy]}	
	/∴ (∃x){Tx•Bx•(∃y)(Ty•My•Pxy)} → (∃w){Tw•Aw•(∃v)(Tv•Dv•Pwv)}	
3.	(x)(y)(z)[(Pxy•Pyz) → Pxz]	Add Premiss
4.	(∃x){Tx•Bx•(∃y)(Ty•My•Pxy)}	Assume
5.	Tx•Bx•(∃y)(Ty•My•Pxy)	Assume
6.	Tx•Bx	5 Simp
7.	(Tx•Bx) → (∃y)[Ty•Ay•Pyx]	1 UI
8.	(∃y)[Ty•Ay•Pyx]	6,7 MP
9.	Tw•Aw•Pwx	Assume
10.	(∃y)(Ty•My•Pxy)	5 Simp
11.	Tu•Mu•Pxu	Assume
12.	Tu•Mu	11 Simp
13.	(Tu•Mu) → (∃y)[Ty•Dy•Puy]	2 UI
14.	(∃y)[Ty•Dy•Puy]	12,13 MP
15.	Tv•Dv•Puv	Assume
16.	Pxu	11 Simp
17.	Puv	15 Simp
18.	(y)(z)[(Pxy•Pyz) → Pxz]	3 UI
19.	(z)[(Pxu•Puz) → Pxz]	18 UI
20.	(Pxu•Puv) → Pxv	19 UI
21.	Pxu•Puv	16,17 Conj
22.	Pxv	20,21 MP
23.	Pwx	9 Simp
24.	(y)(z)[(Pwy•Pyz) → Pwz]	3 UI
25.	(z)[(Pwx•Pxz) → Pwz]	24 UI
26.	(Pwx•Pxv) → Pwv	25 UI
27.	Pwx•Pxv	22,23 Conj
28.	Pwv	26,27 MP
29.	Tv•Dv	15 Simp
30.	Tv•Dv•Pwv	28,29 Conj
31.	(∃v)(Tv•Dv•Pwv)	30 EG
32.	(∃v)(Tv•Dv•Pwv)	14, 15-31 EI
33.	Tw•Aw	9 Simp
34.	Tw•Aw•(∃v)(Tv•Dv•Pwv)	32,33 Conj
35.	(∃w){Tw•Aw•(∃v)(Tv•Dv•Pwv)}	34 EG
36.	(∃w){Tw•Aw•(∃v)(Tv•Dv•Pwv)}	10, 11-35 EI
37.	(∃w){Tw•Aw•(∃v)(Tv•Dv•Pwv)}	8, 9-36 EI
38.	(∃w){Tw•Aw•(∃v)(Tv•Dv•Pwv)}	4, 5-37 EI
39.	(∃x){Tx•Bx•(∃y)(Ty•My•Pxy)} → (∃w){Tw•Aw•(∃v)(Tv•Dv•Pwv)}	4-38 CP

