1. Two blocks are stacked as shown and the bottom block is accelerated by a force $F$. Let $\mu$ be the coefficient of both static and kinetic friction for both contact surfaces as shown in the figure.
   a) Find the maximum force that can be exerted on $M$ so that $m$ does not slip. Find the resulting acceleration.
   b) What is the force on $M$ by $m$ ($x$- and $y$-components) when $F$ has its maximum value (the value found in part a)?

2. A hole is drilled through the center of the earth and a rock is dropped into the hole at the earth’s surface. Assuming that the earth is a sphere of constant density, show that the motion is periodic and find the period. Compare the period with that of an earth satellite in an orbit close to the surface of the earth.

3. A ladder rests against a smooth wall and slides without friction on wall and floor.
   a) Choose appropriate generalized coordinates, write the Lagrangian, and obtain the equations of motion while the ladder is in contact with the wall.
   b) If the ladder is initially at rest at an angle $\theta_0$ with respect to the floor, at what angle does the ladder leave the wall?
   c) Write the Lagrangian and equations of motion for the ladder after it has left the wall. (Do not attempt to solve the equations for this case.)
4. A one-dimensional system has Lagrangian:

\[ L = \frac{m\dot{x}^2}{2} + \lambda \dot{x} \]

where \( \lambda > 0 \).

a) Find the canonical momentum \( p_x \).

b) Find the Hamiltonian for the system.

c) Find the equation of motion using Lagrange’s equation.

d) If \( x(0) = 0 \) and \( \dot{x}(0) = v_0 \), find \( x(t) \).

5. Consider a situation in which initially the magnetic field strength is zero. Now, a magnetic field of strength \( B \) is activated in the \( z \) direction at a linear rate, such that the field for \( t \geq 0 \) has the form

\[ \vec{B} = At\hat{z}, \]

where \( \hat{z} \) is the unit vector oriented in the positive \( z \) direction as shown in the figure below. Initially, there is no current flowing in the loop. The loop is in the \( x-y \) plane, and hence is oriented perpendicular to the \( z \) axis. What is the current which flows in the square loop for \( t > 0 \)? What is the direction (i.e. clockwise or counterclockwise) of the current induced in the loop by the steadily increasing magnetic field? Assume that the sides of the square have length \( a \) and assume each side has an identical resistance \( r \).
6. We examine a very long set of two hollow coaxial cylindrical conductors, which may be taken to be of infinite length for the purpose of this question. The metallic cylinders have azimuthal symmetry, and are circular in cross section. The interior conductor has a radius \( r = a \), while the outer cylinder has a radius \( r = b \). On the inner cylinder, there is a positive charge \(+\lambda\) per unit length, while on the outer cylinder, the charge per unit length is equal in magnitude but opposite in sign, or \(-\lambda\). Consider the case in which the space between the cylinders is evacuated, so the dielectric constant is uniform throughout and equal to unity: \( K = 1 \). Calculate the potential difference between the two cylindrical conductors, and use the result to determine the capacitance per unit length of the coaxial arrangement.

7. 
   a) Write Maxwell’s equations in SI units for \( E \) and \( B \) in a vacuum. Assume that there are no dielectric or magnetic susceptibility properties but that charges and currents are present.
   b) Show that charge conservation is implied by one of the Maxwell equations.
   c) Which of the Maxwell’s equations relates to the nonexistence of magnetic monopoles? Explain.
   d) Which equation expresses Faraday’s law of induction?
   e) Using \( \nabla \times (\nabla \times \mathbf{A}) = \nabla (\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A} \), obtain a wave equation for \( E \). Discuss the “source” term in this wave equation in terms of particle and field energy exchanges.
8. Below are two resistor, inductor, capacitor (RLC) circuits, with voltage source (V), labeled ‘A’ and ‘B’. Please answer the following:

(a) From circuit A shown below, please write an analytic expression to represent the potential difference between points i, ii and iii.

(b) Define impedance phenomenologically.

