Homework 4

problems: 5.61, 5.67, 6.63, 13.21
Problem 5.61

An object of mass $M$ is held in place by an applied force $F$ and a pulley system as shown in the figure. The pulleys are massless and frictionless. Find (a) the tension in each section of rope, $T_1$, $T_2$, $T_3$, $T_4$, and $T$, and (b) the magnitude of $F$. Suggestion: Draw a free-body diagram for each pulley.

Only the vertical components of the vectors result in a non-trivial equation. Symmetry of the arrangement that $T_1 = T_2$ and $T_1 = T_3$. Also the definition of tension (magnitude of the force exerted on each side of the rope) $T_1 = F$. Since the acceleration of the block and each pulley is zero, from Newton’s second law (for the vertical component) we obtain additional three equations relating the tensions. In the following equations, the terms are aligned in such a way that it is easy to use a calculator to find the solution.

\[

t_1 - t_2 = 0 \\
t_1 - t_3 = 0 \\
t_1 - F = 0 \\
t_5 = Mg \\
t_2 + t_3 - t_5 = 0 \\
t_2 - t_3 + t_4 = 0
\]

There are six unknowns in this set of six linear equations. The rest is simple algebra. Solving the equations simultaneously for the unknowns leads to

\[
F = T_1 = T_2 = T_3 = Mg/2; \quad T_4 = 3Mg/2; \quad T_5 = Mg
\]
Problem 5.67

What horizontal force must be applied to the cart shown in Figure P5.61 in order that the blocks remain stationary relative to the cart? Assume that all surfaces, wheels, and the pulley are frictionless.

Three forces are exerted block 1: gravitational force exerted by earth \( W_1 \), tensile force exerted by the cord \( T_{1C} \), and the normal force \( N_{1M} \) exerted by block M. See the free body diagram included in the figure.

Three forces are also exerted on the second block: gravitational force exerted by earth \( W_2 \), tensile force exerted by the cord \( T_{2C} \), and the normal force \( N_{2M} \) exerted by block M. Six objects interact with block M: the earth exerts a gravitational force \( W \), the table and the other two blocks exert normal forces \( (N, N_{M1}, N_{M2}) \), the source on force \( F \), and the cord at the pulley with normal force \( f = -T_{C1} - T_{C2} \).

It is required in the problem that all three blocks move with the same acceleration \( a \) (unknown). We obtain three vector equations (one for each block):

1) \[ N_{1M} + W_1 + T_{1C} = m_1 \vec{a} \]
2) \[ N_{2M} + W_2 + T_{2C} = m_2 \vec{a} \]
3) \[ -N_{1M} - N_{2M} + \vec{N} + \vec{W} + \vec{F} + \vec{f} = M\vec{a} \]

The rest is math. We can write these three equations in scalar components using the information about the directions.

4) \[ [T, N_{1M} - m_1g] = m_1[a,0] \]
Solving by substitution, we can find the horizontal component $F$ of the force.

\[
F = Ma + N_{2M} + T = Ma + m_2a + m_1a = (M + m_2 + m_1)\frac{T}{m_1} = (M + m_1 + m_2)\frac{m_2g}{m_1}
\]

Note that we can find more information about the system. The six equations have six unknowns which we can find: the tension in the cord, all three normal forces, force $F$, and the acceleration of the system.

Comment. In fact, three of the six equations lead to the answer. With better skills, you will be able to solve it without writing all the general relations. You could just relate the magnitude $F$ of the applied force with the acceleration of the system. Then relate this acceleration with the required tension in the cord, necessary to accelerate block 1, and finally relate the tension in the cord with the weight of block 2.

\[
7) \quad F = (M + m_1 + m_2)a
\]
\[
8) \quad T = m_1a
\]
\[
9) \quad T = m_2g
\]
Problem 6.63

A model airplane of a mass 0.75 kg flies in a horizontal circle at the end of a 60-m control wire, with a speed of 35 m/s. Compute the tension in the wire if it makes a constant angle of 20° with the horizontal. The forces exerted on the airplane are the pull in the control wire, its own weight, and the aerodynamic lift, which acts at 20° inward from the vertical as shown in the Figure P6.71.