10.) 1.	(x){Fx → (y)(Sy → Cxy) → (∃z)[Az•(∃w)(Sw•Bzw)•Hzx]}	
2.	(x)[(Gx•Fx) → (y)(Sy → Cxy)]	
	/∴ (x)(Ax → Rx) → (y){Py → (z)[Rz•(∃w)(Mw•Bzw)•~Hzy] → ~Gy}	
3.	(x)(Px → Fx)	Add Premiss
4.	(x)(Sx → Mx)	Add Premiss
5.	(x)(Ax → Rx)	Assume
6.	Py	Assume
7.	(z)[Rz•(∃w)(Mw•Bzw)•~Hzy]	Assume
8.	Rz•(∃w)(Mw•Bzw)•~Hzy	7 UI
9.	~Hzy	8 Simp
10.	~Az v ~(∃w)(Mw•Bzw) v ~Hzy	9 Add
11.	~[Az•(∃w)(Mw•Bzw)•Hzy]	10 DM
12.	(z)~[Az•(∃w)(Mw•Bzw)•Hzy]	11 UG
13.	~(∃z)[Az•(∃w)(Mw•Bzw)•Hzy]	12 QN
14.	(x){Fx → (Sv → Cxv) → (∃z)[Az•(∃w)(Sw•Bzw)•Hzx]}	1 UI
15.	Fy → (Sv → Cyv) → (∃z)[Az•(∃w)(Sw•Bzw)•Hzy]	14 UI
16.	Py → Fy	3 UI
17.	Fy	6,16 MP
18.	(Sv → Cyv) → (∃z)[Az•(∃w)(Sw•Bzw)•Hzy]	15,17 MP
19.	~(Sv → Cyv)	13,18 MT
20.	(x)[(Gx•Fx) → (Sv → Cxv)]	2 UI
21.	(Gy•Fy) → (Sv → Cyv)	20 UI
22.	~(Gy•Fy)	19,21 MT
23.	~Gy v ~Fy	22 DM
24.	~Gy	17,23 DS
25.	(z)[Rz•(∃w)(Mw•Bzw)•~Hzy] → ~Gy	7-24 CP
26.	Py → (z)[Rz•(∃w)(Mw•Bzw)•~Hzy] → ~Gy	6-25 CP
27.	(y){Py → (z)[Rz•(∃w)(Mw•Bzw)•~Hzy] → ~Gy}	26 UG
28.	(x)(Ax → Rx) → (y){Py → (z)[Rz•(∃w)(Mw•Bzw)•~Hzy] → ~Gy}	5-27 CP

Section 5.4

1.)	1.	(∃x){Ax•Dxt•(y)[(Ay•Dyt) → (y=x)]•(z)(Dxz → Oz)}	/∴ Ot
	2.	Ax•Dxt•(y)[(Ay•Dyt) → (y=x)]•(z)(Dxz → Oz)	Assume
	3.	(z)(Dxz → Oz)	2 Simp
	4.	Dxt → Ot	3 UI
	5.	Dxt	2 Simp
	6.	Ot	4,5 MP
	7.	Ot	1, 2-6 EI

3.	1.	(∃x){Sx•(y){[Sy•(y≠x)] → Sxy}•Nx}	
	2.	(x)[(Sx•Nx) → Ix] /∴ (∃x){Sx•(y){[Sy•(y≠x)] → Sxy}•Ix}	
	3.	Sx•(y){[Sy•(y≠x)] → Sxy}•Nx	Assume
	4.	Sx•Nx	3 Simp
	5.	(Sx•Nx) → Ix	2 UI
	6.	Ix	4,5 MP
	7.	Sx•(y){[Sy•(y≠x)] → Sxy}	3 Simp
	8.	Sx•(y){[Sy•(y≠x)] → Sxy}•Ix	6,7 Conj
	9.	(∃x){Sx•(y){[Sy•(y≠x)] → Sxy}•Ix}	8 EG
	10.	(∃x){Sx•(y){[Sy•(y≠x)] → Sxy}•Ix}	1, 3-9 EI

5.)	1.	(x)(Ex → Wx)	
	2.	(x){Wx → (y)[(y≠x) → ~Wy]}	
	3.	(∃x)Ex /∴ (∃x){Ex•(y)[(y≠x) → ~Ey]}	
	4.	Ex	Assume
	5.	Ex → Wx	1 UI
	6.	Wx	4,5 MP
	7.	Wx → (y)[(y≠x) → ~Wy]	2 UI
	8.	(y)[(y≠x) → ~Wy]	6,7 MP
	9.	(v≠x) → ~Wv	8 UI
	10.	Ev → Wv	1 UI
	11.	~Wv → ~Ev	10 Trans
	12.	(v≠x) → ~Ev	9,11 HS
	13.	(y)[(y≠x) → ~Ey]	12 UG
	14.	Ex•(y)[(y≠x) → ~Ey]	4,13 Conj
	15.	(∃x){Ex•(y)[(y≠x) → ~Ey]}	14 EG
	16.	(∃x){Ex•(y)[(y≠x) → ~Ey]}	3, 4-15 EI

7.)

