Homework 2

problems: 3.2, 3.23, 3.53, 3.63
Problem 3.2

Two points in the xy plane have the following polar coordinates \((2.50 \text{ m}, 30.0^\circ)\) and \((3.80 \text{ m}, 120.0^\circ)\). Determine (a) the Cartesian coordinates of these points and (b) the distance between them.

(a)
From geometrical consideration (or from the relationship between the polar and the Cartesian coordinates)
\[
\mathbf{A} = [A_x, A_y] = [2.5 \cos 30^\circ, 2.5 \sin 30^\circ] = [2.17, 1.25] \text{ m}
\]
\[
\mathbf{B} = [B_x, B_y] = [3.8 \cos 120^\circ, 3.8 \sin 120^\circ] = [-1.90, 3.29] \text{ m}
\]

(b)

solution 1
The simplest way is to find the magnitude of vector \(\mathbf{B-A}\) from the scalar components of this vector
\[
\mathbf{B} - \mathbf{A} = [-1.9, 3.29] \text{ m} - [2.17, 1.25] \text{ m} = [4.07, 2.04] \text{ m}
\]
\[
|\mathbf{B} - \mathbf{A}| = \sqrt{(4.07 \text{ m})^2 + (2.04 \text{ m})^2} = 4.55 \text{ m}
\]

solution 2
One can also use the law of cosines
\[
|\mathbf{B} - \mathbf{A}| = \sqrt{(2.5 \text{ m})^2 + (3.8 \text{ m})^2 - 2 \cdot 2.5 \text{ m} \cdot 3.8 \text{ m} \cdot \cos(120^\circ - 30^\circ)} = 4.55 \text{ m}
\]
Problem 3.23

Consider two vectors \( \mathbf{A} = 3 \mathbf{i} - 2 \mathbf{j} \) and \( \mathbf{B} = -\mathbf{i} - 4 \mathbf{j} \). Calculate (a) \( \mathbf{A} + \mathbf{B} \), (b) \( \mathbf{A} - \mathbf{B} \), (c) \( |\mathbf{A} + \mathbf{B}| \), (d) \( |\mathbf{A} - \mathbf{B}| \), (e) the direction of \( \mathbf{A} + \mathbf{B} \) and \( \mathbf{A} - \mathbf{B} \).

(a), (b)
To find the sum and the difference of the two vectors we should use the properties of the vectors (associative and commutative properties of addition and the distributive property of the product over addition of vectors).
\[
\mathbf{A} + \mathbf{B} = (3 \mathbf{i} - 2 \mathbf{j}) + (-\mathbf{i} - 4 \mathbf{j}) = 2 \mathbf{i} - 6 \mathbf{j}
\]
\[
\mathbf{A} - \mathbf{B} = (3 \mathbf{i} - 2 \mathbf{j}) - (-\mathbf{i} - 4 \mathbf{j}) = 4 \mathbf{i} + 2 \mathbf{j}
\]

(c), (d)
The magnitude of a vector can be easily found from its scalar components
\[
|\mathbf{A} + \mathbf{B}| = \sqrt{2^2 + (-6)^2} = 6.32
\]
\[
|\mathbf{A} - \mathbf{B}| = \sqrt{4^2 + 2^2} = 4.47
\]

(e)
Knowing the scalar components also allows one to find the direction of the vectors
\[
\theta_{\mathbf{A} + \mathbf{B}} = \arctan \frac{-6}{3} = -71.6^\circ
\]
\[
\theta_{\mathbf{A} - \mathbf{B}} = \arctan \frac{2}{4} = 26.6^\circ
\]
Problem 3.53

A vector is given by \( \mathbf{R} = 2 \mathbf{i} + \mathbf{j} + 3 \mathbf{k} \). Find (a) the magnitudes of the x, y, and z components, (b) the magnitude of \( \mathbf{R} \), and (c) the angles between \( \mathbf{R} \) and the x, y, and z axes.

(a) First one has to identify the vector components of vector \( \mathbf{R} \):
\[
\begin{align*}
\mathbf{R}_x &= 2.00 \mathbf{i} & \mathbf{R}_y &= 1.00 \mathbf{j} & \mathbf{R}_z &= 3.00 \mathbf{k}
\end{align*}
\]
and to use the scalar components of these three vectors to find their magnitudes
\[
\begin{align*}
R_x &= \sqrt{2^2} = 2 & R_y &= \sqrt{1^2} = 1 & R_z &= \sqrt{3^2} = 3
\end{align*}
\]

(b) Using the scalar components
\[
R = \sqrt{2^2 + 1^2 + 3^2} = 3.74
\]

(c) From the definition of the cosine function
\[
\begin{align*}
\theta_x &= \arccos \frac{R_x}{R} = \arccos \frac{2}{3.74} = 57.7^\circ \\
\theta_y &= \arccos \frac{R_y}{R} = \arccos \frac{1}{3.74} = 74.5^\circ \\
\theta_z &= \arccos \frac{R_z}{R} = \arccos \frac{3}{3.74} = 36.7^\circ
\end{align*}
\]
Problem 3.63

A rectangular parallelepiped has the dimensions $a$, $b$, and $c$ as in Figure 3.63. (a) Obtain a vector expression for the face diagonal vector $\mathbf{R}_1$. What is the magnitude of this vector? (b) Obtain a vector expression for the body diagonal vector $\mathbf{R}_2$. Note that make $\mathbf{R}_1$, $ck$ and $\mathbf{R}_2$ make a right triangle, and prove that the magnitude of $\mathbf{R}_2$ is $\sqrt{a^2 + b^2 + c^2}$.

(a) Imagine vectors $\mathbf{R}_x$, $\mathbf{R}_y$, and $\mathbf{R}_z$ along the edges of the parallelepiped. In the Cartesian coordinate system, marked in the figure,

$$\mathbf{R}_x = a \mathbf{i}, \quad \mathbf{R}_y = b \mathbf{j}, \quad \mathbf{R}_z = c \mathbf{k}. $$

Therefore

$$\mathbf{R}_1 = \mathbf{R}_x + \mathbf{R}_y = a \mathbf{i} + b \mathbf{j} \quad \text{and} \quad \mathbf{R}_1 = \sqrt{\mathbf{R}_x^2 + \mathbf{R}_y^2} = \sqrt{a^2 + b^2}$$

Similarly,

$$\mathbf{R}_2 = \mathbf{R}_x + \mathbf{R}_y + \mathbf{R}_z = a \mathbf{i} + b \mathbf{j} + c \mathbf{k}$$

and

$$\mathbf{R}_2 = \sqrt{\mathbf{R}_x^2 + \mathbf{R}_y^2 + \mathbf{R}_z^2} = \sqrt{a^2 + b^2 + c^2}$$