TEST 3
(PHY 250)

1. a) Define electric current (the quantity describing transport of charge).
   b) Ohm’s law is written as \( J = \sigma E \). Explain this relation.
   c) What is inductance? (Include the defining equation.)
   d) How should be an ammeter connected with an electrical element in order to
determine the current in the element? How should be a voltmeter connected
with an electrical element in order to determine the voltage across the
element? (Make a schematic drawing.)

2. Find current \( I_1 \) (in the weaker battery) in the circuit shown in the Figure. In
order to save time, you may write the equations and solve them on your
calculator. Use the appropriate page. (Hint. For convenience, you may use the
numerical values in the equations and omit the units.)

3. An electric heater is rated at 1,500 W, a toaster at 750 W, and an electric grill
at 1,000 W. the three appliances are connected to a common 120V circuit.
a) How much current does each appliance draw? (Assume that in each
   appliance voltage and current are in phase.)
b) Can all appliances be simultaneously connected to a circuit with a 20A
   breaker?

4. A 300 \( \Omega \) resistor and a 11 \( \mu F \) capacitor are connected in series in to a 500/\( \pi \) Hz
generator producing current with peak value of 20 mA.
a) On the complex plane mark the complex impedance of both elements and the
equivalent impedance of the circuit. (Make one tick to correspond to 100\( \Omega \))
b) Draw to scale a phasor diagram showing the current and the voltage for each
element and for the circuit.
c) Consistently with the phasor diagram, draw the plots of the current in the circuit,
the voltage across the resistor and the voltage across the capacitor.
a) The electric current (I) across a surface is defined as the rate at which charge (q) is transferred through this surface

\[ I = \frac{dq}{dt} \]

b) When a steady flow is achieved in a conductor, current density \( J \) is proportional to the electric field vector \( E \). The proportionality constant \( \sigma \) (dependent on the material) is called the conductivity of the conductor.

c) The proportionality coefficient \( L \), relating the voltage across an inductor with the rate of change in the current through the inductor is called the inductance of the inductor

\[ V = -L \frac{dI}{dt} \]

d) The ammeter should be connected in series with the element. The voltmeter should be connected in parallel with the element.
We can write three independent equations for this circuit. I will use Kirchhoff’s loop rule for loops (1) and (2) and Kirchhoff’s junction rule for junction (a)

\[ I_1 + I_2 - I_3 = 0 \]
\[ 6I_1 - 4I_2 = 24 \]
\[ -6I_1 - 2I_3 = -10 \]

If you have not used your calculator to find the current directly from the above set of equations, you may consider Cramer’s method

\[
I_1 = \frac{\begin{vmatrix} 0 & 1 & -1 \\ 24 & -4 & 0 \\ -10 & 0 & -2 \end{vmatrix}}{\begin{vmatrix} 0 & 1 & -1 \\ 24 & -4 & 0 \\ 1 & 1 & -1 \end{vmatrix}} = \frac{0 + 0 + 0 - (-40) - 0 - (-48)}{8 + 0 + 0 - (-24) - 0 - (-12)} = \frac{88}{44} = 2A
\]
a) All appliances in the household are connected in parallel. So the same voltage of \( V_{\text{rms}} = 120V \) exists across each. Since the phase angle for each appliance is also given, it is possible to determine the currents in the appliances

\[
I_{\text{ms}} = \frac{P}{V_{\text{rms}} \cos \varphi} = \frac{P}{V_{\text{rms}}}
\]

For the heater  \( I_{\text{ms},h} = \frac{1500\text{W}}{120\text{V}} = 12.5\text{A} \)

For the toaster  \( I_{\text{ms},t} = \frac{750\text{W}}{120\text{V}} = 6.25\text{A} \)

For the grill  \( I_{\text{ms},g} = \frac{1000\text{W}}{120\text{V}} = 8.33\text{A} \)

b) Since all currents are in phase with the voltage they are in phase with each other. Therefore

\[
I_{\text{ms}} = I_{\text{ms},h} + I_{\text{ms},t} + I_{\text{ms},g} = 12.5\text{A} + 6.25\text{A} + 8.33\text{A} > 20\text{A}
\]

The current is greater than the circuit can deliver. The breaker will open the circuit.
a) The complex impedance of a resistor is equal to its resistance.

\[ Z_R = R = 300 \Omega \]

The complex impedance of the capacitor is an imaginary number dependent on the angular frequency of the current and the capacitance of the capacitor.

\[ Z_C = \frac{-i}{\omega C} = \frac{-i}{2\pi \cdot \frac{500}{\pi} \cdot 11 \cdot 10^{-6} \text{F}} = -90.9i \Omega \]

Since the elements are connected in series the equivalent complex impedance is equal to the sum of the complex impedance of each element.

\[ Z = Z_R + Z_L + Z_C = 300 \Omega + 90.9i \Omega = (300 + 90.9i) \Omega \]

b) The complex voltage across the resistor can be found by multiplying the complex current (through the resistor) by the complex impedance of the resistor.

\[ V_R(t) = I(t) \cdot Z_R \]
Consistent with the given values, the absolute value of this voltage is
\[ V_{R,m} = I_m \cdot R = 20mA \cdot 300\Omega = 6V \]

Since the phase of the complex impedance of the resistor is zero, the phase of the voltage across the resistor agrees with the phase of the current.

The complex voltage across the capacitor can be found by multiplying the complex current (through the capacitor) by the complex impedance of the capacitor
\[ V_C(t) = I(t) \cdot Z_C \]

Consistent with the given values, the absolute value of this voltage is
\[ V_{C,m} = I_m \cdot X_C = 20mA \cdot 91\Omega = 1.8V \]

Since the phase of the complex impedance of the capacitor is \(-90^\circ\), the voltage across the capacitor lags behind the current by \(90^\circ\).

To find the complex voltage across the entire system, we must add the complex voltages across both elements (they are connected in series). I indicated this operation on the phasor diagram. (That voltage can also be found by multiplying the current by the equivalent impedance of the system.) Notice that the phase difference between the voltage across the system and the current through the system is equal to the phase angle of the system (the phase of the equivalent impedance). The absolute value of the voltage is
\[ V_m = I_m \cdot Z = 20mA \cdot \sqrt{(300\Omega)^2 + (91\Omega)^2} \approx 6.3V \]