1.	Ma•Mb•Da•Db•(x){(Mx•Dx) → [(x=a) v (x=b)]}	
2.	(x)[(Mx•Lx) → Dx]	
3.	~La	
4.	(∃x)(Mx•Dx) → (∃y)(My•Dy•Ly)	
5.	(x)(Dx → Ix)	/∴ (∃x){Mx•Lx•Ix•(y)[(My•Ly) → (y=x)]}
6.	Ma•Da	1 Simp
7.	(∃x)(Mx•Dx)	6 EG
8.	(∃y)(My•Dy•Ly)	4,6 MP
9.	My•Ly	Assume
10.	(My•Ly) → Dy	2 UI
11.	Dy	9,10 MP
12.	My	9 Simp
13.	My•Dy	11,12 Conj
14.	(x){(Mx•Dx) → [(x=a) v (x=b)]	1 Simp
15.	(My•Dy) → [(y=a) v (y=b)]	14 UI
16.	(y=a) v (y=b)	13,15 MP
17.	Ly	9 Simp
18.	y≠a	3,17 Id
19.	y=b	16,18 DS
20.	(My•Ly) → (y=b)	9-19 CP
21.	My•Dy•Ly	Assume
22.	My•Ly	21 Simp
23.	y=b	20,22 MP
24.	Ly	22 Simp
25.	Lb	23,24 Id
26.	Lb	8, 21-25 EI
27.	(y)[(My•Ly) → (y=b)]	20 UG
28.	Mb	1 Simp
29.	Db	1 Simp
30.	Db → Ib	5 UI
31.	Ib	29,30 MP
32.	Mb•Lb	26,28 Conj
33.	Mb•Lb•Ib	31,32 Conj
34.	Mb•Lb•Ib•(y)[(My•Ly) → (y=b)]	27,33 Conj
35.	(∃x){Mx•Lx•Ix•(y)[(My•Ly) → (y=x)]}	34 EG

8.)

1.	(x)[Cx → (y)(~Cy → Bxy)]	
2.	(∃x){Tx•Cx•(y){[Ty• (y≠x)] → ~Cy}•(w){[Tw•(w≠x)] → Owx}}	
3.	(x)(Tx → Vx)	/∴ (∃x)[Tx•Vx•(y){[Ty• (y≠x)] → Bxy}]
4.	Tx•Cx•(y){[Ty• (y≠x)] → ~Cy}•(w){[Tw•(w≠x)] → Owx}	Assume
5.	Ty• (y≠x)	Assume
6.	(y){[Ty• (y≠x)] → ~Cy}	4 Simp
7.	[Ty• (y≠x)] → ~Cy	6 UI
8.	~Cy	5,7 MP
9.	Cx	4 Simp
10.	Cx → (y)(~Cy → Bxy)	1 UI
11.	(y)(~Cy → Bxy)	9,10 MP
12.	~Cy → Bxy	11 UI
13.	Bxy	8,12 MP
14.	[Ty• (y≠x)] → Bxy	5-13 CP
15.	(y){[Ty• (y≠x)] → Bxy}	14 UG
16.	Tx	4 Simp
17.	Tx → Vx	3 UI
18.	Vx	16,17 MP
19.	Tx•Vx	16,18 Conj
20.	Tx•Vx•(y){[Ty• (y≠x)] → Bxy}	15,19 Conj
21.	(∃x)[Tx•Vx•(y){[Ty• (y≠x)] → Bxy}]	20 EG
22.	(∃x)[Tx•Vx•(y){[Ty• (y≠x)] → Bxy}]	2, 4-21 EI

9.) 1. (∃x){Px•Rx•(y)[(Py•Ry) → (y=x)]}

2. (∃x){Px•Lx•(y)[(Py•Ly) → (y=x)]}

3. (x)(~Rx v ~Lx)