(c) From the information given in circuit A, write an analytic expression to represent the current at point iv.

(d) With reference to circuit B, please write an analytic expression showing the difference in current between points v, vi and vii.

(e) For circuit B, when is impedance minimized?
Department of Physics Written Exam

Part II. Quantum Mechanics and Thermodynamics

Saturday Afternoon, 1pm – 5pm
January 15, 2011

1. Assume that the Planck and DeBroglie relations, $E=nh$ and $p=h/\lambda$, hold for free quantum mechanical particles of mass $m$.
   a) Suppose we describe the state of a particle by a plane wave:
      $$\Psi(x,t) = A \exp[ikx - i\omega t]$$
      Show that the frequency and wavelength (or $k$ and $\omega$) are related due to the (non-relativistic) Schroedinger equation for free particles.
   b) Discuss the dispersive properties of the Schroedinger equation (free case) and find the formula for the phase velocity.

2. A spinless particle of mass $m>0$ is incident with momentum $\hbar k$ on a potential energy step of magnitude $+V_0$. Let the incident wave function be
   $$\Psi_{inc}(x,t) = A \exp[ikx - i\omega t]$$
   With energy $E = \hbar \omega > +V_0$. Assume that the reflected and transmitted waves are also plane waves.
   a) Find the transmitted and reflected wave amplitudes in terms of $k$ and $A$.
   b) Find the three quantum mechanical currents $J_{inc}$, $J_{trans}$, and $J_{ref}$.
   c) Define the reflection coefficient $R=|J_{ref}/J_{inc}|$ and the transmission coefficient $T=|J_{trans}/J_{inc}|$. Show that $R<1$, $T<1$, and $R+T=1$ for all $k$.

3. A spin 1 particle has a magnetic moment $\mu = \frac{g e}{2m} \vec{s}$ where $g=1$. The particle is in a uniform $\vec{B}$ field given by $\vec{B} = B_0 \hat{k}$ along the $z$-axis and the Hamiltonian is $\hat{H} = -\vec{\mu} \cdot \vec{B}$.
   a) Write down the explicit matrix representing $\hat{H}$ using
      $$\hat{S}_z = \hbar \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$
   b) Find the three normalized energy eigenstates of $\hat{H}$. (Do not include spatial degrees of freedom.)
4. Show that for the eigenstate $|l, m_l\rangle$ of $\hat{L}^2$ and $\hat{L}_z$, the expectation value of the square of the component of angular momentum perpendicular to the $z$-axis, $(\hat{L}_x^2 + \hat{L}_y^2)$, is $[l(l+1)-m_l^2]h^2$.

5. You add 20g of 5°C cream to 200g of 60°C coffee. The specific heat of both the cream and the coffee is 4.2J/gK and is constant for the considered temperature range.
   a) Show that the final temperature of the mixture is 55°C. (Ignore the cup and the atmosphere.)
   b) How much heat was transferred between the two liquids.

6. Consider the cyclic process presented in the figure (not necessarily for an ideal gas!).
   a) If the heat delivered to the system is negative for the process $BC$, and the change in the internal energy is negative for the process $CA$, determine the signs of the heat delivered to the system, of the work done by the system, and of the change in the internal energy associated with the processes $BC$, $CA$, and $AB$. Explain.
   b) Find the work performed by the system in the cycle.
7. Consider a thermally isolated system consisting of two volumes $V_1$ and $V_2$ of an ideal gas separated by a thermally conducting and moveable partition. The temperature and pressure are shown. The partition is now allowed to move without the gases mixing. When equilibrium is established, what is the change in the total internal energy? What is the change in total entropy? What is the equilibrium temperature? What is the equilibrium pressure?

8. A certain system is found to have a Gibbs free energy given by:

$$ G(p, T) = RT \ln \left( \frac{a p}{(RT)^{5/2}} \right) $$

where $a$ and $R$ are constants. Find the specific heat at constant pressure, $c_p$. 