1. Suppose that two particles having masses $m_1$ and $m_2$ are subject only to forces between these particles. Suppose further that these forces are conservative and are described by some potential energy $V(r)$, where $r = |r|$, $r = r_2 - r_1$, and the vectors $r_1$ and $r_2$ locate the particles with respect to some origin.
   a) Write down and discuss any theorems related to total momentum.
   b) List all conserved quantities for this system, separating these quantities into three groups relating to the whole system, the center of mass, and relative motion.

2. A space platform in the form of a thin circular disc of radius $a$ and mass $m$ is rotating with angular velocity $\omega$ about its symmetry axis. A meteorite strikes the platform at the edge, imparting an impulse $P$ to the platform, parallel to the axis of symmetry. Find the resulting motion of the platform.

3. A particle of mass $m$ rests on a smooth plane. The plane is raised to an inclination angle $\theta$ at a constant rate $k$ ($\theta = 0$ at $t = 0$), causing the particle to move down the plane. Determine the motion of the particle in terms of the distance it travels along the plane from its initial position.

4. A straight wire rotates with constant angular speed $\omega$ about the vertical direction at a fixed polar angle $\theta_o$. A mass $m$ is free to slide along the wire. Using $d$ as the distance along the wire from the point of support, as in the drawing below, construct the Lagrangian for this system. Show that the condition of an equilibrium circular orbit is $d_o = g (\cos \theta_o)/(\omega \sin \theta_o)^2$. Note, if you obtained $d_o = -g (\cos \theta_o)/(\omega \sin \theta_o)^2$ instead of the desired result, explain why.

![Diagram of particle sliding down an inclined plane with a wire attached at an angle $\theta_o$ and distance $d$]
5. Charge $Q$ is uniformly distributed throughout the volume of a solid sphere of radius $R$.
   a) Use Gauss’ law to show that the magnitude of the electric field at distance $r$ from the center of the sphere is
   \[
   E(r) = \begin{cases} 
   \frac{kQ}{R^3} \cdot r & \text{for } r \leq R \\
   \frac{kQ}{r^2} & \text{for } r \geq R
   \end{cases}
   \]

   b) Use the expressions for the magnitude of the electric field to determine electric potential at a distance $r$ from the center of the sphere using the convention that the potential goes to zero at infinity.

6. Consider a charge $Q$ located a distance $D>R$ away from a grounded conducting sphere of radius $R$. Using the method of images, calculate the magnitude and position of the associated image charge. Determine the induced surface charge density of the sphere.

7. An infinite cylindrical conductor of radius $a$ has a hole of radius $b$ bored parallel to, and centered a distance $d$ from the cylinder axis ($d+b < a$). The current density $J$ is uniform throughout the remaining metal of the cylinder and is parallel to the axis. Use Ampere’s Law and the principle of linear superposition to find the magnitude and direction of the magnetic-flux density in the hole.
8. Occasionally, physicists need to calculate quantities using computational programs like Mathcad. Show how you would set up the necessary integrals and any additional calculations to calculate the x-, y-, and z-components of the magnetic field at an arbitrary point in space due to a loosely wound solenoid with $N$ turns, of length $L$, carrying current $I$, whose center is at the origin, and whose axis is along the $z$-axis.
1. “To every physical observable there corresponds an operator in the Hilbert space”.
   a) Why must the operator be Hermitian?
   b) Show that eigenvalues of a Hermitian operator are real.
   c) Show that eigenvectors belonging to different eigenvalues of a Hermitian operator are orthogonal.

2. A spinless particle of mass \( m \) is constrained to remain in the area \( A \) in the \( x\)-\( y \) plane as shown below.

   a) Let \( V(x, y) \), the potential energy, be zero for \( m \) in the area \( A \) and infinite everywhere else. Find the lowest energy level of this system in terms of \( m \), \( a \), \( b \) and \( \hbar \) by solving the Schrödinger equation with appropriate boundary conditions.
   b) Show that if \( a=b=1\text{Å} \) and \( m \) is the electron mass, then the lowest energy level is about 10eV, the order of magnitude of atomic energies.

3. In evaluating the harmonic oscillator, we use ladder operators as follows:

   \[
   \hat{a} = \sqrt{\frac{\alpha}{2}} \left( x + \frac{i}{m\omega} p \right) ; \quad \hat{a}^\dagger = \sqrt{\frac{\alpha}{2}} \left( x - \frac{i}{m\omega} p \right) \\
   \hat{a} |n\rangle = \sqrt{n} |n-1\rangle ; \quad \hat{a}^\dagger |n\rangle = \sqrt{(n+1)} |n+1\rangle
   \]

   Where \( x \) and \( p \) are operators, \( |n\rangle \) represents the \( n \)th eigenstate, and \( \alpha = \sqrt{\frac{m\omega}{\hbar}} \).

   a. Express the \( x \) and \( p \) operators in terms of ladder operators.
b. Find the quantum mechanical uncertainty product $\Delta x \Delta p$ for an eigenstate $|n\rangle$ of the harmonic oscillator. That is, for a particle of mass $m$ subjected to a one-dimensional potential energy $V(x) = (1/2) m \omega^2 x^2$.

4. Describe a modern physics experimental approach to verify, understand or explain the predicted phenomena listed below and how the realization of such information helped to shape quantum mechanics. An example answer to (a) is given as a guide; complete only (b)-(f):
   a) Electrons exist in discrete energy levels when bound to an atom
   b) Electrons have spin angular momentum
   c) Angular momentum of electrons is quantized
   d) Electrons have wave-like properties
   e) Light scatters from matter discretely (in a particle-like manner)
   f) Atoms have internal structure (a nucleus)

Answer to (a): elastic and inelastic electron scattering from gas atoms; only when the proper electron energy is incident on the gas atom does an excitation or inelastic scattering occur. This study verified the Bohr model of quantized energy states.

5. An ideal gas is enclosed in a cylinder. There is a movable piston on top of the cylinder. The piston has a mass of 8.0 kg and an area of 5.0 cm$^2$ and is free to slide up and down, keeping the pressure of the gas constant. How much work is done by the gas as the temperature of 0.20 moles of the gas is raised from 20°C to 300°C?

6. Find the entropy change for
   a. 1.0g of ice melting at 0°C
   b. 1.0g of water evaporating at 100°C
   You will need to use the heats of fusion and vaporization:
   $L_f = 6.0kJ/mole$
   $L_v = 41kJ/mole$

7. The volume compression ratio of a diesel engine is 15:1. The working fluid is air (for which the specific heat capacity ratio $\gamma=1.4$) which enters the cylinder at 7^0C. Assuming the compression is adiabatic, find the temperature to which the air is heated.
8. For an ideal monatomic gas, the number of particles with energy $E$ is determined by the Maxwell-Boltzmann distribution function

$$N(E) = \frac{n N_A}{kT} e^{\frac{E}{kT}}$$

where $n$ is the number of moles of the gas and $N_A$ is Avogadro’s number.

Show that the total energy $U$ of an ideal gas is proportional to its temperature $T$

$$U = n C_v T$$

where $C_v$ is the molar heat capacity at constant volume.