/∴ (∃x)(∃y)[Px•Py•(Lx v Rx)•(Ry v Ly)•(x≠y)•(w){[Pw•(Lw v Rw)] → [(w=x)v(w=y)]}]

4.	Pv•Rv•(y)[(Py•Ry) → (y=v)]	Assume
5.	Px•Lx•(y)[(Py•Ly) → (y=x)]	Assume
6.	Pv	4 Simp
7.	Px	5 Simp
8.	Rv	4 Simp
9.	Lx	5 Simp
10.	Rv v Lv	8 Add
11.	Lx v Rx	9 Add
12.	~Rv v ~Lv	3 UI
13.	Rv → ~Lv	12 MI
14.	~Lv	8,13 MP
15.	x≠v	9,14 Id
16.	Px•Pv	6,7 Conj
17.	Px•Pv•(Lx v Rx)	11,16 Conj
18.	Px•Pv•(Lx v Rx)•(Rv v Lv)	10,17 Conj
19.	Px•Pv•(Lx v Rx)•(Rv v Lv)•(x≠v)	15,18 Conj
20.	Pw•(Lw v Rw)	Assume
21.	(y)[(Py•Ry) → (y=v)]	4 Simp
22.	(y)[(Py•Ly) → (y=x)]	5 Simp
23.	(Pw•Rw) → (w=v)	21 UI
24.	(Pw•Lw) → (w=x)	22 UI
25.	(Pw•Lw) v (Pw•Rw)	20 Dist
26.	[(Pw•Lw) → (w=x)]•[(Pw•Rw) → (w=v)]	23,24 Conj
27.	(w=x) v (w=v)	25,26 CD
28.	[Pw•(Lw v Rw)] → [(w=x)v(w=v)]	20-27 CP
29.	(w){[Pw•(Lw v Rw)] → [(w=x)v(w=v)]}	28 UG
30.	Px•Pv•(Lx v Rx)•(Rv v Lv)•(x≠v)•(w){[Pw•(Lw v Rw)] → [(w=x)v(w=v)]}	19,29 Conj
31.	(∃y)[Px•Py•(Lx v Rx)•(Ry v Ly)•(x≠y)•(w){[Pw•(Lw v Rw)] → [(w=x)v(w=y)]}]	30 EG
32.	(∃x)(∃y)[Px•Py•(Lx v Rx)•(Ry v Ly)•(x≠y)•(w){[Pw•(Lw v Rw)] → [(w=x)v(w=y)]}]	31 EG
33.	(∃x)(∃y)[Px•Py•(Lx v Rx)•(Ry v Ly)•(x≠y)•(w){[Pw•(Lw v Rw)] → [(w=x)v(w=y)]}]	2, 5-32 EI
34.	(∃x)(∃y)[Px•Py•(Lx v Rx)•(Ry v Ly)•(x≠y)•(w){[Pw•(Lw v Rw)] → [(w=x)v(w=y)]}]	1, 4-33 EI

10.) 1. (x)(Ax → Bx)
2. (x)(Bx → Cx)
3. (x)(y)(w){(Cx•Cy•Cw) → [(x=y) v (x=w) v (y=w)]}
4. (∃x)(∃y)[Ax•Ay•(x≠y)] /∴ (∃x)(∃y)[Bx•By•(x≠y)•(w){Bw → [(x=w) v (y=w)]}]

