1. a) How will the ray indicated in the figure on the following page be reflected by the mirror? (Be accurate!)

b) Explain the symbols in the thin lens equation.

c) Recall the laws governing reflection and refraction of light at a boundary between two media.

d) When does total internal reflection occur? Include the expression relating the critical angle with the indices of refraction of the media.

2. Light of wavelength 589 nm traveling through air is incident on a smooth, flat slab of silica (n = 1.46) at an angle of $30^\circ$ to the normal, as sketched in the figure (included inside).

a) Find the angle of refraction.

b) Find the speed of light in silica.

c) What is the wavelength of this light in silica?

3. Two converging lenses having focal lengths of 10 cm and 20 cm are located 50 cm apart as in the attached figure. The final image is to be located between the lenses at the position indicated.

a) Draw the ray diagram for both lenses.

b) From the lens equation find how far to the left of the first lens should the object be?

4. The intensity of sunlight under a clear sky is (on the average) 1 kW/m$^2$.

a) If at noon the light falls at normal angle on a roof of dimensions 8m $\times$ 20m, calculate the total power incident on that roof at this time.

b) At what rate does the Sun (radius $r \approx 7 \cdot 10^8$ m, distance to Earth $R \approx 1.5 \cdot 10^{11}$ m) radiate electromagnetic energy?

c) Calculate the total energy incident on the roof during a full sunny day.
a)  

![Diagram of a mirror and its image](image)

b)  

**The thin lens equation**

\[
\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}
\]

relates the position \( s \) of the object and the position \( s' \) of the image produced by a thin spherical lens with focal length \( f \).

c)  

At the boundary of two media, light undergoes both reflection and refraction. The reflected ray lies in the plane of incidence and the angle of reflection is equal to the angle of incidence:

\[ \theta_1 = \theta_1' \]

The refracted ray lies in the plane of incidence and the angle of refraction is related to the angle of incidence by Snell's law:

\[ n_1 \sin \theta_1 = n_2 \sin \theta_2 \]

d)  

Total internal reflection occurs when light travels from a medium possessing a high index of refraction \( (n_1) \) to one possessing a lower index of refraction \( (n_2) \). The critical angle (the smallest angle of incidence \( \theta_c \) at which total internal refraction occurs) is related to the indices of refraction of both media

\[ \theta_c = \arcsin \frac{n_2}{n_1} \]
a) The angle of refraction can be found directly from Snell’s law
\[ n_1 \sin \theta_1 = n_2 \sin \theta_2 \]
from which
\[ \theta_2 = \arcsin \left( \frac{n_1}{n_2} \sin \theta_1 \right) = \arcsin \left( \frac{1}{1.46} \sin 30^\circ \right) = 20^\circ \]

b) The speed of light in silica can be found directly from the definition of index of refraction. If the index of refraction of the medium is \( n \) than the speed of light in this medium is
\[ v = \frac{c}{n} = \frac{3 \cdot 10^8 \text{ m/s}}{1.46} = 2.1 \cdot 10^8 \text{ m/s} \]

c) In one oscillation period \( T \) the light travels exactly one wavelength. You can see that it is true by analyzing the included figure. Consider one oscillation at point A and the distance traveled by the wave in this time.

Therefore
\[ \lambda_1 = vT = v \cdot \frac{\lambda_0}{c} = \frac{\lambda_0}{n} = \frac{589 \text{ nm}}{1.46} = 403 \text{ nm} \]
(Despite the change in wavelength the light does not change its color!)
a) The algebraic solution can be found from the lens equation applied to both lenses. Consistent with the figure

1) \[ \frac{1}{p} + \frac{1}{q} = \frac{1}{f_1} \]
2) \[ \frac{1}{s} + \frac{1}{s'} = \frac{1}{f_2} \]

Additionally
3) \[ q + s = d \]
4) \[ -s' + l = d \]

We obtained four equations with four unknown. The rest is algebra. We have to solve the above set for \( p \). From the information given (eq. 4) we can determine the position of the final image (with respect to the second lens)

\[ s' = -(d - l) = -(50\text{cm} - 31\text{cm}) = -19\text{cm} \]

We can use this information in equation 2 and find the position (with respect to the second lens) of the first image
\[
s = \left( \frac{1}{f_2} - \frac{1}{s'} \right)^{-1} = \left( \frac{1}{20\text{cm}} - \frac{1}{-19\text{cm}} \right)^{-1} \approx 9.74\text{cm}
\]

The position of the first image with respect to the first lens is therefore (from eq. 3)
\[
q = d - s \approx 50\text{cm} - 9.74 \approx 40.26\text{cm}
\]

Now from equation 1
\[
p = \left( \frac{1}{f_2} - \frac{1}{q} \right)^{-1} = \left( \frac{1}{20\text{cm}} - \frac{1}{40.26\text{cm}} \right)^{-1} \approx 133\text{cm}
\]

(It makes sense to perform calculation step-by-step. (We can check each step for possible discrepancies with the ray diagram.)
a) Intensity is defined in such a way that it is equal to the power of radiation passing a surface of a unit area, perpendicular to the propagation direction. In other words the power falling on a differential surface depends on the size and orientation of the surface according to

2) \[ dP = I \cos \theta \cdot dA \]

Since at the considered instant the surface of the roof is perpendicular to the light rays the integration should be simple

3) \[ P_{\text{max}} = \int_{\text{roof}} dP = I \int_{\text{roof}} dA = IA = 1 \frac{\text{kW}}{\text{m}^2} \cdot (8 \text{m} \cdot 20 \text{m}) = 160 \text{kW} \]

b) The rate \( P_{\text{tot}} \) at which the sun radiates electromagnetic energy is equal to the energy falling, in a unit time, on any closed surface around the sun. The intensity, given in the problem, tells how much energy passes a surface, perpendicular to the direction of the light beams, with an area of one square meter at a distance equal to the distance between the sun and the earth. From the symmetry, we can assume that the intensity of the sun’s light has the same value along any spherical surface concentric with the sun. Therefore

1) \[ P_{\text{tot}} = \int_{\text{sphere}} I \cos 0^\circ dA = 4\pi R^2 I = 4\pi \cdot (1.5 \cdot 10^{11} \text{ m})^2 \cdot 1 \frac{\text{kW}}{\text{m}^2} = 2.8 \cdot 10^{23} \text{ kW} \]

c) This time the angle of incidence is variable (time dependent). Since the Earth rotates at a constant rate, that angle is a linear function of time

4) \[ \theta(t) = \theta_0 + \omega t \]

where \( \omega \) is the magnitude of Earth’s angular velocity. For the time reference in the morning (at six o’clock), the initial angle is \(-90^\circ\). At time \( t \) (between the sunrise and sunset) the total power incident on the roof is

5) \[ P(t) = \int_{\text{roof}} dP = P_{\text{max}} \cos \theta(t) \]
From the definition of power we can find the total energy falling on the roof in a day. We can limit the integration only to the time when the sun is above the roof. It is more convenient to use the angle as the integration variable. From equation (4) we can relate the time and angle differentials

\[ 4') \quad d\theta = \omega \, dt \]

Hence

\[ 6) \quad \Delta E = \int P(t) \, dt = \int_{\text{day}} P_{\text{max}} \cos \theta(t) \, dt = \frac{P_{\text{max}}}{\omega} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos \theta \cdot d\theta = \frac{2P_{\text{max}}}{\omega} = \]

\[ = \frac{2 \cdot 160\text{kW}}{2\pi / 24\text{h}} = 4.4\text{GJ} \]