The plane moves in a uniform circular motion. Consider the forces and the acceleration for the plane at the instant it passes the origin of the inertial reference frame indicated in the figure. There are three forces exerted on the airplane:

1) the gravitational force exerted by the earth \[ \mathbf{W} = [0, -mg], \]
2) the aerodynamic lift exerted by the air \[ \mathbf{F} = F[\sin \alpha, \cos \alpha], \]
3) and the tension force exerted by the control wire \[ \mathbf{T} = T[\cos \alpha, -\sin \alpha]. \]

We can also figure out the components of the acceleration

4) \[ \mathbf{a} = \frac{d}{dt} (\omega \times \mathbf{r}) = \omega \times (\omega \times \mathbf{r}) = -\omega^2 \mathbf{r} = -\left( \frac{v}{r} \right)^2 \mathbf{r} = \left[ \frac{v^2}{l \cos \alpha}, 0 \right] \]

(Note that we know quite a lot about the acceleration. Its two scalar components are zero.)
From Newton's second law we can relate the forces exerted on the airplane with its acceleration.

\[
5) \quad [0, -mg] + F[\sin \alpha, \cos \alpha] + T[\cos \alpha, -\sin \alpha] = m \left[ \frac{v^2}{l \cos \alpha}, 0 \right]
\]

The rest is math. We obtained two equations (for each scalar component) with two unknowns (the magnitude of the lift \(F\) and the tension \(T\)).

\[
F \sin \alpha + T \cos \alpha = \frac{mv^2}{l \cos \alpha}
\]
\[
mg + F \cos \alpha - T \sin \alpha = 0
\]

Eliminating, by substitution, the magnitude of the lift force we find the tension in the wire

\[
T = m \left( \frac{v^2}{l} - g \sin \alpha \right) = 0.75 \text{kg} \cdot \left( \frac{35 \text{m}}{60 \text{s}} \right)^2 - 9.8 \frac{\text{m}}{\text{s}^2} \sin 20^\circ = 12.8 \text{N}
\]
Problem 13.21

A synchronous satellite, which always remains above the same point on a planet’s equator, is put into orbit around Jupiter so that scientists can study the famous red spot. Jupiter rotates once every 9.84 h. Use the data of Table 13.2 (Jupiter: mass \( M = 1.9 \times 10^{27} \) kg, mean radius \( R = 7 \times 10^7 \) m ) to find the altitude of the satellite.

\[
\text{A stationary satellite must move with the same angular velocity} \ \omega \ \text{at which the planet rotates. From information given in the problem it should not be difficult to find the magnitude of that angular velocity. Jupiter rotates at a constant rate, therefore we can express the magnitude of the angular velocity in terms of the given quantities}
\]

\[
1) \quad \omega = \frac{d\alpha}{dt} = \frac{2\pi}{T}
\]

where \( T \) is the time of one revolution of the planet.

Angular velocity and the acceleration of a particle in a circular motion are related. Remembering the (simple) equation relating velocity with angular velocity \( \mathbf{v} = \omega \times \mathbf{r} \), we can derive the expression relating acceleration to angular velocity

\[
2) \quad \mathbf{a} = \frac{d\mathbf{v}}{dt} = \omega \times \mathbf{v} = \omega \times (\omega \times \mathbf{r}) = -\omega^2 \mathbf{r}
\]

where \( \mathbf{r} \) is the position vector of the satellite with respect to the center of the orbit. In terms of the magnitude of the vectors the above equation reads

\[
3) \quad a = \omega^2 r = \omega^2 (R+h)
\]

where \( R \) is Jupiter’s mean radius and \( h \) is the altitude of the satellite above Jupiter’s surface.
There is only one force exerted on the satellite - the gravitational force \( \mathbf{F}_g \) exerted by the planet. The net force \( \mathbf{F}_{\text{net}} \) is therefore equal to the gravitational force. Using Newton’s law of gravity, we can relate the magnitude of the net force with the mass \((m)\) of the satellite and altitude \((h)\) above the planet

4) \[
\mathbf{F}_{\text{net}} = \mathbf{F}_g = G \frac{Mm}{(R + h)^2}
\]

where \( M \) is Jupiter’s mass.

The acceleration and the net force exerted on an object are related by Newton’s second law. In terms of magnitudes

5) \[
\mathbf{F}_{\text{net}} = ma
\]

The rest is math. Eliminating all unknowns not relevant to the question, we get an equation for the altitude

\[
G \frac{Mm}{(R + h)^2} = m \cdot \left( \frac{2\pi}{T} \right)^2 (R + h)
\]

Solving

\[
h = 3 \sqrt{GM \left( \frac{T}{2\pi} \right)^2} - R =
\]

\[
= 3 \sqrt{6.67 \cdot 10^{-11} \text{Nm}^2/\text{kg}^2 \cdot 1.9 \cdot 10^{27} \text{kg} \cdot \left( \frac{9.84 \cdot 3600 \text{s}}{1 \text{h}} \right)^2} - 7 \cdot 10^7 \text{m}
\]

\[
\approx 8.9 \cdot 10^7 \text{m} = 89,000 \text{km}
\]