5.	(∃y)[Ax•Ay•(x≠y)]	Assume
6.	Ax•Ay•(x≠y)	Assume
7.	Ax	6 Simp
8.	Ay	6 Simp
9.	x≠y	6 Simp
10.	Ax → Bx	1 UI
11.	Ay → By	1 UI
12.	Bx	7,10 MP
13.	By	8,11 MP
14.	Bx•By	12,13 Conj
15.	Bx•By•(x≠y)	9,14 Conj
16.	Bw	Assume
17.	Bw → Cw	2 UI
18.	Cw	16,17 MP
19.	(x)(y){(Cx•Cy•Cw) → [(x=y) v (x=w) v (y=w)]}	3 UI
20.	(x){(Cx•Cy•Cw) → [(x=y) v (x=w) v (y=w)]}	19 UI
21.	(Cx•Cy•Cw) → [(x=y) v (x=w) v (y=w)]	20 UI
22.	Bx → Cx	2 UI
23.	By → Cy	2 UI
24.	Cx	12,22 MP
25.	Cy	13,23 MP
26.	Cx•Cy	24,25 Conj
27.	Cx•Cy•Cw	18,26 Conj
28.	(x=y) v (x=w) v (y=w)	21,27 MP
29.	(x=w) v (y=w)	9,28 DS
30.	Bw → [(x=w) v (y=w)]	16-29 CP
31.	(w){Bw → [(x=w) v (y=w)]}	30 UG
32.	Bx•By•(x≠y)•(w){Bw → [(x=w) v (y=w)]}	15,31 Conj
33.	(∃y)[Bx•By•(x≠y)•(w){Bw → [(x=w) v (y=w)]}]	32 EG
34.	(∃x)(∃y)[Bx•By•(x≠y)•(w){Bw → [(x=w) v (y=w)]}]	33 EG
35.	(∃x)(∃y)[Bx•By•(x≠y)•(w){Bw → [(x=w) v (y=w)]}]	5, 6-34 EI
36.	(∃x)(∃y)[Bx•By•(x≠y)•(w){Bw → [(x=w) v (y=w)]}]	4, 5-35 EI

Section 5.5

Part I

1.) ~(∃x)(F)Fx
2.) (∃F)(x)~Fx
4.) (x)(y){(x≠y) → (∃F)(Fx•Fy)}
5.) (x){Gx → (F)(Fx → Fn)}
7.) (F){[**G**F → (Fj ≡ Fs)]•[**B**F → (Fj → ~Fs)] or (F){[(**G**F•Fj) → Fs]•(**G**F•Fs) → Fj]•[(**B**F•Fj) → ~Fs]}
8.) (x){(F)(**R**F → Fx) → ~(∃G)(**O**G•Gx)} or (x){(F)(**R**F → Fx) → (G)(**O**G → ~Gx)}
10.) Sentence #1: (x){Px → (∃F)(**U**F•Fx)}
Sentence #2: (y){[Py•(G)(**U**G → ~Gy)] → Uy} or (y){[Py•~(∃G)(**U**G•Gy)] → Uy}

Part II

1.)	1.	(x)(F)Fx	Assume
	2.	(x)Fx	1 UI
	3.	(F)(x)Fx	2 UG
	4.	[(x)(F)Fx] → [(F)(x)Fx]	1-3 CP
	5.	(F)(x)Fx	Assume
	6.	(F)Fx	5 UI
	7.	(x)(F)Fx	6 UG
	8.	[(F)(x)Fx] → [(x)(F)Fx]	5-7 CP
	9.	[(x)(F)Fx] → [(F)(x)Fx]•[(F)(x)Fx] → [(x)(F)Fx]	4,8 Conj
	10.	[(x)(F)Fx] ≡ [(F)(x)Fx]	9 Equiv

3.)	1.	(∃x)(F)Fx	Assume
	2.	(F)Fx	Assume
	3.	Fx	2 UI
	4.	(∃x)Fx	3 EG
	5.	(∃x)Fx	1, 2-4 EI
	6.	(F)(∃x)Fx	5 UG
	7.	[(∃x)(F)Fx] → [(F)(∃x)Fx]	1-6 CP

4.)	1.	(∃F)(x)Fx	Assume
	2.	(x)Fx	Assume
	3.	Fx	2 UI
	4.	(∃F)Fx	3 EG
	5.	(∃F)Fx	1, 2-4 EI
	6.	(x)(∃F)Fx	5 UG
	7.	[(∃F)(x)Fx] → [(x)(∃F)Fx]	1-6 CP

5.)

1.	(∃R)(x)(∃y)Rxy	Assume
2.	(∃R)(x)Rxy	Assume
3.	(x)Rxy	Assume
4.	Rxy	3 UI
5.	(∃R)Rxy	4 EG
6.	(∃y)(∃R)Rxy	5 EG
7.	(∃y)(∃R)Rxy	2, 3-6 EI
8.	(∃y)(∃R)Rxy	1, 2-7 EI
9.	(x)(∃y)(∃R)Rxy	8 UG
10.	[(∃R)(x)(∃y)Rxy] → [(x)(∃y)(∃R)Rxy]	1-9 CP

7.)

1.	(x)(y)(z)[(Rxy•Ryz) → ~Rxz]	Assume
2.	(x)(y)[(Rxy•Ryx) → ~Rxx]	1 UI
3.	(x)[(Rxx•Rxx) → ~Rxx]	2 UI
4.	(Rxx•Rxx) → ~Rxx	3 UI
5.	Rxx → ~Rxx	4 Taut
6.	~Rxx v ~Rxx	5 MI
7.	~Rxx	6 Taut
8.	(x)~Rxx	7 UG
9.	(x)(y)(z)[(Rxy•Ryz) → ~Rxz] → (x)~Rxx	1-8 CP
10.	(R){(x)(y)(z)[(Rxy•Ryz) → ~Rxz] → (x)~Rxx}	9 UG

8.)

1.	(x)(∃y)Rxy•(x)(y)(z)[(Rxy•Ryz) → Rxz]•(x)(y)(Rxy → Ryx)	Assume
2.	(x)(∃y)Rxy	1 Simp
3.	(x)Rxy	Assume
4.	Rxy	3 UI
5.	(x)(y)(z)[(Rxy•Ryz) → Rxz]	1 Simp
6.	(x)(y)[(Rxy•Ryx) → Rxx]	5 UI
7.	(x)[(Rxy•Ryx) → Rxx]	6 UI
8.	(Rxy•Ryx) → Rxx	7 UI
9.	(x)(y)(Rxy → Ryx)	1 Simp
10.	(x)(Rxy → Ryx)	9 UI
11.	Rxy → Ryx	10 UI
12.	Ryx	4,11 MP
13.	Rxy•Ryx	4,12 Conj
14.	Rxx	8,13 MP
15.	Rxx	2, 3-14 EI
16.	(x)Rxx	15 UG
17.	{(x)(∃y)Rxy•(x)(y)(z)[(Rxy•Ryz) → Rxz]•(x)(y)(Rxy → Ryx)} → (x)Rxx	1-16 CP
18.	(R)[{(x)(∃y)Rxy•(x)(y)(z)[(Rxy•Ryz) → Rxz]•(x)(y)(Rxy → Ryx)} → (x)Rxx**]**	17 UG

9.) 1. (x)(y)[(x=y) ≡ (F)(Fx ≡ Fy)] /∴ (x)(y)[(x=y) → (y=x)]

2.	x=w	Assume
3.	(x)[(x=w) ≡ (F)(Fx ≡ Fw)]	1 UI
4.	(x=w) ≡ (F)(Fx ≡ Fw)	3 UI
5.	[(x=w) → (F)(Fx ≡ Fw)]•[(F)(Fx ≡ Fw) → (x=w)]	4 Equiv
6.	(x=w) → (F)(Fx ≡ Fw)	5 Simp
7.	(F)(Fx ≡ Fw)	2,6 MP
8.	Fx ≡ Fw	7 UI
9.	Fw ≡ Fx	8 Equiv
10.	(F)(Fw ≡ Fx)	9 UG
11.	(y)[(w=y) ≡ (F)(Fw ≡ Fy)]	1 UI
12.	(w=x) ≡ (F)(Fw ≡ Fx)	11 UI
13.	[(w=x) → (F)(Fw ≡ Fx)]•[(F)(Fw ≡ Fx) → (w=x)]	12 Equiv
14.	(F)(Fw ≡ Fx) → (w=x)	13 Simp
15.	w=x	10,14 MP
16.	(x=w) → (w=x)	2-15 CP
17.	(y)[(x=y) → (y=x)]	16 UG
18.	(x)(y)[(x=y) → (y=x)]	17 UG

11.) 1. (x)(y)[(x=y) ≡ (F)(Fx ≡ Fy)] /∴ (x)(y)(z){[(x=y)•(y=z)] → (x=z)}

2.	(x=y)•(y=z)	Assume
3.	x=y	2 Simp
4.	(x)[(x=y) ≡ (F)(Fx ≡ Fy)]	1 UI
5.	(x=y) ≡ (F)(Fx ≡ Fy)	4 UI
6.	[(x=y) → (F)(Fx ≡ Fy)]•[(F)(Fx ≡ Fy) → (x=y)]	5 Equiv
7.	(x=y) → (F)(Fx ≡ Fy)	6 Simp
8.	(F)(Fx ≡ Fy)	3,7 MP
9.	y=z	2 Simp
10.	(x)[(x=z) ≡ (F)(Fx ≡ Fz)]	1 UI
11.	(y=z) ≡ (F)(Fy ≡ Fz)	10 UI
12.	[(y=z) → (F)(Fy ≡ Fz)]•[(F)(Fy ≡ Fz) → (y=z)]	11 Equiv
13.	(y=z) → (F)(Fy ≡ Fz)	12 Simp
14.	(F)(Fy ≡ Fz)	9,13 MP
15.	Fx ≡ Fy	8 UI
16.	Fy ≡ Fz	14 UI
17.	Fx ≡ Fz	15,16 Equiv
18.	(F)(Fx ≡ Fz)	17 UG
19.	(x)[(x=z) ≡ (F)(Fx ≡ Fz)]	1 UI
20.	(x=z) ≡ (F)(Fx ≡ Fz)	19 UI
21.	[(x=z) → (F)(Fx ≡ Fz)]•[(F)(Fx ≡ Fz) → (x=z)]	20 Equiv
22.	(F)(Fx ≡ Fz) → (x=z)	21 Simp
23.	x=z	17,22 MP
24.	[(x=y)•(y=z)] → (x=z)	2-23 CP
25.	(z){[(x=y)•(y=z)] → (x=z)}	24 UG
26.	(y)(z){[(x=y)•(y=z)] → (x=z)}	25 UG
27.	(x)(y)(z){[(x=y)•(y=z)] → (x=z)}	26 UG

12.) * It is unclear exactly what Copi means by the statement in the text. There are two possibilities demonstrated below. The tempting translation is to attempt to prove that: If circles are ellipses, then they have all of the properties of all ellipses. Or in symbolic form: (x)(y)(F){(Cx•Ey•Fy) → Fx]} But clearly this is a false statement, so no proof could be given.

12.)

1.	(x)(Cx → Ex)	Assume
2.	(x)(y)[(x=y) ≡ (F)(Fx ≡ Fy)]	Add Premiss (Def of Id)
3.	Cx	Assume
3.	Cx → Ex	1 UI
4.	Ex	2,3 MP
5.	x=x	4 Id
6.	Ex•(x=x)	4,5 Conj
7.	(∃y)[Ey•(x=y)]	6 EG
8.	Ey•(x=y)	Assume
9.	x=y	8 Simp
10.	(x)[(x=y) ≡ (F)(Fx ≡ Fy)]	2 UI
11.	(x=y) ≡ (F)(Fx ≡ Fy)	10 UI
12.	[(x=y) → (F)(Fx ≡ Fy)]•[(F)(Fx ≡ Fy) → (x=y)]	11 Equiv
13.	(x=y) → (F)(Fx ≡ Fy)	12 Simp
14.	(F)(Fx ≡ Fy)	9,13 MP
15.	(Fx → Fy)•(Fy → Fx)	14 Equiv
16.	Fy → Fx	15 Simp
17.	(F)(Fy → Fx)	16 UG
18.	Ey	8 Simp
19.	Ey•(F)(Fy → Fx)	17,18 Conj
20.	(∃y)[Ey•(F)(Fy → Fx)]	19 EG
21.	(∃y)[Ey•(F)(Fy → Fx)]	7, 8-20 EI
22.	Cx → (∃y)[Ey•(F)(Fy → Fx)]	3-21 CP
23.	(x){Cx → (∃y)[Ey•(F)(Fy → Fx)]}	22 UG
24.	(x)(Cx → Ex) → (x){Cx → (∃y)[Ey•(F)(Fy → Fx)]}	1- 23 CP

*This proves the statement: If circles are ellipses, then every circle possesses all the properties of some ellipse.

12.)

1.	(x)(Cx → Ex)	Assume
2.	Cx	Assume
3.	Ey•Fy•(w)(~Fw → ~Ew)	Assume
4.	Cx → Ex	1 UI
5.	Ex	2,4 MP
6.	(w)(~Fw → ~Ew)	3 Simp
7.	~Fx → ~Ex	6 UI
8.	Fx	5,7 MT
9.	{Ey•Fy•(w)(~Fw → ~Ew)} → Fx	3-8 CP
10.	(y)[{Ey•Fy•(w)(~Fw → ~Ew)} → Fx]	9 UG
11.	(F)(y)[{Ey•Fy•(w)(~Fw → ~Ew)} → Fx]	10 UG
12.	Cx → (F)(y)[{Ey•Fy•(w)(~Fw → ~Ew)} → Fx]	2 – 11 CP
13.	(x){Cx → (F)(y)[{Ey•Fy•(w)(~Fw → ~Ew)} → Fx]}	12 UG
14.	(x)(Cx → Ex) → (x){Cx → (F)(y)[{Ey•Fy•(w)(~Fw → ~Ew)} → Fx]}	1-13 CP

*This proves the statement: If circles are ellipses, then every circle possesses all the properties which are requisite of an ellipse.

13.)

1.	(x)(y)(Rxy → Ryx)•(x)(y)(z)[(Rxy•Ryz) → Rxz]	Assume
2.	(x)(y)(Rxy → Ryx)	1 Simp
3.	(x)(y)(z)[(Rxy•Ryz) → Rxz]	1 Simp
4.	(∃y)(Rxy v Ryx)	Assume
5.	Rxu v Rux	Assume
6.	~Rux → Rxu	5 MI
7.	(y)(Rxy → Ryx)	2 UI
8.	Rxu → Rux	7 UI
9.	~Rux → Rux	4,6 HS
10.	Rux v Rux	9 MI
11.	Rux	10 Taut
12.	~Rxu → Rux	5 MI
13.	(y)(Ruy → Ryu)	2 UI
14.	Rux → Rxu	13 UI
15.	~Rxu → Rxu	12,14 HS
16.	Rxu v Rxu	15 MI
17.	Rxu	16 Taut
18.	Rxu•Rux	11,17 Conj
19.	(x)(y)[(Rxy•Ryx) → Rxx]	3 UI
20.	(x)[(Rxu•Rux) → Rxx]	19 UI
21.	(Rxu•Rux) → Rxx	20 UI
22.	Rxx	18,21 MP
23.	Rxx	4, 5-22 EI
24.	(∃y)(Rxy v Ryx) → Rxx	4-23 CP
25.	(x)[(∃y)(Rxy v Ryx) → Rxx]	24 UG
26.	{(x)(y)(Rxy → Ryx)•(x)(y)(z)[(Rxy•Ryz) → Rxz]} → (x)[(∃y)(Rxy v Ryx) → Rxx]	1-25 CP
27.	(R){{(x)(y)(Rxy → Ryx)•(x)(y)(z)[(Rxy•Ryz) → Rxz]} → (x)[(∃y)(Rxy v Ryx) → Rxx]}	26 UG